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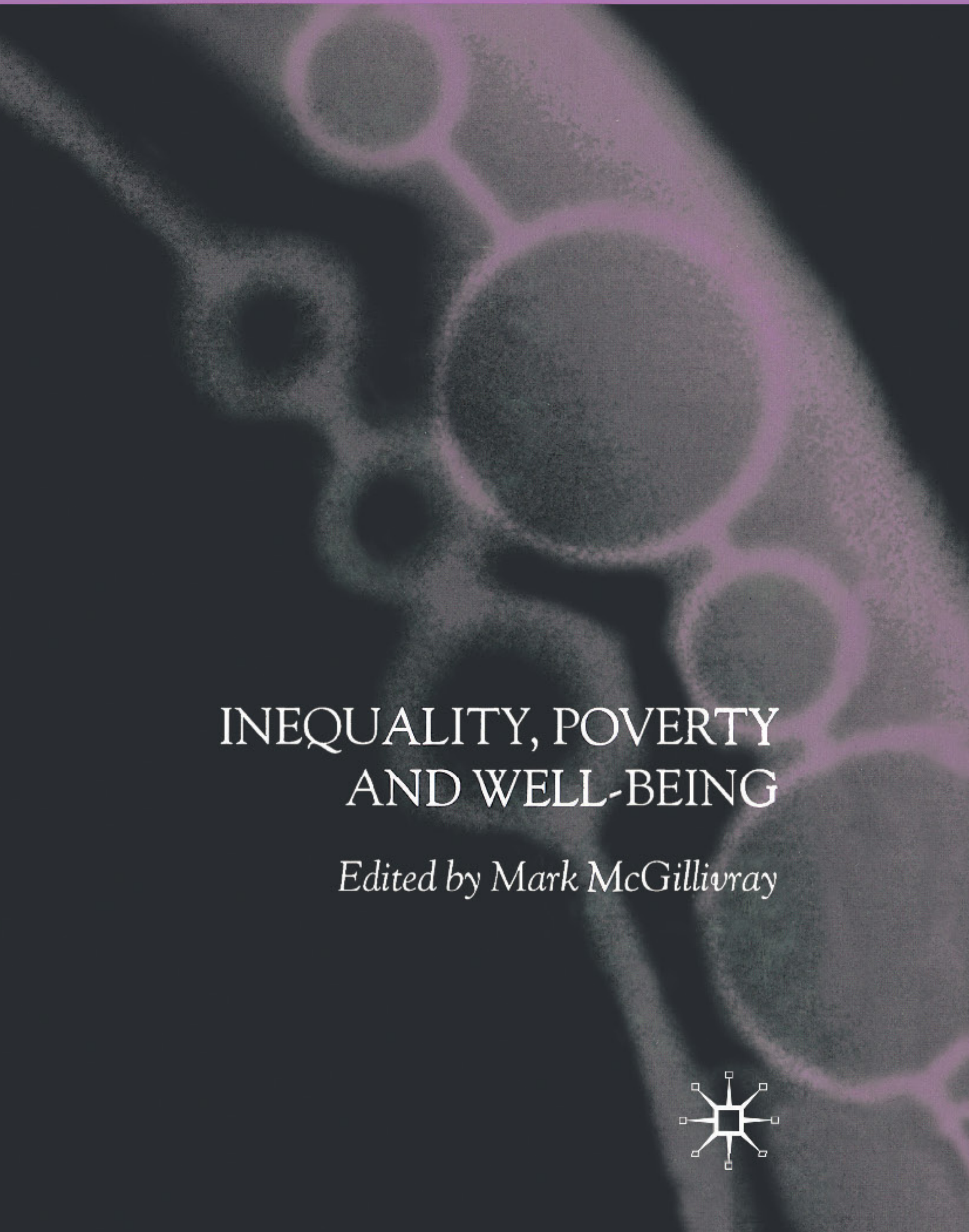

INEQUALITY, POVERTY AND WELL-BEING

Edited by Mark McGillivray

Studies in Development Economics and Policy

General Editor: **Anthony Shorrocks**

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List of Abbreviations and Acronyms

ANY	anonymity
APP	alternative population principle
BF	Basu and Foster
CARA	constant absolute risk aversion
CIREQ	Centre interuniversitaire de recherche en économie quantitative
CIS	Commonwealth of Independent States
CLGU	critical-level generalized-utilitarianism
CLU	critical-level utilitarianism
CON	congestion
CRS	constant returns to scale
DEA	data envelopment analysis
DRS	decreasing returns to scale
DMU	decision making unit
EFF	efficiency
EI	educational attainment indices
EKS	Elteto-Köves-Szulc
EWI	ecosystem well-being index
EXT	externality
FGT	Foster, Greer and Thorbecke
FX	foreign exchange
GDP	gross domestic product
GDI	gender-related development index
GDN	Gender Development Network
GEM	gender empowerment measure
GK	Geary–Khamis
GNP	gross national product
GREQAM	Groupement de Recherche en Economie Quantitative d'Aix-Marseille
HCRS	HDI constant returns to scale
HDI	human development index
HDR	*Human Development Report*
HPI	human poverty index
HPTE	HDI pure technical efficiency
HVRS	HDI variable returns to scale

HWI	human well-being index
ICP	International Comparison Program
IRS	increasing returns to scale
KHS	Kravis, Heston and Summers
KW	Kruskal–Wallis test
LI	life expectancy indices
MDGs	Millennium Development Goals
MOL	measure of literacy
MON	monotonicity
NOM	normalization
NRS	non-increasing returns to scale
NSSO	National Sample Survey Organisation (India)
OECD	Organisation for Economic Co-operation and Development
OLS	ordinary least squares
PPF	principle of population for social welfare functions and inequality indices
POP	principle of population
PPO	principle of population for quasi-orderings
PPP	purchasing power parity
PQLI	Physical Quality of Life Index
PT	principle of transfers
PTE	pure technical efficiency
PWT	Penn World Table
RS	returns to scale
SD	strong disposability
SF	symmetry for social welfare functions and inequality indices
SO	symmetry for quasi-orderings
STICERD	Suntory and Toyota International Centres for Economics and Related Disciplines
SUD	subgroup decomposability
TII	translation invariance for inequality indices
TIO	translation invariance for inequality quasi-orderings
UFO	unit of currency on the planet Utopia
UNDP	United Nations Development Programme
UNU	United Nations University
VRS	variable returns to scale
WI	income (wealth) indices
WIDER	World Institute for Development Economics Research
WD	weak disposability

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Inequality, Poverty and Well-being

Mark McGillivray

Introduction

With more than a billion people living on less than one dollar per day, some evidence of increasing gaps in living conditions within and between countries and clear evidence of substantial declines in life expectancy or other health outcomes in some parts of the world, the related topics of inequality, poverty and well-being are core international issues. The international community has a long history of concern for them and resolve to tackle poverty in the developing world, in income and other well-being dimensions, has evidently heightened in recent years given the commitment to the Millennium Development Goals (MDGs) at the United Nations Millennium Summit in September 2000. Among the many MDG targets are to halve the number of people living in extreme income poverty and to achieve universal primary schooling by 2015 (UN Millennium Project 2005). Concerns for well-being achievement in the developed world are also clearly evident, despite it having achieved ever higher levels of average incomes and improvements in traditional well-being measures. Issues such as social exclusion, human security, well-being sustainability, levels of personal satisfaction and happiness are widely discussed.

The research community has an intense interest in inequality, poverty and well-being. The literature is vast and many different disciplines have contributed to it, aided by the efforts of various national and international agencies in supplying statistical data. Many conceptual and methodological advances have been made and data have increased in availability, coverage and reliability. Social science research, as McGillivray and Shorrocks (2005) point out, has increasingly recognized the importance of non-income dimensions of well-being achievement and of population heterogeneity.

The recognition of non-income dimensions reflects a greater acceptance that well-being and poverty are multidimensional and, in particular, that no single uni-dimensional measure adequately captures the full gamut of well-being achievement. Accordingly, many multidimensional well-being conceptualizations have been developed, and include those of Sen (1982, 1985, 1993), Stewart (1985), Doyal and Gough (1991), Ramsay (1992), Max-Neef (1993), Schwartz (1994), Cummins (1996), Narayan *et al.* (2000) and Nussbaum (2000). The best known indicator of multidimensional well-being achievement at the level of nations is the UNDP's human development index (HDI). Used since 1990, the HDI is updated annually and available for more than 170 countries (UNDP 2005). While often criticized within the academic community, the HDI is used extensively in research and policy work (McGillivray and Shorrocks 2005). The UNDP also publishes country level data on its human poverty index (HPI), gender-related development index (GDI) and gender empowerment measure (GEM) (UNDP 2005).

The increased recognition of population heterogeneity is reflected in the attention given to inequalities in income and other well-being dimensions, both among and within countries (McGillivray and Shorrocks 2005). Most research has tended to focus on income inequality, taking advantage of the much improved income data that have become available in recent years. Changes over time in global income inequality have been hotly debated with some studies showing evidence of increasing inequality since the 1960s in national income per capita among countries (Jackman 1982, Barro and Sala-i-Martin 1992, Sheehey 1996, Jones 1997, Korzeniewicz and Moran 1997, Ram 1999). Other studies report evidence of stability (Berry *et al.* 1983, Peacock *et al.* 1988), while some find evidence of decreasing national income inequality among nations (Melchior *et al.* 2000, Sala-i-Martin 2002). Firebaugh (1999) demonstrates that results from studies of inequality among countries are dependent on whether each country's income is weighted by its share in world population. Studies that do not do this tend to report increasing world inequality.

More is now known about inequality, poverty and well-being than ever before, as a result of the studies cited above and the many more studies that comprise the research literature on these topics. Yet despite progress, the vitality of underlying concepts and quality of empirical measures are still challenged. Plenty of debates are yet to be settled, in addition to that on global income inequality, and there remain many interesting and exciting challenges for the international research community.

Volume contents

This book looks at advances in underlying inequality, poverty and well-being concepts and corresponding empirical measures. It has a strong analytical orientation, consisting of a mix of conceptual and empirical papers that constitute new and often highly innovative contributions to the literature. It consists of a further ten chapters, each of which attempts to push the literature into new and inspiring directions, addressing some of the challenges alluded to above.

Public policies often involve prioritizing or ranking alternatives in which the size and the composition of the population may vary. Examples are the allocation of resources to particular intended well-being outcomes and the design of aid packages to developing countries. In order normatively to assess the corresponding feasible choices, criteria for social evaluation that are capable of performing variable-population comparisons are required. Chapter 2, by Charles Blackorby, Walter Bossert and David Donaldson addresses this issue. Entitled 'Population Ethics and the Value of Life', the chapter reviews some of the properties of criteria for conducting variable-population comparisons.

Chapter 3 examines inequality measurement. It is written by Alain Chateauneuf and Patrick Moyes and is entitled 'A Non-welfarist Approach to Inequality Measurement'. An important principle in inequality measurement is the transfer principle. This principle requires that inequality decreases and welfare increases as a result of a progressive transfer. Recent experimental studies have shown that the principle of transfers is, to a large extent, rejected by the public, something that raises doubts about the ability of the Lorenz criterion to capture the very notion of inequality. Chapter 3 introduces different generalizations of the concept of a progressive transfer, where the solidarity among the individuals taking part in the transfer plays a crucial role. It shows how these transformations are related to the measures of deprivation and satisfaction that are proposed in the literature. Finally, it identifies classes of the Yaari (1987, 1988) social welfare functions that are consistent with these weaker notions of the principle of transfers.

Chapter 4, written by Christian Seidl, Stefan Traub and Andrea Morone, is entitled 'Relative Deprivation, Personal Income Satisfaction and Average Well-being under Different Income Distributions: An Experimental Investigation'. An individual's perception or categorization of their income will depend on the context in which it is viewed. For example, the level of satisfaction individuals derive from their incomes will depend in part on the incomes of other individuals. This and similar issues relating to the categorization of incomes is examined in Chapter 4. More precisely, the chapter looks at income categorization effects of uniform, normal, bimodal, positively skewed and negatively skewed income distributions. Among the chapter's findings is a paradox between individual income satisfaction and aggregate well-being, with the latter being higher for a negatively-skewed distribution than a positive one.

The focus on income inequality continues with Chapter 5. That chapter, written by Steve Dowrick and Muhammad Akmal, is entitled 'Contradictory Trends in Global Income Inequality: A Tale of Two Biases'. It asks whether global income inequality, defined in terms of population-weighted income inequality among nations, rose or fell during the last decades of the twentieth century. The chapter initially provides results consistent with those discussed earlier, suggesting that income inequality rose if the comparisons are based on exchange rates but fell if purchasing power comparisons based on information from the Penn World Tables are employed. The chapter then argues that both measures of real incomes lead to biased international comparisons, because observations based on exchange rates ignore the relative price of non-tradeables, while the fixed price PPP method underlying the Penn World Tables is subject to substitution bias. The contradictory inequality trends reflect growing dissimilarity between national price structures that increase the degree of bias in each method of comparison. Dowrick and Akmal use the multilateral true index methodology of Dowrick and Quiggin (1997) to yield 'true' PPP income comparisons that are free of both substitution bias and traded sector bias to examine changes in income inequality during the period 1980 to 1993. No evidence of a significant change in global income inequality is reported.

Mobility is closely related to inequality. Economic mobility refers to changes over time in the economic status of an individual or group. Are, for example, the income poor upwardly mobile or do they stay in the same economic position over time? The economic mobility literature investigated this and many similar questions, using a variety of indices. Mobility indices and their conceptual underpinnings are examined in Chapter 6. Written by Gary S. Fields, it is entitled 'The Many Facets of Economic Mobility'. It observes that the different mobility indices used by different authors are not measures of the same underlying conceptual entity; rather, different mobility indices measure fundamentally different concepts. Using data for the United States and France, Chapter 6 addresses such fundamental mobility questions as whether mobility has been rising or falling over time and which groups in the population have more mobility than others. The results show that the answers to even these most fundamental questions depend on the mobility concept used. The implication is that before social scientists 'do a mobility study', we need to be very clear about the mobility concept or concepts we wish to study. The choice can and does make a vital difference.

The book then focuses on the issue of poverty, commencing with Chapter 7. The chapter is entitled 'Social Groups and Economic Poverty: A Problem in Measurement' and written by S. Subramanian. This chapter points to some elementary conflicts between the claims of interpersonal and inter-group justice as they manifest themselves in the process of seeking a real-valued index of poverty which is required to satisfy certain seemingly desirable properties. It indicates how 'group-sensitive' poverty measures, similar to the Anand–Sen (1995) gender-adjusted human development index and the Subramanian–Majumdar (2002) group-disparity adjusted deprivation index, may be constructed. Some properties of a specific 'group-sensitive' poverty index are appraised, and the advantage of having a 'flexible' measure that is capable of effecting a trade-off between the claims of interpersonal and inter-group equality is spelt out. The implications of directly incorporating group disparities into the measurement of poverty for poverty comparisons and anti-poverty policy are also discussed.

A non-income dimension of well-being and poverty is examined in Chapter 8, which is written by Satya R. Chakravarty and Amita Majumder. The chapter is entitled 'Intersociety Literacy Comparisons'. Basu and Foster (1998) characterized a sophisticated literacy measure using five axioms. These axioms are externality, anonymity, monotonicity, normalization and decomposability. The essential idea underlying the first of these axioms is that a literate person confers positive externalities on all illiterate persons of the household to which they belong; appropriate situations where normalization and decomposability do not hold. Chapter 8 provides a class of literacy measures that may or may not satisfy decomposability but satisfies the remaining four axioms. This class can then be extended to a larger class of measures whose members will fulfil anonymity, monotonicity and externality but not necessarily the other two Basu–Foster axioms. The chapter also illustrates different measures numerically using the national sample survey household level data for the rural sectors of seven states in India for the year 1993–94, and derives some policy implications of intra-household positive externality from literacy.

Chapters 9 to 11 look at topics relating to multi-dimensional well-being measurement. Chapter 9 is entitled 'The Human Development Index Adjusted for Efficient Resource Utilization' and is written by F.J. Arcelus, Basu Sharma and G. Srinivasan. The UNDP obtains HDI values by taking the average of three equally weighted measures of longevity, education and income. However, this computational process is independent of the resources being devoted by each country to the achievement of the three outcome levels. Hence, it is conceivable that two different countries may consume vastly different amounts of resources in achieving the same outcome, say, longevity. This difference in the efficiency of resource utilization is not reflected in the HDI. Chapter 9 addresses this efficiency issue, providing HDI values adjusted for the efficiency of resource allocation processes.

Chapter 10 looks at the incorporation of environmental factors into well-being indices. It is entitled 'A Framework for Incorporating Environmental Indicators in the Measurement of Human Well-being' and is written by Osman Zaim. Relying on an economic–theoretical approach to index numbers, it also proposes an improvement index which alleviates the well-known deficiency of across time comparison of deprivation index. A feature of the index proposed is that it does not require any normative judgements in the selection of weights to aggregate over constituent indices. Rather, within an activity analysis framework optimally chosen weights are determined by the data. In developing the index which incorporates environmental indicators to the measurement of human well-being, due emphasis is put on production with negative externalities and a very recent analytical device – directional distance functions – is employed as a major tool in constructing quantity indexes and improvement indexes.

The final chapter, Chapter 11, is entitled 'Measuring Non-economic Well-being Achievement' and written by Mark McGillivray. The chapter observes that income per capita and most widely reported non-income based human well-being indicators are highly correlated among countries. Yet many countries exhibit higher achievement in the latter than predicted by the former. The reverse is true for many other countries. Chapter 11 commences by extracting the inter-country variation in a composite of various widely reported, non-income based well-being indices not accounted for by variations in income per capita. This extraction is interpreted as an aggregate measure of not only non-economic well-being achievement, but also of achievement in converting economic into non-economic well-being. The chapter then looks at correlations between this extraction and a number of new or less widely used well-being measures, in an attempt to find the measure that best captures these achievements. Various empirical procedures are performed, which *inter alia* allow for measurement error in the non-income-based measures. A number of indicators are examined, including measures of poverty, inequality, health status, education status, gender bias, empowerment, governance and subjective well-being.

The topics covered in this book address important issues relating to research on inequality, poverty and well-being. They are useful in their own right, but beyond that it is hoped that they will stimulate further research along similar lines.

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2

Population Ethics and the Value of Life*

Charles Blackorby, Walter Bossert and David Donaldson

Introduction

Public policies frequently involve choices of alternatives in which the size and the composition of the population may vary. Examples are the allocation of resources to prenatal care and the design of aid packages to developing countries. In order to assess the corresponding feasible choices on normative grounds, criteria for social evaluation that are capable of ranking alternatives with different populations and population sizes are required.

Such criteria, which we call population principles, are extensions of fixed population social evaluation principles. The purpose of this chapter is to discuss some of their properties. In particular, we examine the consequences and the mutual compatibility of several requirements regarding the addition of individuals to a given society.

The principles discussed in this chapter are welfarist: the ranking of any two alternatives depends on the well-being of those who ever live in them only.[1] Thus, knowledge of all those who ever live together with their levels of lifetime utility (interpreted as levels of lifetime well-being) is sufficient to establish a welfarist social ranking. Because of the significance of utility information, it is important to employ a comprehensive account of well-being such as that of Griffin (1986) or of Sumner (1996). The interpretation of individual utilities as indicators of lifetime (as opposed to per period) well-being is essential to avoid counter-intuitive recommendations regarding the termination of lives.

For an individual, a neutral life is one which is as good as one in which he or she has no experiences. The term 'experiences' encompasses everything that is perceived by the individual throughout his or her lifetime. Thus, a life without any experiences can be envisaged as a life that is spent in a permanent state of unconsciousness. Above neutrality, life, as a whole, is worth living; below neutrality, it is not. Following standard practice, we assign a utility level of zero to neutrality. It is possible to use other normalizations but, in that case, the definitions of the principles discussed here must be adjusted accordingly.

Same-number generalized utilitarianism ranks any two alternatives with the same population size by comparing their total or average transformed utilities. The transformation is increasing, continuous and preserves the zero normalization for a neutral life. If the transformation is strictly concave, the principle is strictly averse to utility inequality, giving priority to the interests of those whose utility levels are low. There are many ways of extending same-number generalized utilitarianism to a variable-number framework, and we call a population principle whose same-number subprinciples are generalized-utilitarian a same-number generalized-utilitarian principle.

Critical-level generalized utilitarianism (Blackorby and Donaldson 1984; Blackorby, Bossert and Donaldson 1995, 1997, 2005b) is a class of same-number generalized-utilitarian principles. Each of its members uses the sum of the differences between transformed individual utility levels and a transformed fixed critical level to make comparisons.[2] If the critical level is equal to zero, classical generalized utilitarianism results. For each value of the critical-level parameter, a different principle is obtained.

Parfit (1976, 1982, 1984) criticizes classical utilitarianism (the special case of classical generalized utilitarianism where the transformation is the identity mapping) on the grounds that it implies the repugnant conclusion. A principle leads to the repugnant conclusion if population size can always be substituted for quality of life, no matter how close to neutrality the well-being of a large population is. More precisely, a population principle implies the repugnant conclusion if and only if, for any population size, for any positive level of utility and for any level of utility strictly between zero and the specified level, there exists a larger population size such that an alternative in which everyone in the larger population has the lower level of utility is better than any alternative with the smaller population and the higher utility for everyone.[3] The higher utility level can be arbitrarily large and the lower utility level can be arbitrarily close to zero, the level that represents a neutral life. An implication of the repugnant conclusion is that there are situations where mass poverty is considered better than some alternatives in which fewer people lead very good lives. The generalized counterpart of classical utilitarianism suffers from the same problem.

The Pareto plus principle (see Sikora 1978) extends the strong Pareto principle to variable-population comparisons. It requires the addition of an individual with a lifetime utility above neutrality to a utility-unaffected population to be ranked as a social improvement. In conjunction with several standard conditions, this axiom is inconsistent with avoidance of the repugnant conclusion. In addition, Pareto plus appears to rest on the idea that individuals who do not exist – potential people – have interests, a view that is not easy to defend. Thus, we accept violations of Pareto plus in order to be able to avoid the repugnant conclusion.

In this chapter, we summarize some important aspects of welfarist population ethics that are discussed in detail in some earlier contributions. In addition, new results analyze some implications of Pareto plus and avoidance of the repugnant conclusion. Two impossibility theorems regarding the compatibility of Pareto plus and avoidance of the repugnant conclusion are presented. In response to those impossibilities, we discuss an alternative to Pareto plus, the negative expansion principle. It requires any alternative to be ranked as better than an expansion in which no one in the existing population is affected and an added individual is below neutrality.

In the next section, we introduce population principles and, as a special case, same-number generalized utilitarianism. The notions of a neutral life and critical levels are then discussed. In addition, we examine the restrictions that are imposed on critical levels by the Pareto plus principle, avoidance of the repugnant conclusion and the negative expansion principle. We go on to present and discuss critical-level generalized utilitarianism and close the chapter with our conclusions.

Population principles

A population principle ranks alternatives according to their social goodness. Each social alternative is a complete history of the world (or universe) and is associated with all information that may be relevant to the ranking. In particular, information about the well-being of all individuals who ever live (counting individuals who are not yet born) is included. The social ranking is assumed to be an ordering; that is, a reflexive, transitive and complete at-least-as-good-as relation. Two alternatives are equally good if and only if each is at least as good as the other. An alternative is better than another if and only if it is at least as good and the converse is not true.

A utility distribution consists of the lifetime utility levels of all the people who ever live in the corresponding alternative. Because we consider anonymous principles only, it is not necessary to keep track of individual identities. Consequently, the utility levels in an alternative can be numbered from one to the number of individuals alive. Thus, if there are n people alive in an alternative, a utility distribution is an n-tuple $u = (u_1, \dots, u_n)$ where each number in the list is the utility level of one of the members of society. The utility distribution $\mathbf{1}_n$ is a distribution where all n people alive have a utility of one.

We restrict attention to welfarist population principles.[4] A principle is welfarist if and only if there is a single ordering defined on utility distributions that can be used to rank all alternatives: one alternative is at least as good as another if and only if the utility distribution corresponding to the first is at least as good as the distribution corresponding to the second according to this ordering. In order to be a population principle, the ordering of utility distributions must be capable of different-number comparisons: any two distributions $u = (u_1, \ldots, u_n)$ and $v = (v_1, \ldots, v_m)$ are ranked, even if the population sizes n and m are different. Because we consider welfarist principles only, we formulate all axioms and principles in terms of the ordering of utility distributions.

In this section, we introduce properties of population principles that impose restrictions on same-number comparisons only. Our first requirement is anonymity: if we relabel the utility levels in a utility distribution u, the resulting distribution is as good as u. Such a relabelling is called a permutation of a utility distribution. A permutation of $u = (u_1, \ldots, u_n)$ is a utility distribution $v = (v_1, \ldots, v_n)$ such that there exists a way of matching each index i in u to exactly one index j in v such that $u_i = v_j$. For example, (u_2, u_1, u_3) is a permutation of (u_1, u_2, u_3).

Anonymity For all population sizes n, for all utility distributions $u = (u_1, \ldots, u_n)$ and $v = (v_1, \ldots, v_n)$, if v is a permutation of u, then u and v are equally good.

The strong Pareto principle is a fixed-population axiom. If everyone alive in two distributions u and v has a utility in u that is at least as high as that in v with at least one strict inequality, u is better than v.

Strong Pareto For all population sizes n and for all utility distributions $u = (u_1, \ldots, u_n)$ and $v = (v_1, \ldots, v_n)$, if $u_i \geq v_i$ for all i with at least one strict inequality, then u is better than v.

It is possible to criticize strong Pareto on the grounds that increases in some or all utility levels may increase utility inequality. A weaker principle that avoids this objection is minimal increasingness. It applies to utility distributions in which all utility levels are equal and declares increases in the common level to be social improvements.

Minimal increasingness For all population sizes n and for all utility levels b and d, if $b > d$, then $b\mathbf{1}_n$ is better than $d\mathbf{1}_n$.

Continuity is a condition that prevents the goodness relation from exhibiting 'large' changes in response to 'small' changes in the utility distribution. It rules out fixed-population principles such as lexicographic maximin (leximin).

Continuity For all population sizes n, for all utility distributions $u = (u_1, \ldots, u_n)$ and $v = (v_1, \ldots, v_n)$ and for all sequences of utility distributions $(u^j)_{j=1,2,\ldots}$ where $u^j = (u^j_1, \ldots, u^j_n)$ for all j,

(a) if the sequence $(u^j)_{j=1,2,\ldots}$ approaches v and u^j is at least as good as u for all j, then v is at least as good as u;
(b) if the sequence $(u^j)_{j=1,2,\ldots}$ approaches v and u is at least as good as u^j for all j, then u is at least as good as v.

A population principle is weakly inequality averse if and only if it ranks an equal distribution as at least as good as any distribution that has the same total utility.

Weak inequality aversion For all population sizes n and for all utility distributions $u = (u_1, \ldots, u_n)$, $\left((\sum_{i=1}^n u_i)/n\right) \mathbf{1}_n$ is at least as good as u.

We conclude this section with a definition of same-number generalized utilitarianism. The members of this class of principles use the sum of transformed utilities to perform all same-number comparisons. The transformation applied to individual utilities is the same for everyone and it is continuous and increasing. Without loss of generality, we assume that the transformation preserves neutrality; that is, its value at zero is equal to zero. The principle is minimally inequality-averse if and only if the transformation is concave and strictly inequality-averse if and only if the transformation is strictly concave. The latter case gives priority to the interests of those whose levels of well-being are low (see Parfit 1997, Blackorby, Bossert and Donaldson 2002, Broome 2005 and Fleurbaey 2005). According to same-number generalized utilitarianism with a transformation g, a utility distribution $u = (u_1, \ldots, u_n)$ is at least as good as a distribution $v = (v_1, \ldots, v_n)$ with the same population size if and only if

$$\sum_{i=1}^{n} g(u_i) \geq \sum_{i=1}^{n} g(v_i)$$

It is easy to verify that same-number generalized utilitarianism satisfies all of the same-number axioms introduced in this section.

Population expansions

A life is worth living if and only if it is better, from the viewpoint of the individual leading it, than a life without any experiences.[5] Similarly, a life is not worth living if and only if it is worse than a life without experiences. A neutral life is one which is neither worth living nor not worth living. Following the standard normalization employed in population ethics, we associate a utility level of zero with a neutral life. Thus, if a person has a positive (negative) level of lifetime well-being, his or her life is (is not) worth living.

Because people who do not exist do not have interests or preferences, it does not make sense to say that an individual gains by being brought into existence with a utility level above neutrality. It makes perfect sense, of course, to say that an individual gains or loses by continuing to live because of surviving a life-threatening illness, say. Such a change affects length of life, not existence itself.[6] We therefore take the view that, unless an individual is alive in two alternatives, comparisons of individual goodness are meaningless.[7] We follow the standard convention and identify the value of a neutral life with a lifetime-utility level of zero.

The axioms introduced in the previous section are same-number axioms because they impose restrictions on same-number comparisons only. One way of establishing links between utility distributions of different dimensions is to assume that, for any distribution of any population size, there exists a level of utility – the critical level – which, if experienced by an additional person, leads to a distribution that is equally good, provided that the utilities of the common population are unchanged. The following axiom postulates the existence of a critical level for every utility distribution.

Existence of critical levels For all population sizes n and for all utility distributions $u = (u_1, \ldots, u_n)$, there exists a critical level c such that u and $(u, c) = (u_1, \ldots, u_n, c)$ are equally good.

A critical level c for a utility distribution u is a level of well-being c such that, if an individual with the critical level is added to u, all other utilities unchanged, the augmented distribution and the original are equally good. As an immediate consequence of strong Pareto and transitivity, each utility distribution can have at most one critical level. In that case, it is possible to define a critical-level function C which provides a critical level for every utility distribution. Thus, any distribution u and the distribution $(u, C(u))$ are equally good. It follows that the overall ordering of utility distributions is completely determined by the same-number orderings and the critical-level function.

Sikora (1978) proposes to extend the strong Pareto principle to variable-population comparisons. He calls the resulting axiom Pareto plus, and it is usually defined as the conjunction of strong Pareto and the requirement that the addition of an individual above neutrality to a utility-unaffected population is a social improvement. Because we want to retain strong Pareto as a separate axiom, we state the second part of the condition only.

Pareto plus For all population sizes n, for all utility distributions $u = (u_1, \ldots, u_n)$ and for all positive utility levels a, $(u, a) = (u_1, \ldots, u_n, a)$ is better than u.

In the axiom statement, the common population in u and (u, a) is unaffected and, thus, in order to defend the axiom, it must be argued that a level of well-being above neutrality is better than non-existence. Thus, the axiom extends the Pareto condition to situations where a person is not alive in all alternatives that are compared. While it is possible to compare alternatives with different populations from a social point of view (which is the issue addressed in population ethics), it is questionable to make such a comparison from the viewpoint of an individual if the person is not alive in one of the alternatives. It is therefore difficult to interpret this axiom as a Pareto condition because it appears to be based on the idea that people who do not exist have interests that should be respected.

There is, therefore, an important asymmetry that applies to the assessment of alternatives with different populations. Although it is perfectly reasonable to say that an individual considers his or her life worth living if he or she is alive with a positive level of lifetime well-being, it does not make sense to say that a person who does not exist gains from being brought into existence with a life above neutrality: such a person cannot experience gains or losses.

The following result illustrates the requirements on critical levels imposed by Pareto plus, provided strong Pareto and existence of critical levels are satisfied. Not surprisingly, Pareto plus is equivalent to the requirement that all critical levels be non-positive.

Theorem 1 *Suppose that an anonymous population principle satisfies strong Pareto and existence of critical levels. The principle satisfies Pareto plus if and only if all critical levels are non-positive.*

Proof Suppose all critical levels are non-positive. By existence of critical levels and strong Pareto, critical levels are unique and the critical-level function C is well-defined. By definition of a critical level, u and $(u, C(u))$ are equally good for all utility distributions u. Let a be a positive utility level. Because all critical levels are non-positive, it follows that $a > 0 \geq C(u)$ and, thus, $a > C(u)$. By strong Pareto, (u, a) is better than $(u, C(u))$ and, because $(u, C(u))$ and u are equally good, transitivity implies that (u, a) is better than u. Thus, Pareto plus is satisfied.

Now suppose there exists a utility distribution u such that the critical level $C(u)$ for u is positive. By definition, $(u, C(u))$ and u are equally good. Let a be such that $0 < a < C(u)$. Strong Pareto implies that $(u, C(u))$ is better than (u, a). Using transitivity again, it follows that u is better than (u, a) and, thus, Pareto plus is violated because a is positive. □

Another property that imposes restrictions on variable-population comparisons is avoidance of the repugnant conclusion. A principle leads to the repugnant conclusion (Parfit 1976, 1982, 1984) if population size can always be substituted for quality of life, no matter how close to neutrality the well-being of a large population is. That is, there are situations where mass poverty is considered better than some alternatives in which fewer people lead very good lives. We share Parfit's view regarding the unacceptability of the repugnant conclusion and we therefore require a population principle to avoid it.

Avoidance of the repugnant conclusion There exists a population size n, a positive utility level ξ and a utility level ε strictly between zero and ξ such that, for all population sizes $m > n$, a utility distribution in which each of n

individuals has the utility level ξ is at least as good as a utility distribution in which each of m individuals has a utility of ε.

An important criticism of Pareto plus is that all anonymous, weakly inequality-averse population principles that satisfy it lead to the repugnant conclusion. Similar theorems can be found in Parfit (1976, 1982, 1984), Blackorby and Donaldson (1991), McMahan (1996), Blackorby, Bossert, Donaldson and Fleurbaey (1998), Carlson (1998) and Blackorby, Bossert and Donaldson (2005b).

Theorem 2 *There exists no anonymous population principle that satisfies minimal increasingness, weak inequality aversion, Pareto plus and avoidance of the repugnant conclusion.*

Proof Suppose that an anonymous population principle satisfies minimal increasingness, weak inequality aversion and Pareto plus. For any population size n, let ξ, ε and δ be utility levels such that $0 < \delta < \varepsilon < \xi$. Choose the integer r such that

$$r > n\frac{[\xi - \varepsilon]}{[\varepsilon - \delta]} \tag{2.1}$$

Because the numerator and denominator are both positive, r is positive. By Pareto plus, $(\xi\mathbf{1}_n, \delta\mathbf{1}_r)$ is better than $\xi\mathbf{1}_n$. Average utility in $(\xi\mathbf{1}_n, \delta\mathbf{1}_r)$ is $(n\xi + r\delta)/(n+r)$ so, by minimal inequality aversion, $[(n\xi + r\delta)/(n+r)]\mathbf{1}_{n+r}$ is at least as good as $(\xi\mathbf{1}_n, \delta\mathbf{1}_r)$. By (2.1),

$$\varepsilon > \frac{n\xi + r\delta}{n+r}$$

and, by minimal increasingness, $\varepsilon\mathbf{1}_{n+r}$ is better than $[(n\xi + r\delta)/(n+r)]\mathbf{1}_{n+r}$. Using transitivity, it follows that $\varepsilon\mathbf{1}_{n+r}$ is better than $\xi\mathbf{1}_n$ and avoidance of the repugnant conclusion is violated. □

If weak inequality aversion is dropped from the list of axioms in Theorem 2, the remaining axioms are compatible. For example, a principle proposed by Sider (1991) – which he calls geometrism – satisfies minimal increasingness, Pareto plus and avoidance of the repugnant conclusion. It uses a positive constant k between zero and one which ranks alternatives with a weighted sum of utilities: the jth-highest non-negative utility level receives a weight of k^{j-1} and the lth-lowest negative utility receives a weight of k^{l-1}. Critical levels are all zero and the repugnant conclusion is avoided but, because weights on higher positive utilities exceed weights on lower ones, the principle prefers inequality of positive utilities over equality (see Arrhenius and Bykvist 1995).

If a population principle is same-number generalized-utilitarian, the inequality-aversion requirement of Theorem 2 can be dropped.

Theorem 3 *There exists no same-number generalized-utilitarian population principle that satisfies Pareto plus and avoidance of the repugnant conclusion.*

Proof Suppose that a same-number generalized-utilitarian population principle satisfies Pareto plus. For any population size n, let ξ, ε and δ be utility levels such that $0 < \delta < \varepsilon < \xi$. Choose the integer r such that

$$r > n\frac{[g(\xi) - g(\varepsilon)]}{[g(\varepsilon) - g(\delta)]} \tag{2.2}$$

Because g is increasing, the numerator and denominator of (2.2) are both positive and, therefore, r is positive. (2.2) implies that

$$(n + r)g(\varepsilon) > ng(\xi) + rg(\delta)$$

so, by same-number generalized utilitarianism, $\varepsilon\mathbf{1}_{n+r}$ is better than $(\xi\mathbf{1}_n, \delta\mathbf{1}_r)$. By Pareto plus, $(\xi\mathbf{1}_n, \delta\mathbf{1}_r)$ is better than $\xi\mathbf{1}_n$ and, by transitivity, $\varepsilon\mathbf{1}_{n+r}$ is better than $\xi\mathbf{1}_n$. Consequently, avoidance of the repugnant conclusion is violated. □

We now show that anonymous population principles that satisfy strong Pareto, weak inequality aversion, existence of critical levels and avoidance of the repugnant conclusion must have at least one positive critical level.

Theorem 4 *If an anonymous population principle satisfies strong Pareto, weak inequality aversion, existence of critical levels and avoidance of the repugnant conclusion, then there exists a utility distribution u with a positive critical level.*

Proof Suppose that an anonymous population principle satisfies strong Pareto, weak inequality aversion, existence of critical levels and avoidance of the repugnant conclusion. Then all critical levels exist and are unique. Now, suppose that all critical levels are non-positive. Theorem 1 implies that Pareto plus is satisfied and, because strong Pareto implies minimal increasingness, Theorem 2 implies that avoidance of the repugnant conclusion is violated, a contradiction. Therefore, there must be at least one utility distribution u with a positive utility level. □

A variant of Theorem 4 shows that same-number generalized-utilitarian principles that satisfy existence of critical levels and avoidance of the repugnant conclusion must have some positive critical levels. Because the proof uses Theorems 1 and 3 and is similar to the proof of Theorem 4, it is omitted.

Theorem 5 *If a same-number generalized-utilitarian population principle satisfies existence of critical levels and avoidance of the repugnant conclusion, then there exists a utility distribution u with a positive critical level.*

The negative expansion principle is the dual version of Pareto plus. It requires any utility distribution to be ranked as better than one with the *ceteris paribus* addition of an individual whose life is not worth living.

Negative expansion principle For all population sizes n, for all utility distributions $u = (u_1, \dots, u_n)$ and for all negative utility levels a, u is better than $(u, a) = (u_1, \dots, u_n, a)$.

If a population principle satisfies strong Pareto and all critical levels exist, this axiom requires them to be non-negative. Because the theorem is parallel to Theorem 1, it is not proved.

Theorem 6 *Suppose that an anonymous population principle satisfies strong Pareto and existence of critical levels. The principle satisfies the negative expansion principle if and only if all critical levels are non-negative.*

There are many population principles that satisfy minimal increasingness, weak inequality aversion, the negative expansion principle and avoidance of the repugnant conclusion. Among these are all of the critical-level generalized-utilitarian principles with positive critical levels.

The negative expansion principle does rule out some principles that avoid the repugnant conclusion, however. If average utility is negative, average utilitarianism approves of the *ceteris paribus* addition of a person with a negative utility level above the average. If all critical levels exist, all same-number generalized-utilitarian principles with some negative critical levels are similarly ruled out. These include the number-dampened utilitarian principles (Ng 1986) other than classical utilitarianism and their generalized counterparts (see Blackorby, Bossert and Donaldson 2003).

Critical-level generalized utilitarianism

If strong Pareto is satisfied, a critical level represents a minimally acceptable level of utility such that the *ceteris paribus* addition of a single individual with a greater lifetime utility is a social improvement. Because no one in the existing population is affected, it is natural to choose a constant critical-level function.

This choice is implied by adding a weakening of existence of critical levels and an independence condition to the same-number axioms introduced earlier. Existence independence requires the ranking of any two utility distributions to be independent of the existence (and, thus, the utilities) of individuals who have the same utility levels in both. A principle that satisfies this condition is capable of performing comparisons by restricting attention to affected individuals – the utilities of the unconcerned are irrelevant to establish the ranking of utility distributions.

Existence independence For all population sizes n, m, r and for all utility distributions $u = (u_1, \ldots, u_n)$, $v = (v_1, \ldots, v_m)$ and $w = (w_1, \ldots, w_r)$, the utility distribution (u, w) is at least as good as the utility distribution (v, w) if and only if u is at least as good as v.

Existence of critical levels can be weakened to the following requirement. Unlike the stronger axiom, it requires the existence of only one critical level.

Weak existence of critical levels There exists a utility distribution $u = (u_1, \ldots, u_n)$ and a utility level c such that u and $(u, c) = (u_1, \ldots, u_n, c)$ are equally good.

According to critical-level generalized utilitarianism, utility distribution $u = (u_1, \ldots, u_n)$ is at least as good as distribution $v = (v_1, \ldots, v_m)$ if and only if

$$\sum_{i=1}^{n}[g(u_i) - g(\alpha)] \geq \sum_{i=1}^{m}[g(v_i) - g(\alpha)]$$

where α is a fixed critical level. Without loss of generality, we can again assume that the continuous and increasing transformation g preserves the utility level representing neutrality; that is, it satisfies $g(0) = 0$. Classical generalized utilitarianism is obtained for the special case where the critical-level parameter α is equal to zero, the utility level representing a neutral life.

A subclass of the critical-level generalized-utilitarian class is the critical-level utilitarian (CLU) class in which the transformation g is the identity mapping. According to CLU, utility distribution $u = (u_1, \ldots, u_n)$ is at least as good as distribution $v = (v_1, \ldots, v_m)$ if and only if

$$\sum_{i=1}^{n}[u_i - \alpha] \geq \sum_{i=1}^{m}[v_i - \alpha]$$

where α is a fixed critical level. Classical utilitarianism is obtained when $\alpha = 0$.

The critical-level generalized-utilitarian (CLGU) principles are the only ones that satisfy the axioms anonymity, strong Pareto, continuity, existence independence and weak existence of critical levels. If the negative expansion principle is added, the fixed critical level must be non-negative and, if avoidance of the repugnant conclusion is added instead, the critical level must be positive. This result, which is proved in Blackorby, Bossert and Donaldson (1998, 2005b), provides a strong case in favour of the CLGU principles with positive critical levels.[8] Because we consider the repugnant conclusion unacceptable, we add its avoidance to the list of axioms to obtain a characterization of the subclass of critical-level generalized-utilitarian principles with a positive critical level.

Theorem 7 *A welfarist population principle satisfies anonymity, strong Pareto, continuity, existence independence, weak existence of critical levels and avoidance* *of the repugnant conclusion if and only if it is critical-level generalized-utilitarian with a positive critical level α.*

If, in Theorem 7, avoidance of the repugnant conclusion is replaced with Pareto plus and the negative expansion principle, a characterization of classical generalized utilitarianism results.

Theorem 8 *A welfarist population principle satisfies anonymity, strong Pareto, continuity, existence independence, weak existence of critical levels, Pareto plus and the negative expansion principle if and only if it is classical generalized-utilitarian.*

Conclusion

Parfit (1976, 1982, 1984) argues that the repugnant conclusion should be avoided and we concur. Because all reasonable population principles that satisfy Pareto plus lead to the repugnant conclusion (Theorems 2 and 3), we reject Pareto plus.

An ethically attractive alternative to Pareto plus is the negative expansion principle. It prevents the *ceteris paribus* addition of a person whose life is not worth living from being ranked as a social improvement. It requires critical levels, if they exist, to be non-negative and, in addition, is compatible with avoidance of the repugnant conclusion. It also rules out some principles, such as average utilitarianism, that do not lead to the repugnant conclusion.

It is important that lifetime utilities rather than per period utilities are considered if principles with positive critical levels are employed. This means that, contrary to a widespread misconception, the termination of a life does not change population size: instead, it changes the affected person's lifetime and may change her or his lifetime utility. Thus, a positive critical level does *not* recommend that a life with a lifetime utility between zero and the critical level should be terminated. Suppose we use critical-level utilitarianism with a critical level of two. Consider first a situation where two individuals are alive, one with a lifetime utility of 4, the other with a lifetime utility of 1. The sum of utility gains over the critical level is $(4-2)+(1-2)=1$. Now suppose terminating the second person's life would reduce her or his lifetime utility to 0. In this case, the relevant sum is $(4-2)+(0-2)=0$ and, thus, this alternative is worse. Note that, once a person exists, the person has full moral standing and his or her utility must count in the criterion for social evaluation. Suppose now that the first person is the only one alive and we ask whether a new person with a lifetime utility of 1 should be brought into being. The one-person society has a sum of utility gains of $(4-2)=2$ and if the second person is brought into existence, the corresponding sum is $(4-2)+(1-2)=1$ and, thus, it is better that the second (non-existing) person not be born. The different treatment of existing and non-existing individuals in this example cannot be obtained if the critical level is equal to zero.

The critical-level generalized-utilitarian principles with positive critical levels are not the only ones that satisfy anonymity, strong Pareto, continuity, existence of critical levels, avoidance of the repugnant conclusion and the negative expansion principle. However, all of the others that do necessarily violate existence independence. Because space constraints prevent us from examining them here, we refer the interested reader to Blackorby, Bossert and Donaldson (2003).

Notes

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1. See, for instance, Blackorby, Bossert and Donaldson (2005a, 2005b) for a detailed account and defence of welfarism.
2. Fixed critical levels are proposed by Parfit (1976, 1982, 1984).
3. Parfit's statement of the repugnant conclusion is weaker.
4. See, for example, Blackorby, Bossert and Donaldson (2005a, 2005b) for a case in favour of welfarist social evaluation.
5. See Broome (1993).
6. For further discussions, see Parfit (1984, app. G), Heyd (1992, ch. 1), McMahan (1996) and Blackorby, Bossert and Donaldson (1997, 2005b).
7. See Parfit (1984, app. G), Heyd (1992, ch. 1), Broome (1993, 2004, ch. 8) and McMahan (1996).
8. See also Blackorby, Bossert and Donaldson (1995, 2005b) for an intertemporal formulation. An alternative characterization can be found in Blackorby and Donaldson (1984).

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3

A Non-welfarist Approach to Inequality Measurement*

Alain Chateauneuf and Patrick Moyes

Introduction

Motivation and relationship to the literature

Following Kolm (1969) and Atkinson (1970) there is a wide agreement in the literature to appeal to the Lorenz curve for measuring inequality. A distribution of income is typically considered as being no more unequal than another distribution if its Lorenz curve lies nowhere below that of the latter distribution. Besides its simple graphical representation, much of the popularity of the so-called Lorenz criterion originates in its relationship with the notion of progressive transfers. It is traditionally assumed that inequality is reduced by a progressive transfer; that is, when income is transferred from a richer to a poorer individual without affecting their relative positions on the ordinal income scale. The principle of tranfers, which captures this judgement, is closely associated with the Lorenz quasi-ordering of distributions of equal means. Indeed half a century ago, Hardy *et al.* (1952) demonstrated that if a distribution Lorenz dominates another distribution, then the former can be obtained from the latter by means of a finite sequence of progressive transfers, and conversely.[1] This relationship between progressive transfers and the Lorenz quasi-ordering constitutes the cornerstone of the modern theory of welfare and inequality measurement. As a consequence, the literature has concentrated on Lorenz consistent inequality measures; that is, indices such that a progressive transfer is always recorded as reducing inequality or increasing welfare.

Notwithstanding its wide application in theoretical and empirical work, the approach based on the Lorenz curve is not immune to criticism. Whereas most of the literature on inequality and welfare measurement imposes the principle of transfers, one may however raise doubts about the ability of such a condition to capture the very idea of inequality in general. Though a progressive transfer unambiguously reduces inequality between the individuals involved in the transfer, it is far from obvious that everyone would agree that inequality on the whole has declined as a result. This is due to the fact that, in general, making two incomes closer increases the gap between each of these two incomes and the incomes of the rest of society, so it is difficult to acknowledge that inequality on the whole has declined. It is to some extent surprising that the profession has been assimilating overall inequality reduction with local pairwise inequality reduction for such a long time. The fact that progressive transfers are not universally approved has been confirmed by recent experimental studies (see, for example, Amiel and Cowell 1992, Ballano and Ruiz-Castillo 1993, Harrison and Seidl 1994, Gaertner and Namezie 2003, among others).

However, the experimental studies fail to provide information about the subjects' preferences towards equality, with the exception that these preferences are at variance with the views captured by the principle of tranfers used by the theory of inequality measurement. Different ideas come to mind in order to reconcile the theory with the conclusions of these experimental studies. A first possibility would be to declare that inequality unambiguously decreases if and only if the income differentials between *any* two individuals in the population are reduced, assuming that all individuals occupy the same positions on the income scale in the situations under comparison. This is a kind of unanimity point of view: overall inequality decreases if and only if the inequalities between any two individuals in society decrease. This rules out the limitation of the principle of transfers we pointed out above since now, not only should the gap between the donor and the recipient of a transfer be reduced, but also the gaps between these two individuals and the individuals not taking part in the transfer. This still leaves open the question of knowing which kind of income differentials are thought of relevance when making inequality judgements. The relative and absolute differentials quasi-orderings introduced by Marshall *et al.* (1967) constitute two possible candidates. But there are other possible views – for example, along the lines suggested by Bossert and Pfingsten (1990) – that might constitute alternative grounds for constructing a theory of inequality measurement more in line with common sense.

On the other hand, there is evidence that the social status of an individual – approximated by her position in the social hierarchy – plays an important role in the determination of her well-being (see, for example, Weiss and Fershtman 1998). Attitudes such as envy, deprivation, resentment and satisfaction have been argued to be important components of individual judgements and they might be taken into account as far as distributive justice is concerned. In particular, the notion of individual deprivation originating in the work of Runciman (1966) accommodates such views, making the individual's assessment of a given social state depend on her situation compared with the situations of all the individuals who are treated more favourably than her. The deprivation profile, which indicates the level of deprivation felt by each individual, might therefore constitute the basis of social judgement. Drawing upon previous work by Yitzhaki (1979), Hey and

Lambert (1980), Kakwani (1984), Chakravarty *et al.* (1995), Chakravarty (1997), and Chakravarty and Moyes (2003), one can propose two deprivation quasi-orderings depending on the way individual deprivation is defined. Individual deprivation in a given state formally ressembles the aggregate poverty gap where the poverty line is set equal to other individuals' incomes.[2] So stated, one may conceive of absolute individual deprivation, which is simply the sum of the gaps between the individual's income and the incomes of all individuals richer than her, and relative deprivation, where the income gaps are deflated by the individual's income. Then the deprivation quasi-ordering is based on the comparisons of the individual deprivation curves and social deprivation unambiguously decreases as the individual deprivation curve is moving downwards.

Rather than comparing herself with individuals who are richer than her – equivalently who occupy a higher position on the social status scale – an individual can consider those who are poorer. The larger the aggregate gap between her income and the incomes of poorer individuals, the higher her satisfaction will be. More precisely, one may conceive of absolute individual satisfaction, which is simply the sum of the gaps between the individual's income and the incomes of all individuals poorer than her, and relative individual satisfaction, where the income gaps are deflated by the individual's income (see Chakravarty 1997). The notion of satisfaction may be considered the dual of the notion of deprivation. Then, the satisfaction quasi-ordering is based on the comparisons of the individual satisfaction curves and social satisfaction unambiguously decreases as the individual satisfaction curve moves downwards. A natural objective of society will be to make individual satisfaction and deprivation as small as possible, the minimum being attained when all incomes are equal.

The theoretical approach

Assuming that we subscribe to these more primitive notions of inequality, the next step is to identify the welfare and inequality indices that are consistent with the differentials, deprivation and/or satisfaction quasi-orderings. In this chapter, we restrict attention to ethical inequality indices, which means that we start with a given welfare ordering of income distributions – more precisely a given social welfare function – and derive an inequality index accordingly.[3] We assume, in addition, that this ordering can be represented by a member of the class of rank-dependent expected utility social welfare functions introduced by Quiggin (1993), which admits as particular cases the utilitarian and the generalized Gini social welfare functions known as the expected utility and the Yaari models respectively in the theory of choice under risk.[4] Then, we look for the restrictions that have to be imposed on the social welfare function – equivalent to social welfare ordering – that guarantee that the implied ethical inequality index is consistent with the primitive views captured by the differentials, deprivation and satisfaction quasi-orderings. It is argued that the standard expected utility model does not permit the different concepts of inequality discussed above to be distinguished. In other words, the utilitarian social welfare function is not sufficiently flexible to accommodate such distinct attitudes as those encompassed by the differentials, deprivation and satisfaction quasi-orderings.[5] On the contrary, the dual model of choice introduced by Yaari (1987, 1988) permits measures consistent with the differentials, deprivation and satisfaction quasi-orderings to be derived.[6] More precisely, we identify the restrictions to be imposed on the weighting function that guarantee that inequality will not increase when incomes are more equally distributed according to the three former quasi-orderings.

The next section introduces our conceptual framework, consisting of distributions for finite populations of possibly different sizes where every individual is associated with a given income. In addition to the Lorenz criterion, we distinguish different inequality views which we identify with quasi-orderings defined on the set of income distributions. The quasi-orderings we consider are all weaker than the Lorenz quasi-ordering as they all imply it. We then defines the inequality quasi-orderings used, explore their relationships and hint at some connections with progressive transfers. We go on to examine different ways of weakening the notion of equalizing transfer – equivalent to strengthening the principle of transfers – which are related to our inequality quasi-orderings. Our main results follow and we investigate the implications for the social welfare functions of the inequality views captured by the differentials, deprivation and satisfaction quasi-orderings in the particular case where distributions have equal means. It is shown that the utilitarian model, which frames most of the theory of welfare and inequality measurement, does not allow distinction between these views and the traditional one captured by the Lorenz quasi-ordering. On the contrary, the model proposed by Yaari (1987, 1988) allows the ethical planner to make a distinction between these competing views. We go on to indicate how the analysis could be extended in order to cover the general case where the distributions under comparison do not necessarily have the same mean. We close the chapter with a summary of our results and suggest some directions for future work.

Preliminary notation and definitions

We assume throughout that incomes are drawn from an interval D, which is a compact subset of $\mathbb{R}$. An income distribution or situation for a population consisting of n identical individuals ($n \geq 2$) is a list $\mathbf{x} := (x_1, x_2, \ldots, x_n)$ where $x_i \in D$ is the income of individual i. We indicate by $\mathbf{1_n} := (1, \ldots, 1)$ the unit vector in $\mathbb{R}^n$. We are typically interested in the comparison of distributions for populations of varying sizes. Letting $\mathcal{Y}_n(D)$ represent the set of income distributions for a population of size n, the set of all income distributions of finite size will be denoted as $\mathcal{Y}(D) := \bigcup_{n=2}^{\infty} \mathcal{Y}_n(D)$. The dimension of distribution $\mathbf{x} \in \mathcal{Y}(D)$ is indicated by $n(\mathbf{x})$ and the arithmetic mean by $\mu(\mathbf{x}) := \sum_{i=1}^{n(\mathbf{x})} x_i / n(\mathbf{x})$. Given $\mathbf{x} := (x_1, \ldots, x_{n(\mathbf{x})}) \in \mathcal{Y}(D)$, we use $\mathbf{x}_{(\,)} := (x_{(1)}, x_{(2)}, \ldots, x_{(n(x))})$ to indicate its non-decreasing rearrangement defined by $\mathbf{x}_{(\,)} = \Pi\mathbf{x}$ for some $n(\mathbf{x}) \times n(\mathbf{x})$ permutation matrix Π such that $x_{(1)} \leq x_{(2)} \leq \cdots \leq x_{(n(\mathbf{x}))}$. We denote as $F(\cdot\,;\mathbf{x})$ the cumulative distribution function of $\mathbf{x} \in \mathcal{Y}(D)$ defined by $F(z;\mathbf{x}) := q(z;\mathbf{x})/n(\mathbf{x})$, for all $z \in (-\infty, +\infty)$, where $q(z;\mathbf{x}) := \#\left\{i \in \{1, 2, \ldots, n(\mathbf{x})\} \,\middle|\, x_{(i)} \leq z\right\}$. We let $F^{-1}(\cdot\,;\mathbf{x})$ represent the inverse cumulative distribution function – equivalently the quantile function – of $\mathbf{x}$ obtained by letting $F^{-1}(0;\mathbf{x}) := x_{(1)}$ and

$$F^{-1}(p;\mathbf{x}) := \operatorname{Inf}\left\{z \in (-\infty, +\infty) \,\middle|\, F(z;\mathbf{x}) \geq p\right\}, \ \forall p \in (0,1) \tag{3.1}$$

(see Gastwirth 1971).

We are interested in the comparisons of alternative income distributions from the point of view of social welfare and inequality. A *social welfare function* $W : \mathcal{Y}(D) \longrightarrow \mathbb{R}$ associates to every distribution a real number $W(\mathbf{x})$ that represents the social welfare attained in situation $\mathbf{x} \in \mathcal{Y}(D)$. When $W(\mathbf{x}) \geq W(\mathbf{y})$, then we will say that situation $\mathbf{x}$ is at least as good as $\mathbf{y}$ from the point of view of W. Similarly an *inequality index* $I : \mathcal{Y}(D) \longrightarrow \mathbb{R}$ indicates for every distribution the degree of inequality attained with the convention that $I(\mathbf{x}) \leq I(\mathbf{y})$ means that situation $\mathbf{x}$ is no more unequal than situation $\mathbf{y}$. Here social welfare functions and inequality indices are considered as particular cardinal representations of orderings – complete, reflexive and transitive binary relations – on the set of income distributions, and no cardinal significance should be attributed to the values taken by the measures. We denote as $\mathbb{W}(D)$ and $\mathbb{I}(D)$ the set of social welfare functions and the set of inequality indices, respectively. Although we are interested in making comparisons of arbitrary distributions whose dimensions may differ, it is worth emphasizing that there is no loss of generality restricting attention to distributions with the same dimension. This is a consequence of the *principle of population*, according to which a replication does not affect inequality and welfare (see Dalton 1920). We will say that $\mathbf{x} \in \mathcal{Y}(D)$ is a *replication* of $\mathbf{y} \in \mathcal{Y}(D)$ if

$$\exists\, r \in \mathbb{N}\ (r \geq 2) \,:\, \mathbf{x} = \underbrace{(\mathbf{y}; \ldots; \mathbf{y})}_{r} =: \mathbf{y}^r \tag{3.2}$$

Principle of population for social welfare functions and inequality indices (PPF) The cardinal measure $M \in \mathbb{W}(D) \cup \mathbb{I}(D)$ satisfies the principle of population if, for all $\mathbf{x} \in \mathcal{Y}(D)$ and all $r \in \mathbb{N}\ (r \geq 2) : M\left(\mathbf{x}^r\right) = M(\mathbf{x})$.

Similarly because all individuals are identical in all respects other than their incomes, it is assumed that exchanging incomes between two individuals would not affect the levels of welfare and inequality.

Symmetry for social welfare functions and inequality indices (SF) The cardinal measure $M \in \mathbb{W}(D) \cup \mathbb{I}(D)$ satisfies the condition of symmetry if, for all $\mathbf{x} \in \mathcal{Y}(D)$ and all $n(\mathbf{x}) \times n(\mathbf{x})$ permutation matrices $\Pi : M(\Pi\mathbf{x}) = M(\mathbf{x})$.

Actually, as will be shown, all the social welfare functions and the inequality indices we will consider throughout will satisfy PPF and SF.

In a number of cases, it is impossible to reach a unanimous agreement regarding the appropriate ordering of situations and the only consensus that it is possible to achieve yields a partial ranking. A *quasi-ordering* is a reflexive and transitive binary relation defined on the set of distributions that may result in a partial ranking of the situations under consideration.[7] Given the quasi-ordering $\geq_J$ over $\mathcal{Y}(D)$, we denote as $>_J$ and $\sim_J$ its asymmetrical and symmetrical components defined in the usual way.

Principle of population for quasi-orderings (PPO) The binary relation $\geq_J$ over $\mathcal{Y}(D)$ satisfies the principle of population if, for all $\mathbf{x} \in \mathcal{Y}(D)$ and all $r \in \mathbb{N}$ $(r \geq 2) : \mathbf{x}^r \sim_J \mathbf{x}$.

Symmetry for quasi-orderings (SO) The binary relation $\geq_J$ over $\mathcal{Y}(D)$ satisfies the condition of symmetry if, for all $\mathbf{x} \in \mathcal{Y}(D)$ and all $n(\mathbf{x}) \times n(\mathbf{x})$ permutation matrices $\Pi : \Pi\mathbf{x} \sim_J \mathbf{x}$.

As we will see, all the quasi-orderings we consider in this chapter satisfy PPO and SO so there is no loss of generality in restricting attention to distributions of the same dimension.[8]

We are mostly concerned with welfare and inequality indices that are compatible with certain given inequality views that will be expressed by means of quasi-orderings. Precisely, given a quasi-ordering $\geq_J$ over $\mathcal{Y}(D)$ and a social welfare function $W \in \mathbb{W}(D)$ [resp. an inequality index $I \in \mathbb{I}(D)$], we will say that *W [resp. I] is consistent with $\geq_J$*, if

$$\forall\, \mathbf{x}, \mathbf{y} \in \mathcal{Y}(D) \; : \; \mathbf{x} \geq_J \mathbf{y} \Longrightarrow W(\mathbf{x}) \geq W(\mathbf{y}) \text{ [resp. } I(\mathbf{x}) \leq I(\mathbf{y})] \tag{3.3}$$

From Lorenz to more primitive inequality views

Introductory example and definitions

It is typically assumed in normative economics that inequality is reduced and welfare increased by a transfer of income from a richer individual to a poorer individual. More precisely, we have

Definition 1 Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathcal{Y}(D)$ with $n(\mathbf{x}) = n(\mathbf{y})$, we will say that **x** *is obtained from* **y** *by means of a progressive transfer*, if there exists $\Delta > 0$ and two individuals i, j such that

$$x_k = y_k, \; \forall\, k \neq i, j \tag{3.4a}$$

$$x_i = y_i + \Delta; \quad x_j = y_j - \Delta \tag{3.4b}$$

$$\Delta \leq (y_j - y_i)/2 \tag{3.4c}$$

By definition, a progressive transfer does not reverse the relative positions of the individuals involved. However, although the donor cannot be made poorer than the recipient, it may be the case that their positions relative to the positions of the other individuals are modified. It is convenient to assume that the progressive transfer is *rank-preserving* in the sense that the relative positions of *all* the individuals are unaffected, which amounts to imposing the additional condition

$$\left(x_k - x_h\right)\left(y_k - y_h\right) \geq 0, \quad \forall\, h \neq k \tag{3.5}$$

Principle of transfers (PT) For all $\mathbf{x}, \mathbf{y} \in \mathcal{Y}(D)$ with $n(\mathbf{x}) = n(\mathbf{y})$, we have $W(\mathbf{x}) \geq W(\mathbf{y})$ and $I(\mathbf{x}) \leq I(\mathbf{y})$, whenever $\mathbf{x}$ is obtained from $\mathbf{y}$ by a (rank-preserving) progressive transfer.

The notion of a progressive transfer is closely associated with that of the Lorenz quasi-ordering. The *Lorenz curve* of distribution $\mathbf{x} \in \mathcal{Y}(D)$ – denoted as $L(p; \mathbf{x})$ – is defined by

$$L(p; \mathbf{x}) := \int_0^p F^{-1}(s; \mathbf{x})\, ds, \quad \forall p \in (0, 1) \tag{3.6}$$

so that $L(p; \mathbf{x})$ represents the total income possessed by the fraction p of poorest individuals deflated by the population size in situation $\mathbf{x}$.

Definition 2 Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathcal{Y}(D)$, we will say that $\mathbf{x}$ *Lorenz dominates* $\mathbf{y}$, which we write $\mathbf{x} \geq_L \mathbf{y}$, if and only if

$$L(p; \mathbf{x}) \geq L(p; \mathbf{y}),\ \forall\, p \in (0, 1) \text{ and } L(1; \mathbf{x}) = L(1; \mathbf{y}) \tag{3.7}$$

The higher its associated Lorenz curve, the less unequal a distribution is according to the Lorenz criterion. In the particular case where $n(\mathbf{x}) = n(\mathbf{y}) = n$, condition (3.7) reduces to

$$\frac{1}{n}\sum_{j=1}^{k} x_{(j)} \geq \frac{1}{n}\sum_{j=1}^{k} y_{(j)}, \ \forall k = 1, 2, \ldots, n-1, \text{ and } \mu(\mathbf{x}) = \mu(\mathbf{y}) \tag{3.8}$$

As we have already asserted, much of the popularity of the Lorenz criterion originates in the fact that it is closely associated with progressive transfers. Hardy *et al.* (1952) were the first to show that a distribution Lorenz dominates another one if and only if it can be obtained from the latter by means of successive applications of progressive transfers (see also Berge 1963, Kolm 1969, Fields and Fei 1978, Marshall and Olkin 1979, among others). Precisely, they proved the following:

Proposition 1 *Let* $\mathbf{x}, \mathbf{y} \in \mathcal{Y}(D)$ *such that* $\mu(\mathbf{x}) = \mu(\mathbf{y})$ *and* $n(\mathbf{x}) = n(\mathbf{y})$. *Then, the following two statements are equivalent:*

(a) $\mathbf{x}$ *is obtained from* $\mathbf{y}$ *by means of a finite sequence of progressive transfers.*
(b) $\mathbf{x} \geq_L \mathbf{y}$.

In the result above, it is important to note that there is no particular restriction on the way the progressive transfers are combined: any sequence of progressive transfers results in an improvement in terms of Lorenz dominance.[9] A direct implication of Proposition 1 is that any measure that verifies the principle of transfers is Lorenz-consistent (see Foster 1985).

The problem is that it is not obvious that everyone would agree on the fact that a progressive transfer decreases inequality in all circumstances. This has been exemplified in a number of experimental studies by means of questionnaires, where it has been demonstrated that the principle of transfers is largely rejected by the respondents (see Amiel and Cowell 1992, Ballano and Ruiz-Castillo 1993, Harrison and Seidl 1994, Gaertner and Namezie 2003, among others). The following example, which captures the main features of the situations presented to the interviewees in these experiments, might help to convince the reader that the principle of transfers is debatable.

Example 1 Let $n = 4$ and consider the distributions $\mathbf{x}^1 = (1,3,5,7)$, $\mathbf{x}^2 = (1,3,6,6)$, $\mathbf{x}^3 = (1,4,4,7)$, $\mathbf{x}^4 = (2,2,5,7)$, and $\mathbf{x}^5 = (2,3,5,6)$. It is immediate that each of the distributions $\mathbf{x}^2$, $\mathbf{x}^3$, $\mathbf{x}^4$ and $\mathbf{x}^5$ obtains from $\mathbf{x}^1$ by means of a single (rank-preserving) progressive transfer of one income unit. It follows from Proposition 1 that $\mathbf{x}^g \geq_L \mathbf{x}^1$, for all $g = 2,3,4,5$, so that everyone who subscribes to the principle of transfers – equivalently to the Lorenz criterion – will consider that distributions $\mathbf{x}^2$, $\mathbf{x}^3$, $\mathbf{x}^4$ and $\mathbf{x}^5$ are less unequal than distribution $\mathbf{x}^1$.

Inspection of the distributions reveals that $\mathbf{x}^2$ is obtained from $\mathbf{x}^1$ by transferring one unit of income from the richest individual to the second richest, which actually amounts to equalizing the incomes of the two richest individuals. Inequality between individuals 3 and 4 has therefore been eliminated but at the same time the income gap – or income differentials – between individuals 1 and 3 on the one hand, and individuals 2 and 3 on the other hand, has been widened. This is actually true, whatever the way we measure these pairwise income differentials.[10] Although the Lorenz criterion would say that $\mathbf{x}^2$ is unambiguously more equal than $\mathbf{x}^1$, there might be – and there actually are – people who disagree with this conclusion, invoking the fact that pairwise income differentials are not all made smaller as a result of the progressive transfer. Here, we propose to take the absolute income difference as a measure of the income differential between two individuals. Precisely, given the income distribution $\mathbf{x} \in \mathcal{Y}(D)$, we define

$$AD(p,s;\mathbf{x}) := F^{-1}(s;\mathbf{x}) - F^{-1}(p;\mathbf{x}), \quad \forall\, 0 \leq p < s \leq 1 \tag{3.9}$$

Thus, $AD(p,s;\mathbf{x})$ measures the absolute income gap between the richer individual occupying rank s and the poorer individual ranked p in situation $\mathbf{x}$. It is our contention that nobody would object to the judgement that inequality does not increase when the absolute income gaps between any richer and any poorer individuals are made smaller. Actually, it is a simple matter to verify that the following definition captures precisely this idea.

Definition 3 Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathcal{Y}(D)$, we will say that $\mathbf{x}$ *dominates* $\mathbf{y}$ *in absolute differentials*, which we write $\mathbf{x} \geq_{AD} \mathbf{y}$, if and only if

$$AD(p, s; \mathbf{x}) \leq AD(p, s; \mathbf{x}), \quad \forall\, 0 \leq p < s \leq 1 \tag{3.10}$$

According to condition (3.10), the differences between any two adjacent individuals' incomes are no larger in situation $\mathbf{x}$ than in situation $\mathbf{y}$. In the particular case where $n(\mathbf{x}) = n(\mathbf{y}) = n$, condition (3.10) reduces to

$$x_{k+1} - x_k \leq y_{k+1} - y_k, \quad \forall\, k = 1, 2, \ldots, n-1 \tag{3.11}$$

This quasi-ordering, first introduced by Marshall *et al.* (1967) in the fields of majorization (see also Bickel and Lehmann 1976), has been considered a suitable inequality criterion (see, for example, Thon 1987, Preston 1990, and Moyes 1994, 1999).

When comparing two distributions by means of the absolute differentials quasi-ordering, every individual compares her situation with that of all the individuals richer than her. It might be that what is important is not really by how much every poorer individual falls below every richer individual, but rather by how much on average she is away from the richer individuals. This is reminiscent of the notion of deprivation introduced by Runciman (1966), according to whom the individual's assessment of a given social state depends on her situation compared with the situations of individuals who are treated more favourably than her. Given distribution $\mathbf{x} \in \mathcal{Y}(D)$, we let

$$\begin{aligned} ADP(p; \mathbf{x}) &:= \int_p^1 \left[F^{-1}(s; \mathbf{x}) - F^{-1}(p; \mathbf{x})\right] ds \\ &\equiv \int_p^1 AD(p, s; \mathbf{x})\, ds, \quad \forall p \in (0, 1) \end{aligned} \tag{3.12}$$

We can interpret $ADP(p; \mathbf{x})$ as a measure of the absolute deprivation felt by individual with rank p in situation $\mathbf{x}$. By definition, the best-off individual is never deprived and $ADP(1; \mathbf{x}) = 0$, for all $\mathbf{x} \in \mathcal{Y}(D)$. Following Chakravarty *et al.* (1995), and Chakravarty (1997), we introduce

Definition 4 Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathcal{Y}(D)$ such that $n(\mathbf{x}) = n(\mathbf{y}) = n$, we will say that *there is no more absolute deprivation in* $\mathbf{x}$ *than in* $\mathbf{y}$, which we write $\mathbf{x} \geq_{ADP} \mathbf{y}$, if and only if

$$ADP(p; \mathbf{x}) \leq ADP(p; \mathbf{y}), \quad \forall p \in (0, 1) \tag{3.13}$$

Actually condition (3.13) simply states that overall deprivation decreases if the individual deprivation felt by any member of society decreases. In the particular case where $n(\mathbf{x}) = n(\mathbf{y}) = n$, condition (3.13) reduces to

$$\frac{1}{n}\sum_{j=k+1}^{n}\left[x_j - x_k\right] \leq \frac{1}{n}\sum_{j=k+1}^{n}\left[y_j - y_k\right], \quad \forall\, k = 1,2,\ldots,n-1 \tag{3.14}$$

Rather than comparing herself with the individuals richer than her, an individual might find some comfort in comparing her situation with the situations of the individuals who are in a worse position than her. Given distribution $\mathbf{x} \in \mathcal{Y}(D)$, we let

$$\begin{aligned} ASF(p;\mathbf{x}) &:= \int_0^p \left[F^{-1}(p;\mathbf{x}) - F^{-1}(s;\mathbf{x})\right] ds \\ &\equiv \int_0^p AD(p,s;\mathbf{x})\, ds, \quad \forall p \in (0,1) \end{aligned} \tag{3.15}$$

We can interpret $ASF(k;\mathbf{x})$ as a measure of the absolute satisfaction felt by individual ranked p in situation $\mathbf{x}$. By definition, the worst-off individual is never satisfied and $ASF(0;\mathbf{x}) = 0$, for all $\mathbf{x} \in \mathcal{Y}(D)$. Following Chakravarty (1997), we introduce

Definition 5 Given two income distributions $\mathbf{x},\mathbf{y} \in \mathcal{Y}(D)$ such that $n(\mathbf{x}) = n(\mathbf{y}) = n$, we will say that *there is no more absolute satisfaction in* $\mathbf{x}$ *than in* $\mathbf{y}$, which we write $\mathbf{x} \geq_{ASF} \mathbf{y}$, if and only if

$$ASF(p;\mathbf{x}) \leq ASF(p;\mathbf{y}), \quad \forall p \in (0,1) \tag{3.16}$$

Actually condition (3.16) simply states that overall satisfaction decreases if the individual satisfaction felt by any member of the society decreases. In the particular case where $n(\mathbf{x}) = n(\mathbf{y}) = n$, condition (3.16) reduces to

$$\frac{1}{n}\sum_{i=1}^{k-1}\left[x_k - x_i\right] \leq \frac{1}{n}\sum_{i=1}^{k-1}\left[y_k - y_i\right], \quad \forall\, k = 2,3,\ldots,n \tag{3.17}$$

Applying the preceding quasi-orderings to the comparisons of the distributions introduced in Example 1 gives the rankings shown in Table 3.1.

The symbol '1' at the intersection of row i and column j means that '$\mathbf{x}^i >_J \mathbf{x}^j$', while a '0' means that '$\mathbf{x}^j >_J \mathbf{x}^i$'. The occurence of the symbol '#' indicates that the distributions $\mathbf{x}^i$ and $\mathbf{x}^j$ are not comparable.

Table 3.1 Inequality rankings of distributions of Example 1

$\geq_{AD}$

	x^1	x^2	x^3	x^4
x^2	#			
x^3	#	#		
x^4	#	#	#	
x^5	1	#	#	#

$\geq_{ADP}$

	x^1	x^2	x^3	x^4
x^2	1			
x^3	#	#		
x^4	#	#	#	
x^5	1	#	#	1

$\geq_{ASF}$

	x^1	x^2	x^3	x^4
x^2	#			
x^3	#	#		
x^4	1	#	#	
x^5	1	1	#	#

$\geq_{L}$

	x^1	x^2	x^3	x^4
x^2	1			
x^3	1	#		
x^4	1	#	#	
x^5	1	1	1	1

According to Table 3.1, depending on the way we measure it, the change in inequality caused by a progressive transfer may be ambiguous. In particular, anyone who subscribes to the views captured by the differentials, the deprivation or the satisfaction quasi-ordering may feel unable to accept the common view that inequality decreases as a result of a progressive transfer. This is in accordance with the findings of the experimental studies, according to which the public rejects, to a large extent, the principle of transfers.

Properties and relationships between the inequality quasi-orderings

Due to the properties of the inverse distribution function, one can easily check that all the quasi-orderings we have defined above are invariant with respect to a permutation and/or a replication of distributions. Hence:

Remark 1 *Let $J \in \{AD, ADP, ASF, L\}$. Then, the inequality quasi-ordering $\geq_J$ verifies PPO and SO.*

Thus, there is no loss of generality to restrict attention to the comparison of distributions in the set $\mathcal{Y}_n(D)$ with $n > 2$ and such that incomes are nondecreasingly arranged. The next remark points out two properties of the absolute deprivation curve and the absolute satisfaction curve that will be useful later on.

Remark 2 *Let $n > 2$ and $\mathbf{x} \in \mathcal{Y}_n(D)$. Then we have*

$$\text{(a) } ADP\left(\frac{k}{n};\mathbf{x}\right) = ADP\left(\frac{k+1}{n};\mathbf{x}\right) + \frac{n-k}{n}\left[x_{k+1} - x_k\right], \textit{ for all } k = 1,2,\dots,n-1;$$

$$\text{(b) } ASF\left(\frac{k}{n};\mathbf{x}\right) = ASF\left(\frac{k-1}{n};\mathbf{x}\right) + \frac{k-1}{n}\left[x_k - x_{k-1}\right], \textit{ for all } k = 2,3,\dots,n.$$

Proof By definition of the absolute deprivation curve and upon manipulation, we have

$$\begin{aligned}
ADP\left(\frac{k}{n};\mathbf{x}\right) &= \frac{x_{k+1} - x_k}{n} + \frac{x_{k+2} - x_k}{n} + \dots + \frac{x_n - x_k}{n} \\
&= \frac{x_{k+1} - x_k}{n} + \frac{x_{k+2} - x_{k+1} + x_{k+1} - x_k}{n} + \dots + \frac{x_n - x_{k+1} + x_{k+1} - x_k}{n} \\
&= \frac{(n-k)\left(x_{k+1} - x_k\right)}{n} + \frac{x_{k+2} - x_{k+1}}{n} + \dots + \frac{x_n - x_{k+1}}{n} \\
&= \frac{n-k}{n}\left(x_{k+1} - x_k\right) + ADP\left(\frac{k+1}{n};\mathbf{x}\right) \qquad (3.18)
\end{aligned}$$

for all $k = 1,2,\dots,n-1$. Using a similar reasoning, we obtain

$$\begin{aligned}
ASF\left(\frac{k}{n};\mathbf{x}\right) &= \frac{x_k - x_1}{n} + \dots + \frac{x_k - x_{k-2}}{n} + \frac{x_k - x_{k-1}}{n} \\
&= \frac{x_k - x_{k-1} + x_{k-1} - x_1}{n} + \dots + \frac{x_k - x_{k-1} + x_{k-1} - x_{k-2}}{n} + \frac{x_k - x_{k-1}}{n} \\
&= \frac{(k-1)\left(x_k - x_{k-1}\right)}{n} + \frac{x_{k-1} - x_1}{n} + \dots + \frac{x_{k-1} - x_{k-2}}{n} \\
&= \frac{k-1}{n}\left(x_k - x_{k-1}\right) + ASF\left(\frac{k-1}{n};\mathbf{x}\right) \qquad (3.19)
\end{aligned}$$

for all $k = 2,3,\dots,n$, in the case of absolute satisfaction. □

Since by definition income distributions are non-decreasingly arranged, a direct implication of Remark 2 is that the absolute deprivation curve and the absolute satisfaction curve are respectively non-increasing and non-decreasing.

Table 3.1 suggests that our inequality quasi-orderings might be nested as the rankings obtained are more or less fine depending on the chosen quasi-ordering. Leaving aside the case $n = 2$, where the preceding quasi-orderings provide the same ranking of distributions, we have:

Remark 3 *Let $n > 2$ and suppose all the distributions under comparison have equal means. Then, we have:* (a) $\geq_{AD} \subset \geq_{ADP}$; (b) $\geq_{AD} \subset \geq_{ASF}$; (c) $\geq_{ADP} \subset \geq_L$; (d) $\geq_{ASF} \subset \geq_L$; *and* (e) $\geq_{ADP} \neq \geq_{ASF}$.

Proof Suppose that $\mathbf{x} \geq_{AD} \mathbf{y}$, so that

$$x_{k+1} - x_k \leq y_{k+1} - y_k, \quad \forall\, k = 1, 2, \ldots, n-1 \tag{3.20}$$

Summing the inequalities above over h for $h = 1, 2, \ldots, k-1$ and $k = 1, 2, \ldots, n-1$, we obtain

$$x_k - x_h \leq y_k - y_h, \quad \forall\, h = 1, 2, \ldots, k-1, \quad \forall\, k = 2, 3, \ldots, n \tag{3.21}$$

(a) $\geq_{AD} \subseteq \geq_{ADP}$ Suppose that $\mathbf{x} \geq_{AD} \mathbf{y}$, so that (3.21) holds. Summing the inequalities in (3.21) over j for $j = h+1, h+2, \ldots, n-1$ and $h = 1, 2, \ldots, n-1$, and upon simplifying, we obtain

$$\sum_{j=h+1}^{n} \left[x_j - x_h\right] \leq \sum_{j=h+1}^{n} \left[y_j - y_h\right], \quad \forall\, h = 1, 2 \ldots, n-1 \tag{3.22}$$

so that $\mathbf{x} \geq_{ADP} \mathbf{y}$.

(b) $\geq_{AD} \subseteq \geq_{ASF}$ Suppose that $\mathbf{x} \geq_{AD} \mathbf{y}$, so that (3.21) holds. Summing the inequalities in (3.21) over i for $i = 1, 2, \ldots, k-1$ and $k = 2, 3, \ldots, n$, and upon simplifying, we obtain

$$\sum_{i=1}^{k-1} [x_k - x_i] \leq \sum_{i=1}^{k-1} [y_k - y_i], \quad \forall\, k = 2, 3, \ldots, n \tag{3.23}$$

so that $\mathbf{x} \geq_{ASF} \mathbf{y}$.

(c) $\geq_{ADP} \subseteq \geq_L$ Developing (3.22), we obtain

$$\begin{aligned} &(n-h+1)\,x_h - \left[x_h + x_{h+1} + \cdots + x_n\right] \\ &\quad \geq (n-h+1)\,y_h - \left[y_h + y_{h+1} + \cdots + y_n\right] \end{aligned} \tag{3.24}$$

for all $h = 1, 2, \ldots, n-1$. The proof then proceeds in $(n-1)$ successive steps.

STEP 1: $h = 1$. Then (3.24) reduces to

$$n x_1 - [x_1 + x_2 + \cdots + x_n] \geq n y_1 - [y_1 + y_2 + \cdots + y_n] \tag{3.25}$$

Since by assumption $x_1 + x_2 + \cdots + x_n = y_1 + y_2 + \cdots + y_n$, we deduce from (3.25) that $x_1 \geq y_1$.

STEP 2: $h = 2$. Then (3.24) reduces to

$$(n-1)\,x_2 - [x_2 + x_3 + \cdots + x_n] \geq (n-1)\,y_2 - [y_2 + y_3 + \cdots + y_n] \tag{3.26}$$

Adding $x_1 - x_1$ and $y_1 - y_1$ to the *lhs* and the *rhs* respectively of (3.26) and since by assumption $x_1 + x_2 + \cdots + x_n = y_1 + y_2 + \cdots + y_n$, we obtain

$$x_1 + (n-1)\,x_2 \geq y_1 + (n-1)\,y_2 \tag{3.27}$$

Adding $(n-2)x_1 \geq (n-2)y_1$, which follows from Step 1, to inequality (3.27), we reach

$$(n-1)\,[x_1 + x_2] \geq (n-1)\,[y_1 + y_2] \tag{3.28}$$

hence $x_1 + x_2 \geq y_1 + y_2$.

⋮

STEP h: $h = h$. Then (3.24) reduces to

$$\begin{aligned}&(n-h+1)\,x_h - \left[x_h + x_{h+1} + \cdots + x_n\right]\\&\quad\geq (n-h+1)\,y_h - \left[y_h + y_{h+1} + \cdots + y_n\right]\end{aligned} \tag{3.29}$$

Adding $\left[x_1 + \cdots + x_{h-1}\right] - \left[x_1 + \cdots + x_{h-1}\right]$ and $\left[y_1 + \cdots + y_{h-1}\right] - \left[y_1 + \cdots + y_{h-1}\right]$ to the *lhs* and the *rhs* respectively of (3.29) and since by assumption $x_1 + x_2 + \cdots + x_n = y_1 + y_2 + \cdots + y_n$, we obtain

$$x_1 + \cdots + x_{h-1} + (n-h+1)\,x_h \geq y_1 + \cdots + y_{h-1} + (n-h+1)\,y_h \tag{3.30}$$

Adding $(n-h)\left[x_1 + \cdots + x_{h-1}\right] \geq (n-h)\left[y_1 + \cdots + y_{h-1}\right]$, which follows from Step $h-1$, to inequality (3.30), we reach

$$(n-h)\left[x_1 + x_2 + \cdots + x_{n-h+1}\right] \geq (n-h)\left[y_1 + y_2 + \cdots + y_{n-h+1}\right] \tag{3.31}$$

hence $x_1 + x_2 + \cdots + x_{n-h+1} \geq y_1 + y_2 + \cdots + y_{n-h+1}$.

⋮

STEP $n-1$: $h = n-1$. Then (3.24) reduces to

$$2x_{n-1} - \left[x_{n-1} + x_n\right] \geq 2y_{n-1} - \left[y_{n-1} + y_n\right] \tag{3.32}$$

Adding $\left[x_1 + \cdots + x_{n-2}\right] - \left[x_1 + \cdots + x_{n-2}\right]$ and $\left[y_1 + \cdots + y_{n-2}\right] - \left[y_1 + \cdots + y_{n-2}\right]$ to the *lhs* and the *rhs* respectively of (3.32) and since by assumption $x_1 + x_2 + \cdots + x_n = y_1 + y_2 + \cdots + y_n$, we obtain

$$x_1 + \cdots + x_{n-2} + 2x_{n-1} \geq y_1 + \cdots + y_{n-2} + 2y_{n-1} \tag{3.33}$$

Adding $[x_1 + \cdots + x_{n-2}] \geq [y_1 + \cdots + y_{n-2}]$, which follows from Step $n-2$, to inequality (3.33), we reach

$$2[x_1 + x_2 + \cdots + x_{n-1}] \geq 2[y_1 + y_2 + \cdots + y_{n-1}] \tag{3.34}$$

hence $x_1 + x_2 + \cdots + x_{n-1} \geq y_1 + y_2 + \cdots + y_{n-1}$.

We have shown that $x_1 + x_2 + \cdots + x_h \geq y_1 + y_2 + \cdots + y_h$, for all $h = 1, 2, \ldots, n-1$. Since by assumption $x_1 + x_2 + \cdots + x_n = y_1 + y_2 + \cdots + y_n$, we conclude that $\mathbf{x} \geq_L \mathbf{y}$.

(d) $\geq_{ASF} \subseteq \geq_L$ Developing (3.23), we obtain

$$k x_k - [x_1 + x_2 + \cdots + x_k] \leq k y_k - [y_1 + y_2 + \cdots + y_k] \tag{3.35}$$

for all $k = 2, 3, \ldots, n$. The proof then proceeds in $(n-1)$ successive steps.

STEP 1: $k = n$. Then (3.35) reduces to

$$n x_n - [x_1 + x_2 + \cdots + x_n] \leq n y_n - [y_1 + y_2 + \cdots + y_n] \tag{3.36}$$

Since by assumption $x_1 + x_2 + \cdots + x_n = y_1 + y_2 + \cdots + y_n$, we deduce from (3.36) that $x_n \leq y_n$.

STEP 2: $k = n-1$. Then (3.35) reduces to

$$\begin{aligned} &(n-1) x_{n-1} - [x_1 + x_2 + \cdots + x_{n-1}] \\ &\quad \leq (n-1) y_{n-1} - [y_1 + y_2 + \cdots + y_{n-1}] \end{aligned} \tag{3.37}$$

Adding $x_n - x_n$ and $y_n - y_n$ to the *lhs* and the *rhs* respectively of (3.37) and since by assumption $x_1 + x_2 + \cdots + x_n = y_1 + y_2 + \cdots + y_n$, we obtain

$$x_n + (n-1) x_{n-1} \leq y_n + (n-1) y_{n-1} \tag{3.38}$$

Adding $(n-2) x_n \leq (n-2) y_n$, which follows from Step 1, to inequality (3.38) we reach

$$(n-1)[x_{n-1} + x_n] \leq (n-1)[y_{n-1} + y_n] \tag{3.39}$$

hence $x_{n-1} + x_n \leq y_{n-1} + y_n$.

$$\vdots$$

STEP $n-k+1$: $k = k$. Then (3.35) reduces to

$$k x_k - [x_1 + x_2 + \cdots + x_k] \leq k y_k - [y_1 + y_2 + \cdots + y_k] \tag{3.40}$$

Adding $[x_{k+1} + \cdots + x_n] - [x_{k+1} + \cdots + x_n]$ and $[y_{k+1} + \cdots + y_n] - [y_{k+1} + \cdots + y_n]$ to the *lhs* and the *rhs* respectively of (3.40) and since by assumption $x_1 + x_2 + \cdots + x_n = y_1 + y_2 + \cdots + y_n$, we obtain

$$k x_k + [x_{k+1} + \cdots + x_n] \leq k y_k + [y_{k+1} + \cdots + y_n] \tag{3.41}$$

Adding $(k-1)\left[x_{k+1}+\cdots+x_n\right] \leq (k-1)\left[y_{k+1}+\cdots+y_n\right]$, which follows from Step $n-k$, to inequality (3.41) and since by assumption $x_1+x_2+\cdots+x_n = y_1+y_2+\cdots+y_n$, we reach

$$k\left[x_k+x_{k+1}+\cdots+x_n\right] \leq k\left[y_k+y_{k+1}+\cdots+y_n\right] \tag{3.42}$$

hence $x_k+x_{k+1}+\cdots+x_n \leq y_k+y_{k+1}+\cdots+y_n$.

⋮

STEP $n-1$: $k=2$. Then (3.35) reduces to

$$2x_2-[x_1+x_2] \leq 2y_2-[y_1+y_2] \tag{3.43}$$

Adding $[x_3+\cdots+x_n]-[x_3+\cdots+x_n]$ and $[y_3+\cdots+y_n]-[y_3+\cdots+y_n]$ to the *lhs* and the *rhs* respectively of (3.43) and since by assumption $x_1+x_2+\cdots+x_n = y_1+y_2+\cdots+y_n$, we obtain

$$2x_2+[x_3+\cdots+x_n] \leq 2y_2+[y_3+\cdots+y_n] \tag{3.44}$$

Adding $[x_3+\cdots+x_n] \leq [y_3+\cdots+y_n]$, which follows from Step $n-2$, to inequality (3.44) and since by assumption $x_1+x_2+\cdots+x_n = y_1+y_2+\cdots+y_n$, we obtain finally

$$2[x_2+x_3+\cdots+x_n] \leq 2[y_2+y_3+\cdots+y_n] \tag{3.45}$$

hence $x_2+x_3+\cdots+x_n \leq y_2+y_3+\cdots+y_n$.

We have shown that $x_k+x_{k+1}+\cdots+x_n \leq y_k+y_{k+1}+\cdots+y_n$, for all $k=2,3,\ldots,n$. Since by assumption $x_1+x_2+\cdots+x_n = y_1+y_2+\cdots+y_n$, we conclude that $\mathbf{x} \geq_L \mathbf{y}$.

(e) $\geq_{ADP} \neq \geq_{ASF}$ Consider Table 3.2, where we have made use of the distributions defined in Example 1. By convention the symbol '1' at the intersection of line '$\{\mathbf{x}^i,\mathbf{x}^j\}$' and row '$\geq_J$' indicates that '$\mathbf{x}^i >_J \mathbf{x}^j$', while a

Table 3.2 Comparisons of selected pairs of distributions

Pairs of distributions	$\geq_{AD}$	$\geq_{ADP}$	$\geq_{ASF}$	$\geq_L$
$\{\mathbf{x}^5,\mathbf{x}^1\}$		1	1	
$\{\mathbf{x}^2,\mathbf{x}^1\}$	#	1	#	
$\{\mathbf{x}^5,\mathbf{x}^2\}$	#	#	1	
$\{\mathbf{x}^3,\mathbf{x}^1\}$		#	#	1
$\{\mathbf{x}^4,\mathbf{x}^1\}$			#	1

'#' means that $\mathbf{x}^i$ and $\mathbf{x}^j$ are not comparable, where $J \in \{AD, ADP, ASF, L\}$. We conclude that $\geq_{ADP}$ and $\geq_{ASF}$ are logically independent.

Finally to make the proof complete, it remains to prove that the inclusions in statements (a) to (d) are strict, which is easily checked by inspection of Table 3.2. □

The preceding discussion demonstrates that the three quasi-orderings we have considered so far are at variance with the Lorenz criterion, and thus capture dimensions of inequality that are not embedded in the latter criterion. It is expected that some of the views represented by these new criteria are more in line with the public's perception of inequality. The extent to which these criteria are closer to the public's view is a matter of empirical investigation which lies outside the scope of this chapter. However, identifying the kind of elementary transformations – analogous to progressive transfers in the case of the Lorenz criterion – that imply inequality reduction according to each of these quasi-orderings will shed light on the significance of these new criteria.[11] A second issue is to determine the structure of the individuals' preferences towards more equality which support the views expressed by the differentials, satisfaction and deprivation quasi-orderings and which, at the same time, are not consistent with those captured by the Lorenz quasi-ordering. Both issues are of particular importance for understanding the normative content of these three quasi-orderings.

Inequality, solidarity and equalizing transformations

A preliminary result

Although it is typically assumed that inequality is reduced and welfare increased by a progressive transfer, Example 1 points at good reasons for challenging this common view. On the one hand, depending on the way we measure inequality, the effect of a transfer of income from a richer to a poorer individual may be ambiguous. In particular, anyone who subscribes to the views captured by the differentials quasi-ordering may feel unable to accept the common view that inequality unambiguously decreases as a result of a progressive transfer. On the other hand, one might even consider that inequality has increased as the result of an elementary progressive transfer. However, it must be stressed that $\mathbf{x} \geq_L \mathbf{y}$ and $\mathbf{y} >_{AD} \mathbf{x}$ cannot hold simultaneously. This follows from Remark 1 according to which, for all $\mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D)$, one has that $\mathbf{x} >_{AD} \mathbf{y}$ implies $\mathbf{x} >_L \mathbf{y}$. But this does not preclude the possibility that $I(\mathbf{x}) > I(\mathbf{y})$ for some inequality index $I \in \mathbb{I}$ consistent with the absolute differentials quasi-ordering. This is because for $\mathbf{x} >_{AD} \mathbf{y}$ it is necessary that all inequality indices in a given class – to be determined – declare distribution $\mathbf{x}$ as being less unequal than distribution $\mathbf{y}$ (see p. 54). Since these three quasi-orderings are all subrelations of the Lorenz quasi-ordering as indicated by Remark 3, it is clear that if a distribution is ranked above another one according to either of the former quasi-orderings, then a sequence of progressive transfers will be needed in order to construct the dominating distribution starting from the dominated one. However, the precise way these progressive transfers have to be combined for such domination to hold has to be determined.

It might be helpful to begin with a benchmark result that constitutes a first step towards a more general solution. By construction, distributions $\mathbf{x}^2$, $\mathbf{x}^3$, $\mathbf{x}^4$ and $\mathbf{x}^5$ in Example 1 obtain from $\mathbf{x}^1$ by means of a *single* rank-preserving progressive transfer of one income unit. Close inspection reveals that the only case where the resulting distribution dominates the original distribution according to the differentials quasi-ordering is when the progressive transfer involves the richest and the poorest individual. The three cases where the progressive transfers generate an improvement according to the deprivation quasi-ordering is when income is taken from the richest individual and given to someone poorer. Finally, a transfer from any richer individual to the poorest results in a reduction of inequality as measured by the satisfaction quasi-ordering. The positions of the recipients and of the donors on the income scale seem to play a crucial role for the impact on inequality of a progressive transfer. The result below – which we state without proof – indicates the restrictions one has to introduce for inequality to decrease as a result of a single progressive transfer in a very particular case.

Remark 4 *Let* $n > 2$ *and* $\mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D)$ *such that* $y_1 < y_2 \leq \cdots \leq y_{n-1} < y_n$. *Suppose we are only permitted to use a single rank-preserving progressive transfer in order to obtain* $\mathbf{x}$ *from* $\mathbf{y}$. *Then, we have*

(a) $\mathbf{x} \geq_{AD} \mathbf{y}$ *if and only if* $1 = i < j = n$;
(b) $\mathbf{x} \geq_{ADP} \mathbf{y}$ *if and only if* $i < j = n$;
(c) $\mathbf{x} \geq_{ASF} \mathbf{y}$ *if and only if* $1 = i < j$;
(d) $\mathbf{x} \geq_{L} \mathbf{y}$ *if and only if* $i < j$.

This result uncovers the rationale behind the construction of Example 1 and it also hints at potential explanations for why the public might reject the principle of transfers in some given situations. But, above all, Remark 4 confirms that there is little room for reducing inequality as measured by either of the differentials, deprivation and satisfaction quasi-orderings if one is only permitted to make use of elementary progressive transfers.

General results

Given Remark 4, we know that for inequality as measured by any of the differentials, deprivation and satisfaction quasi-orderings to decrease, a single progressive transfer will generally not be sufficient and progressive transfers will have to be combined in one way or another. Therefore, our first task is to identify possible transformations of distributions that combine progressive transfers in such a way that domination in terms of the differentials, deprivation and satisfaction quasi-orderings obtains. A related task is to derive the appropriate sequence of such transformations that permits to obtain the dominating distribution from the dominated one. We impose two requirements upon ourselves: (i) the transformations must admit as a particular case the progressive transfers exhibited in Remark 4; and (ii) the transformations must be elementary in the sense that they are as simple as possible. The latter requirement has the effect that in general a single transformation would not suffice to convert the dominated distribution into the dominanting one: successive applications of such transformations will be needed.[12]

We first consider those transformations from which successive applications result in a distributional improvement according to the differentials quasi-ordering. We propose:

Definition 6 Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathcal{Y}(D)$ with $n(\mathbf{x}) = n(\mathbf{y}) = n$, we will say that $\mathbf{x}$ *is obtained from* $\mathbf{y}$ *by means of a* T_1*-transformation,* if there exists $\delta, \epsilon > 0$ and two individuals h, k $(1 \leq h < k \leq n)$ such that condition (3.5) holds and

$$x_g = y_g, \quad \forall\, g \in \{h+1, \ldots, k-1\} \tag{3.46a}$$

$$x_i = y_i + \delta,\ \forall\, i \in \{1, \ldots, h\}; \quad x_j = y_j - \epsilon,\ \forall\, j \in \{k, \ldots, n\} \tag{3.46b}$$

$$h\,\delta = (n - k + 1)\,\epsilon \tag{3.46c}$$

We note that the transformations identified in statement (a) of Remark 4 are particular instances of a T_1-transformation. Such elementary transformations impose considerable solidarity in society. There is solidarity among the rich: if some income is taken from a rich individual, then the same amount has to be taken from every individual who is as rich or richer. Symmetrically, there is solidarity among the poor: if some income is given to a poor individual, then the same amount has to be given to every individual who is as poor or poorer. This solidarity among the donors and the beneficiaries is typically broken down in the progressive transfer. The following result identifies the relationship between the absolute differentials quasi-ordering and T_1-transformations.

Proposition 2 *Let* $n > 2$ *and* $\mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D)$ *such that* $\mu(\mathbf{x}) = \mu(\mathbf{y})$. *Then, the following two statements are equivalent:*

(a) $\mathbf{x}$ *is obtained from* $\mathbf{y}$ *by means of a finite sequence of* T_1*-transformations.*
(b) $\mathbf{x} \geq_{AD} \mathbf{y}$.

Proof

(a) $\Longrightarrow$ (b) Suppose that $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a T_1-transformation. Then, we have

$$x_i = y_i + \delta, \quad \forall\, i = 1, 2, \ldots, h \tag{3.47a}$$

$$x_i = y_i, \quad \forall\, i = h+1, h+2, \ldots, k-1 \tag{3.47b}$$

$$x_i = y_i - \epsilon, \quad \forall\, i = k, k+1, \ldots, n \tag{3.47c}$$

for some $1 \leq h < k \leq n$ and $\delta, \epsilon > 0$ such that $h\delta = (n-k+1)\,\epsilon$ and condition (3.5) holds. This implies that

$$x_1 - y_1 = \cdots = x_h - y_h = \delta > 0 = \cdots = 0 > -\epsilon = x_k - y_k = \cdots = x_n - y_n \tag{3.48}$$

hence $\mathbf{x} \geq_{AD} \mathbf{y}$. The result generalizes to an arbitrary sequence of T_1-transformations invoking the transitivity of the differentials quasi-ordering.

(b) $\Longrightarrow$ (a) Suppose that $\mathbf{x} \geq_{AD} \mathbf{y}$, in which case there are two possibilities. If $\mathbf{x} \sim_{AD} \mathbf{y}$, then $\mathbf{x} = \mathbf{y}$ and the implication is trivially true: the sequence of T_1-transformations is empty. Assume that $\mathbf{x} >_{AD} \mathbf{y}$, in which case it is possible to find two indices h and k such that $1 \leq h < k \leq n$ and

$$x_1 - y_1 \geq \cdots \geq x_h - y_h > 0 = \cdots = 0 > x_k - y_k \geq \cdots \geq x_n - y_n \tag{3.49}$$

Consider the distribution $\mathbf{z} := (z_1, \ldots, z_n)$ defined by

$$z_i = y_i + \delta, \quad \forall\, i = 1, 2, \ldots, h \tag{3.50a}$$

$$z_i = y_i, \quad \forall\, i = h+1, h+2, \ldots, k-1 \tag{3.50b}$$

$$z_i = y_i - \epsilon, \quad \forall\, i = k, k+1, \ldots, n \tag{3.50c}$$

where δ and ϵ are arbitrary real numbers.

In order to determine the appropriate values of δ and ϵ, we consider successively two cases. If $0 < h\left[x_h - y_h\right] \leq -(n-k+1)\left[x_k - y_k\right]$, then we let $\delta = x_h - y_h$ and $\epsilon = (h/(n-k+1))\left[x_h - y_h\right]$. It follows that $h\delta = (n-k+1)\,\epsilon$ and it can be verified that distribution $\mathbf{z}$ is non-decreasingly arranged. Therefore distribution $\mathbf{z}$ is obtained from distribution $\mathbf{y}$ by means of a T_1-transformation and we deduce from the sufficiency part of the proof that $\mathbf{z} >_{AD} \mathbf{y}$. Using the fact that $\mathbf{x} >_{AD} \mathbf{y}$ and since $x_h = z_h$ by definition, it can be shown that

$$x_1 - z_1 \geq \cdots \geq x_h - z_h = x_{h+1} - z_{h+1} = \cdots = x_{k-1} - z_{k-1} = 0 \tag{3.51a}$$

$$x_k - z_k \geq x_{k+1} - z_{k+1} \geq \cdots \geq x_n - z_n \tag{3.51b}$$

Therefore a sufficient condition for $\mathbf{x} \geq_{AD} \mathbf{z}$ is that $x_h - z_h = 0 \geq x_k - z_k$, equivalently

$$0 \geq x_k - z_k = x_k - y_k + \frac{h}{n-k+1}\left[x_h - y_h\right] \tag{3.52}$$

which is a direct implication of our assumption. If $h\left[x_h - y_h\right] \geq -(n-k+1)\left[x_k - y_k\right]$, then we let $\epsilon = -\left[x_k - y_k\right]$ and $\delta = -((n-k+1)/h)\left[y_k - x_k\right]$. It follows again that $h\delta = (n-k+1)\epsilon$ and again one can check that distribution **z** is non-decreasingly arranged. Therefore distribution **z** is obtained from distribution **y** by means of a T_1-transformation and we deduce from the sufficiency part of the proof that $\mathbf{z} >_{AD} \mathbf{y}$. Using the fact that $\mathbf{x} >_{AD} \mathbf{y}$ and since $x_h = z_h$ by definition, it can be shown that

$$x_1 - z_1 \geq \cdots \geq x_{h-1} - z_{h-1} \geq x_h - z_h \tag{3.53a}$$

$$0 = x_{h+1} - z_{h+1} = \cdots = x_k - z_k \geq x_{k+1} - z_{k+1} \geq \cdots \geq x_n - z_n \tag{3.53b}$$

Therefore, a sufficient condition for $\mathbf{x} \geq_{AD} \mathbf{z}$ is that $x_h - z_h \geq 0 = x_k - z_k$, equivalently

$$x_h - z_h = x_h - y_h + \frac{n-k+1}{h}\left[x_k - y_k\right] \geq 0 \tag{3.54}$$

which is a direct implication of our assumption.

We have identified a distribution **z** that obtains from **y** by means of a T_1-transformation and such that $\mathbf{x} \geq_{AD} \mathbf{z} >_{AD} \mathbf{y}$. If $\mathbf{x} = \mathbf{z}$, then the algorithm stops and a single T_1-transformation is needed in order to transform **y** into **x**. If $\mathbf{x} \neq \mathbf{z}$, then $\mathbf{x} >_{AD} \mathbf{z}$ and we apply the procedure described above to distributions **x** and **z**. Given two distributions $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$, let $d(\mathbf{u}, \mathbf{v}) := \#\left\{i \in \{1, 2, \ldots, n\} \mid u_i - v_i \neq 0\right\}$. Since we have, either $x_h = z_h$ when $\delta = x_h - y_h$ and $\epsilon = (h/(n-k+1))\,\delta$, or $x_k = z_k$ when $\epsilon = -\left[x_k - y_k\right]$ and $\delta = ((n-k+1)/h)\epsilon$, it follows that $d(\mathbf{x}, \mathbf{z}) \leq d(\mathbf{x}, \mathbf{y}) - 1$. Therefore one can obtain **x** starting from **y** by means of a finite sequence of at most n T_1-transformations. □

Turning to those transformations which successive applications of would result in a distributional improvement according to the deprivation quasi-ordering, we propose:

Definition 7 Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathcal{Y}(D)$ with $n(\mathbf{x}) = n(\mathbf{y}) = n$, we will say that **x** *is obtained from* **y** *by means of a* T_2*-transformation,* if there exists $\delta, \epsilon > 0$ and two individuals h, k $(1 \leq h < k \leq n)$ such that condition (3.5) holds and

$$x_g = y_g, \ \forall\, g \in \{1, \ldots, h-1\} \cup \{h+1, \ldots, k-1\} \tag{3.55a}$$

$$x_h = y_h + \delta; \quad x_j = y_j - \epsilon, \ \forall\, j \in \{k, \ldots, n\} \tag{3.55b}$$

$$\delta = (n-k+1)\,\epsilon \tag{3.55c}$$

A T_2-transformation admits as a limiting case the transformation identified in statement (b) of Remark 4. Although solidarity is still present in a T_2-transformation, it is now limited to the rich donors participating in the transfers. If some income is taken from a rich individual, then the same amount has to be taken from every non-poorer individual. However, it is no longer necessary that individuals poorer than the transfer recipient also benefit from some (equal) additional income. A progressive transfer constitutes a particular case of a T_2-transformation, which in turn is a particular T_1-transformation. The result below establishes the connection between dominance in terms of the absolute deprivation quasi-ordering and T_2-transformations.

Proposition 3 *Let* $n > 2$ *and* $\mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D)$ *such that* $\mu(\mathbf{x}) = \mu(\mathbf{y})$. *Then, the following two statements are equivalent:*

(a) $\mathbf{x}$ *is obtained from* $\mathbf{y}$ *by means of a finite sequence of* T_2*-transformations.*
(b) $\mathbf{x} \geq_{ADP} \mathbf{y}$.

Proof
(a) $\Longrightarrow$ (b) Suppose that $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a T_2-transformation so that

$$x_i = y_i, \quad \forall\, i = 1, 2, \ldots, h-1 \tag{3.56a}$$

$$x_h = y_h + \delta \tag{3.56b}$$

$$x_i = y_i, \quad \forall\, i = h+1, h+2, \ldots, k-1 \tag{3.56c}$$

$$x_i = y_i - \epsilon, \quad \forall\, i = k, k+1, \ldots, n \tag{3.56d}$$

for some $1 \leq h < k \leq n$ and $\delta, \epsilon > 0$ such that $\delta = (n-k+1)\epsilon$ and condition (3.5) holds. Given two distributions $\mathbf{u}, \mathbf{v} \in \mathcal{Y}_n(D)$, we find convenient to let $\psi(i; (\mathbf{u}; \mathbf{v})) := n\,[ADP(i; \mathbf{u}) - ADP(i, \mathbf{v})]$, for all $i = 1, 2, \ldots, n-1$. We have to show that (3.56) and (3.5) imply that

$$\psi(i; (\mathbf{x}; \mathbf{y})) = (n-i)[x_i - y_i] - \sum_{j=i+1}^{n} [x_j - y_j] \leq 0, \quad \forall\, i = 1, 2, \ldots, n-1 \tag{3.57}$$

Using (3.56) and upon substitution into (3.57), we obtain

$$\psi(i; (\mathbf{x}; \mathbf{y})) = \delta - (n-k+1)\,\epsilon = 0, \quad \forall\, i = 1, 2, \ldots, h-1 \tag{3.58a}$$

$$\psi(h; (\mathbf{x}; \mathbf{y})) = -(n-h)\,\delta - (n-k+1)\,\epsilon = -(n-h+1)\,\delta < 0 \tag{3.58b}$$

$$\psi(i; (\mathbf{x}; \mathbf{y})) = -(n-k+1)\,\epsilon = -\delta < 0, \quad \forall\, i = h+1, \ldots, k-1 \tag{3.58c}$$

$$\psi(i; (\mathbf{x}; \mathbf{y})) = (n-i)\,\epsilon - (n-i)\,\epsilon = 0, \quad \forall\, i = k, \ldots, n-1 \tag{3.58d}$$

hence $\mathbf{x} \geq_{ADP} \mathbf{y}$. The result generalizes to an arbitrary sequence of T_2-transformations invoking the transitivity of the deprivation quasi-ordering.

(b) $\Longrightarrow$ (a) Suppose that $\mathbf{x} \geq_{ADP} \mathbf{y}$, in which case there are two possibilities. If $\mathbf{x} \sim_{ADP} \mathbf{y}$, then $\mathbf{x} = \mathbf{y}$ and the implication is trivially true: the sequence of T$_2$-transformations is empty. Assume that $\mathbf{x} >_{ADP} \mathbf{y}$, in which case it is possible to find three indices h, k and q such that $1 \leq h < k \leq q \leq n$ and

$$x_h - y_h > 0 \tag{3.59a}$$

$$x_i - y_i = 0, \quad \forall\, i = h+1, h+2, \ldots, k-1 \tag{3.59b}$$

$$x_i - y_i \leq x_q - y_q < 0, \quad \forall\, i = k, k+1, \ldots, n \quad (i \neq q) \tag{3.59c}$$

Consider the distribution $\mathbf{z} := (z_1, \ldots, z_n)$ defined by

$$z_i = y_i, \quad \forall\, i = 1, 2, \ldots, h-1 \tag{3.60a}$$

$$z_h = y_h + \delta \tag{3.60b}$$

$$z_i = y_i, \quad \forall\, i = h+1, h+2, \ldots, k-1 \tag{3.60c}$$

$$z_i = y_i - \epsilon, \quad \forall\, i = k, k+1, \ldots, n \tag{3.60d}$$

where δ and ϵ are two arbitrary real numbers.

In order to determine the appropriate values of δ and ϵ, there are two cases to consider. If $0 < x_h - y_h \leq -(n-k+1)\left[x_q - y_q\right]$, then we let $\delta = x_h - y_h$ and $\epsilon = \left[x_h - y_h\right] / (n-k+1)$. If $-\left[x_q - y_q\right] \leq \left[x_h - y_h\right] / (n-k+1) < 0$, then we let $\delta = -(n-k+1)\left[x_q - y_q\right]$ and $\epsilon = -\left[x_q - y_q\right]$. In both cases we have $\delta = (n-k+1)\,\epsilon$ and it can be verified that distribution $\mathbf{z}$ is non-decreasingly arranged. We conclude that distribution $\mathbf{z}$ is obtained from distribution $\mathbf{y}$ by means of a T$_2$-transformation and it follows from the sufficiency part of the proof that $\mathbf{z} >_{ADP} \mathbf{y}$. Considering distribution $\mathbf{z}$, one easily checks that

$$ADP(i; \mathbf{z}) = ADP(i; \mathbf{y}), \quad \forall\, i \in \{1, 2, \ldots, h-1\} \cup \{k, k+1, \ldots, n-1\} \tag{3.61}$$

which, since by assumption $\mathbf{x} \geq_{ADP} \mathbf{y}$, implies that

$$ADP(i; \mathbf{x}) - ADP(i; \mathbf{z}) \leq 0, \quad \forall\, i \in \{1, 2, \ldots, h-1\} \cup \{k, k+1, \ldots, n-1\} \tag{3.62}$$

There are two possibilities: either $k = h+1$, or $k > h+1$. Suppose that $k = h+1$, which upon appealing to Remark 2 implies that

$$\psi(h; (\mathbf{x}, \mathbf{z})) = \psi(h+1; (\mathbf{x}, \mathbf{z})) + (n-h)\left[(x_{h+1} - z_{h+1}) - (x_h - z_h)\right] \tag{3.63}$$

Making use of (3.62) for $i = h = k-1$ and since by definition $x_k - z_k \leq 0$ and $x_h - z_h \geq 0$, (3.63) implies that

$$ADP(h; \mathbf{x}) - ADP(h; \mathbf{z}) \leq ADP(h+1; \mathbf{x}) - ADP(h+1; \mathbf{z}) \leq 0 \tag{3.64}$$

Combining (3.62) and (3.64), we conclude that $\mathbf{x} \geq_{ADP} \mathbf{z}$. Suppose next that $k > h+1$ and consider successively any $i \in \{h, h+1, \ldots, k-2, k-1\}$. Appealing again to Remark 2 for $i = k - 1$, we obtain

$$\psi(k-1;(\mathbf{x},\mathbf{z})) = \psi(k;(\mathbf{x},\mathbf{z})) + (n-k+1)[(x_k - z_k) - (x_{k-1} - z_{k-1})] \tag{3.65}$$

Making use again of (3.62) and the fact that by definition $x_{k-1} - z_{k-1} = 0$ and $x_k - z_k \leq 0$, we deduce from (3.65) that

$$ADP(k-1;\mathbf{x}) - ADP(k-1;\mathbf{z}) \leq ADP(k;\mathbf{x}) - ADP(k;\mathbf{z}) \leq 0 \tag{3.66}$$

Similarly for $i = k - 2$, we obtain

$$\begin{aligned} &\psi(k-2;(\mathbf{x},\mathbf{z})) \\ &\quad = \psi(k-1;(\mathbf{x},\mathbf{z})) + (n-k+2)[(x_{k-1} - z_{k-1}) - (x_{k-2} - z_{k-2})] \end{aligned} \tag{3.67}$$

which, upon using (3.66) and the fact that by definition $x_{k-1} - z_{k-1} = x_{k-2} - z_{k-2} = 0$, implies that

$$ADP(k-2;\mathbf{x}) - ADP(k-2;\mathbf{z}) \leq ADP(k-1;\mathbf{x}) - ADP(k-1;\mathbf{z}) \leq 0 \tag{3.68}$$

Repeating the argument, we obtain

$$\begin{aligned} &ADP(i;\mathbf{x}) - ADP(i;\mathbf{z}) \leq ADP(i-1;\mathbf{x}) - ADP(i-1;\mathbf{z}) \leq 0, \\ &\quad \forall\, i = k-3,\ldots,h+1 \end{aligned} \tag{3.69}$$

Finally for $i = h$, we have

$$\psi(h;(\mathbf{x},\mathbf{z})) = \psi(h+1;(\mathbf{x},\mathbf{z})) + (n-h)[(x_{h+1} - z_{h+1}) - (x_h - z_h)] \tag{3.70}$$

which, upon using (3.69) and the fact that by definition $x_h - z_h \geq 0$ and $x_{h+1} - z_{h+1} = 0$, implies that

$$ADP(h;\mathbf{x}) - ADP(h;\mathbf{z}) \leq ADP(h+1;\mathbf{x}) - ADP(h+1;\mathbf{z}) \leq 0 \tag{3.71}$$

Summing up we have shown that

$$ADP(i;\mathbf{x}) - ADP(i;\mathbf{z}) \leq 0, \quad \forall\, i \in \{h, h+1, \ldots, k-1\} \tag{3.72}$$

Combining (3.62) and (3.72), we conclude again that $\mathbf{x} \geq_{ADP} \mathbf{z}$.

We have identified a distribution $\mathbf{z}$ that obtains from $\mathbf{y}$ by means of a T_2-transformation and such that $\mathbf{x} \geq_{ADP} \mathbf{z} >_{ADP} \mathbf{y}$. If $\mathbf{x} = \mathbf{z}$, then the algorithm stops and a single T_2-transformation has been needed in order to transform $\mathbf{y}$ into $\mathbf{x}$. If $\mathbf{x} \neq \mathbf{z}$, then $\mathbf{x} >_{ADP} \mathbf{z}$ and we apply the procedure described above to distributions $\mathbf{x}$ and $\mathbf{z}$. By repeated application of this reasoning we obtain a sequence $\{\mathbf{z}^s\}$ such that

$$\mathbf{x} >_{ADP} \cdots >_{ADP} \mathbf{z}^s >_{ADP} \mathbf{z}^{s-1} >_{ADP} \cdots >_{ADP} \mathbf{z}^1 >_{ADP} \mathbf{z}^0 \tag{3.73}$$

where $\mathbf{z}^0 := \mathbf{y}$. Letting $ADP(\mathbf{u}) = (ADP(1;\mathbf{u}), \ldots, ADP(n;\mathbf{u}))$, for all $\mathbf{u} \in \mathcal{Y}_n(D)$, this is equivalent to

$$ADP(\mathbf{x}) < \cdots < ADP(\mathbf{z}^s) < ADP(\mathbf{z}^{s-1}) < \cdots < ADP(\mathbf{z}^1) < ADP(\mathbf{y}) \quad (3.74)$$

Therefore the sequence $\{ADP(\mathbf{z}^s)\}$ is monotone and bounded, which implies that there exists a distribution $\mathbf{z}^*$ such that $\mathbf{x} \geq_{ADP} \mathbf{z}^* = \lim_{s\to\infty} \{\mathbf{z}^s\} >_{ADP} \mathbf{z}^0$. If $\mathbf{z}^* \neq \mathbf{x}$, then $\mathbf{x} >_{ADP} \mathbf{z}^*$ and it is possible to construct a distribution $\mathbf{z}^\circ$ starting from $\mathbf{z}^*$ by means of a T_2-transformation such that $\mathbf{x} \geq_{ADP} \mathbf{z}^\circ >_{ADP} \mathbf{z}^*$. But this contradicts the fact that $\mathbf{z}^* = \lim_{s\to\infty} \{\mathbf{z}^s\}$ and we conclude that $\mathbf{z}^* = \mathbf{x}$. Therefore our algorithm is converging and it can be shown that the number of steps is bounded. □

Finally, we consider the transformations from which successive applications would result in a distributional improvement according to the satisfaction quasi-ordering. We propose:

Definition 8 Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathcal{Y}(D)$ with $n(\mathbf{x}) = n(\mathbf{y}) = n$, we will say that $\mathbf{x}$ *is obtained from* $\mathbf{y}$ *by means of a* T_3*-transformation,* if there exists $\delta, \epsilon > 0$ and two individuals h, k $(1 \leq h < k \leq n)$ such that condition (3.5) holds and

$$x_g = y_g, \ \forall\, g \in \{h+1, \ldots, k-1\} \cup \{k+1, \ldots, n\} \quad (3.75a)$$

$$x_i = y_i + \delta, \ \forall\, i \in \{1, \ldots, h\}; \quad x_k = y_k - \epsilon \text{ and} \quad (3.75b)$$

$$h\delta = \epsilon \quad (3.75c)$$

We note that the transformations identified in statement (a) of Remark 4 are particular instances of a T_3-transformation. To some extent a T_3-transformation is the dual of a T_2-transformation: the principle of solidarity is only invoked for those individuals benefiting from the transfers. If some additional income is given to a poor individual, then the same amount has to be given to every non-richer individual. But there is no need for individuals richer than the donor to give away some (equal) amount of income. A progressive transfer constitutes a particular case of a T_3-transformation, which in turn is a particular T_1-transformation. Dominance in terms of the absolute satisfaction quasi-ordering and T_3-transformations are related as it shown below.

Proposition 4 *Let* $n > 2$ *and* $\mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D)$ *such that* $\mu(\mathbf{x}) = \mu(\mathbf{y})$. *Then, the following two statements are equivalent:*

(a) $\mathbf{x}$ *is obtained from* $\mathbf{y}$ *by means of a finite and non-empty sequence of* T_3*-transformations.*
(b) $\mathbf{x} \geq_{ASF} \mathbf{y}$.

Proof

(a) $\Longrightarrow$ (b) Suppose that $\mathbf{x}$ is obtained from $\mathbf{y}$ by means of a T_3-transformation so that

$$x_i = y_i + \delta, \quad \forall\, i = 1, 2, \ldots, h \tag{3.76a}$$

$$x_i = y_i, \quad \forall\, i = h+1, h+2, \ldots, k-1 \tag{3.76b}$$

$$x_k = y_k - \epsilon \tag{3.76c}$$

$$x_i = y_i, \quad \forall\, i = k+1, k+2, \ldots, n \tag{3.76d}$$

for some $1 \leq h < k \leq n$ and $\delta, \epsilon > 0$ such that $h\delta = \epsilon$ and condition (3.5) holds. Given two distributions $\mathbf{u}, \mathbf{v} \in \mathcal{Y}_n(D)$, we let $\chi(i; (\mathbf{u}; \mathbf{v})) := n\,[ASF(i; \mathbf{u}) - ASF(i, \mathbf{v})]$, for all $i = 1, 2, \ldots, n-1$. We have to show that (3.76) and (3.5) imply that

$$\chi(i; (\mathbf{x}; \mathbf{y})) = (i-1)[x_i - y_i] - \sum_{j=1}^{i-1}[x_j - y_j] \leq 0, \quad \forall\, i = 2, 3, \ldots, n \tag{3.77}$$

Using (3.76) and upon substitution into (3.77), we obtain

$$\chi(i; (\mathbf{x}; \mathbf{y})) = (i-1)\delta - (i-1)\,\delta = 0, \quad \forall\, i = 2, 3, \ldots, h \tag{3.78a}$$

$$\chi(i; (\mathbf{x}; \mathbf{y})) = -h\,\delta < 0, \quad \forall\, i = h+1, \ldots, k-1 \tag{3.78b}$$

$$\chi(k; (\mathbf{x}; \mathbf{y})) = -(k-1)\,\epsilon - h\,\delta = -k\,\epsilon < 0 \tag{3.78c}$$

$$\chi(i; (\mathbf{x}; \mathbf{y})) = -h\,\delta + \epsilon = 0, \quad \forall\, i = k+1, \ldots, n \tag{3.78d}$$

hence $\mathbf{x} \geq_{ASF} \mathbf{y}$. The result generalizes to an arbitrary sequence of T_3-transformations invoking the transitivity of the satisfaction quasi-ordering.

(b) $\Longrightarrow$ (a) Suppose that $\mathbf{x} \geq_{ASF} \mathbf{y}$, in which case there are two possibilities. If $\mathbf{x} \sim_{ASF} \mathbf{y}$, then $\mathbf{x} = \mathbf{y}$ and the implication is trivially true: the sequence of T_3-transformations is empty. Assume that $\mathbf{x} >_{ASF} \mathbf{y}$, in which case it is possible to find three indices q, h and k such that $1 \leq q \leq h < k \leq n$ and

$$0 \leq x_q - y_q \leq x_i - y_i, \quad \forall\, i = 1, 2, \ldots, h \quad (i \neq q) \tag{3.79a}$$

$$x_i - y_i = 0, \quad \forall\, i = h+1, h+2, \ldots, k-1 \tag{3.79b}$$

$$x_k - y_k < 0 \tag{3.79c}$$

Consider the distribution $\mathbf{z} := (z_1, \ldots, z_n)$ defined by

$$z_i = y_i + \delta, \quad \forall\, i = 1, 2, \ldots, h \tag{3.80a}$$

$$z_i = y_i, \quad \forall\, i = h+1, h+2, \ldots, k-1 \tag{3.80b}$$

$$z_k = y_k - \epsilon \tag{3.80c}$$

$$z_i = y_i, \quad \forall\, i = k, k+1, \ldots, n \tag{3.80d}$$

where δ and ϵ are two arbitrary real numbers.

In order to determine the appropriate values of δ and ϵ, there are two cases to consider. If $0 < [x_q - y_q] \leq -[x_k - y_k]/h$, then we let $\delta = x_q - y_q$ and $\epsilon = h[x_q - y_q]$. If $[x_q - y_q] \geq -[x_k - y_k]/h > 0$, then we let $\delta = -[x_k - y_k]/h$ and $\epsilon = -[x_k - y_k]$. In both cases we have $\epsilon = h\delta$ and it can be verified that distribution **z** is non-decreasingly arranged. We conclude that distribution **z** is obtained from distribution **y** by means of a T$_3$-transformation and it follows from the sufficiency part of the proof that $\mathbf{z} >_{ASF} \mathbf{y}$. Considering distribution **z**, one easily checks that

$$ASF(i; \mathbf{z}) = ASF(i; \mathbf{y}), \ \forall\, i \in \{2, 3, \ldots, h\} \cup \{k+1, \ldots, n-1, n\} \tag{3.81}$$

which, since by assumption $\mathbf{x} \geq_{ASF} \mathbf{y}$, implies that

$$ASF(i; \mathbf{x}) - ASF(i; \mathbf{z}) \leq 0, \ \forall\, i \in \{2, 3, \ldots, h\} \cup \{k+1, \ldots, n-1, n\} \tag{3.82}$$

There are two possibilities: either $k = h+1$, or $k > h+1$. Suppose that $k = h+1$, which upon appealing to Remark 2, implies that

$$\chi(k; (\mathbf{x}, \mathbf{z})) = \chi(k-1; (\mathbf{x}, \mathbf{z})) + (k-1)\left[(x_k - z_k) - (x_{k-1} - z_{k-1})\right] \tag{3.83}$$

Making use of (3.82) for $i = k = h+1$ and since by definition $x_k - z_k \leq 0$ and $x_h - z_h \geq 0$, (3.83) implies that

$$ASF(k; \mathbf{x}) - ASF(k; \mathbf{z}) \leq ASF(h; \mathbf{x}) - ASF(h; \mathbf{z}) \leq 0 \tag{3.84}$$

Combining (3.82) and (3.84), we conclude that $\mathbf{x} \geq_{ASF} \mathbf{z}$. Suppose next that $k > h+1$ and consider successively any $i \in \{h, h+1, \ldots, k-2, k-1\}$. Appealing again to Remark 2 for $i = h+1$, we obtain

$$\chi(h+1; (\mathbf{x}, \mathbf{z})) = \chi(h; (\mathbf{x}, \mathbf{z})) + h\left[(x_h - z_h) - (x_{h+1} - z_{h+1})\right] \tag{3.85}$$

Making use again of (3.82) and since by definition $x_h - z_h \geq 0$ and $x_{h+1} - z_{h+1} \leq 0$, we deduce from (3.85) that

$$ASF(h+1; \mathbf{x}) - ASF(h+1; \mathbf{z}) \leq ASF(h; \mathbf{x}) - ASF(h; \mathbf{z}) \leq 0 \tag{3.86}$$

Similarly for $i = h+2$, we obtain

$$\begin{aligned}\chi(h+2; (\mathbf{x}, \mathbf{z})) = \chi(h+1; (\mathbf{x}, \mathbf{z})) + (h+1)\big[&(x_{h+1} - z_{h+1}) \\ &- (x_{h+2} - z_{h+2})\big]\end{aligned} \tag{3.87}$$

which, upon using (3.82) and the fact that by definition $x_{h+1} - z_{h+1} = x_{h+2} - z_{h+2} = 0$ implies that

$$ASF(h+2; \mathbf{x}) - ASF(h+2; \mathbf{z}) \leq ASF(h+1; \mathbf{x}) - ASF(h+1; \mathbf{z}) \leq 0 \tag{3.88}$$

Repeating the argument, we obtain

$$ASF(i;\mathbf{x}) - ASF(i;\mathbf{z}) \leq ASF(i-1;\mathbf{x}) - ASF(i-1;\mathbf{z}) \leq 0, \quad \forall\, i = h+3,\ldots,k-1 \tag{3.89}$$

Finally for $i = k$, we have

$$\chi(k;(\mathbf{x},\mathbf{z})) = \chi(k-1;(\mathbf{x},\mathbf{z})) + (k-1)\left[(x_{k-1} - z_{k-1}) - (x_k - z_k)\right] \tag{3.90}$$

which, upon using (3.89) and the fact that by definition $x_{k-1} - z_{k-1} = 0$ and $x_k - z_k \leq 0$, implies that

$$ASF(k;\mathbf{x}) - ASF(k;\mathbf{z}) \leq ASF(k-1;\mathbf{x}) - ASF(k-1;\mathbf{z}) \leq 0 \tag{3.91}$$

Summing up, we have shown that

$$ASF(i;\mathbf{x}) - ASF(i;\mathbf{z}) \leq 0, \quad \forall\, i \in \{h, h+1,\ldots,k-1\} \tag{3.92}$$

Combining (3.82) and (3.92), we conclude again that $\mathbf{x} \geq_{ASF} \mathbf{z}$.

We have identified a distribution $\mathbf{z}$ that obtains from $\mathbf{y}$ by means of a T_3-transformation and such that $\mathbf{x} \geq_{ASF} \mathbf{z} >_{ASF} \mathbf{y}$. If $\mathbf{x} = \mathbf{z}$, then the algorithm stops and a single T_3-transformation is needed to transform $\mathbf{y}$ into $\mathbf{x}$. If $\mathbf{x} \neq \mathbf{z}$, then $\mathbf{x} >_{ASF} \mathbf{z}$ and we apply the procedure described above to distributions $\mathbf{x}$ and $\mathbf{z}$. By repeated application of this reasoning we obtain a sequence $\{\mathbf{z}^s\}$ such that

$$\mathbf{x} >_{ASF} \cdots >_{ASF} \mathbf{z}^s >_{ASF} \mathbf{z}^{s-1} >_{ASF} \cdots >_{ASF} \mathbf{z}^1 >_{ASF} \mathbf{z}^0 \tag{3.93}$$

where $\mathbf{z}^0 := \mathbf{y}$. Letting $ASF(\mathbf{u}) = (ASF(1;\mathbf{u}),\ldots,ASF(n;\mathbf{u}))$, for all $\mathbf{u} \in \mathcal{Y}_n(D)$, this is equivalent to

$$ASF(\mathbf{x}) < \cdots < ASF(\mathbf{z}^s) < ASF(\mathbf{z}^{s-1}) < \cdots < ASF(\mathbf{z}^1) < ASF(\mathbf{y}) \tag{3.94}$$

Therefore the sequence $\{ASF(\mathbf{z}^s)\}$ is monotone and bounded, which implies that there exists a distribution $\mathbf{z}^*$ such that $\mathbf{x} \geq_{ASF} \mathbf{z}^* = \lim_{s\to\infty}\{\mathbf{z}^s\} >_{ASF} \mathbf{z}^0$. If $\mathbf{z}^* \neq \mathbf{x}$, then $\mathbf{x} >_{ASF} \mathbf{z}^*$ and it is possible to construct a distribution $\mathbf{z}^\circ$ starting from $\mathbf{z}^*$ by means of a T_3-transformation such that $\mathbf{x} \geq_{ASF} \mathbf{z}^\circ >_{ASF} \mathbf{z}^*$. But this contradicts the fact that $\mathbf{z}^* = \lim_{s\to\infty}\{\mathbf{z}^s\}$ and we conclude that $\mathbf{z}^* = \mathbf{x}$. Therefore our algorithm is converging and it can be shown that the number of steps is bounded. □

Welfare comparisons for equal mean distributions

Two classes of social welfare functions

Basically, two general families of social welfare functions have been studied in the inequality literature up to now. The first approach – the expected utility model or the utilitarian social welfare function – assumes linearity in the weights so that social welfare in situation $\mathbf{x} \in \mathcal{Y}(D)$ is given by

$$W_U(\mathbf{x}) = \frac{1}{n(\mathbf{x})} \sum_{i=1}^{n(\mathbf{x})} U\left(x_i\right) \tag{3.95}$$

where the *utility function U* is increasing and defined up to an increasing and affine transformation. It is convenient to associate to distribution $\mathbf{x} \in \mathcal{Y}(D)$ the vector $\mathbf{p}^{\mathbf{x}} := \left(p_1^{\mathbf{x}}, \ldots, p_{n(\mathbf{x})}^{\mathbf{x}}\right)$ where $p_i^{\mathbf{x}} := 1/n(\mathbf{x})$, for all $i = 1, 2, \ldots, n(\mathbf{x})$. Then we define

$$P_i^{\mathbf{x}} := p_1^{\mathbf{x}} + \cdots + p_i^{\mathbf{x}} = \frac{i}{n(\mathbf{x})} \tag{3.96a}$$

$$Q_i^{\mathbf{x}} := 1 - P_{i-1}^{\mathbf{x}} = p_i^{\mathbf{x}} + p_{i+1}^{\mathbf{x}} + \cdots + p_{n(\mathbf{x})}^{\mathbf{x}} = \frac{n(\mathbf{x}) - i + 1}{n(\mathbf{x})} \tag{3.96b}$$

for all $i = 1, 2, \ldots, n(\mathbf{x})$, where $P_0^{\mathbf{x}} = 0$. Letting $x_0 := 0$ and $Q_{n(\mathbf{x})+1}^{\mathbf{x}} := 0$, the second approach – the Yaari model – assumes linearity in incomes so that

$$W_f(\mathbf{x}) = \sum_{i=1}^{n(\mathbf{x})} \left[f\left(Q_i^{\mathbf{x}}\right) - f\left(Q_{i+1}^{\mathbf{x}}\right)\right] x_{(i)} \equiv \sum_{i=1}^{n(\mathbf{x})} f\left(Q_i^{\mathbf{x}}\right)\left[x_{(i)} - x_{(i-1)}\right] \tag{3.97}$$

where $f \in \mathcal{F} := \{f : (0,1) \longrightarrow (0,1) \mid f$ continuous, non-decreasing, $f(0) = 0$ and $f(1) = 1\}$ is the *weighting function*.[13] The two former models are actually particular cases of the rank-dependent expected utility model introduced by Quiggin (1993).

Remark 5 *Both the utilitarian and the Yaari social welfare functions verify PPF and SF.*

Proof It follows from the definitions that $W_U(\Pi\mathbf{x}) = W_U(\mathbf{x})$ and $W_f(\Pi\mathbf{x}) = W_f(\mathbf{x})$, for all $\mathbf{x} \in \mathcal{Y}(D)$ and all $n(\mathbf{x}) \times n(\mathbf{x})$ permutation matrices Π. It is also a direct consequence of the definition that $W_U\left(\mathbf{x}^r\right) = W_U\left(\mathbf{x}\right)$, for all $\mathbf{x} \in \mathcal{Y}(D)$ and all $r \in \mathbb{N}$ $(r \geq 2)$. To show that the Yaari social welfare function is population invariant is less immediate but it can be easily established. Consider a distribution $\mathbf{x} := \left(x_1, \ldots, x_n\right)$ and its r-replicate $\mathbf{x}^r$, where $n > 2$ and $r > 1$. Given $i \in \{1, 2, \ldots, n\}$ and $j \in \{1, 2, \ldots, r\}$, define the index $k(i,j) := (i-1)r+j$, and observe that $x_{(k(i,j))}^r = x_i$, for all $j = 1, 2, \ldots, r$ and all $i = 1, 2, \ldots, n$, where

$\mathbf{x}^r_{(\,)}$ is the non-decreasing rearrangement of $\mathbf{x}^r$. It follows that

$$p^{\mathbf{x}}_{k(i,j)} := \frac{1}{nr} \tag{3.98a}$$

$$P^{\mathbf{x}}_{k(i,j)} := \frac{(i-1)r+j}{nr} \tag{3.98b}$$

$$Q^{\mathbf{x}}_{k(i,j)} := 1 - P^{\mathbf{x}}_{k(i,j)-1} = \frac{(n-i+1)r-j+1}{nr} \tag{3.98c}$$

for $i = 1, 2, \dots, n$ and $j = 1, 2, \dots, r$. We note that

$$P^{\mathbf{x}}_{k(i,r)} = \frac{ir}{nr} = P_i, \quad \forall\, i = 1, 2, \dots, n \tag{3.99a}$$

$$Q^{\mathbf{x}}_{k(i,1)} = \frac{(n-i+1)r}{nr} = Q^{\mathbf{x}}_i, \quad \forall\, i = 1, 2, \dots, n \tag{3.99b}$$

Substituting into the definition of the Yaari social welfare function, we have

$$W_f(\mathbf{x}^r) = \sum_{i=1}^{n} \left[\sum_{j=1}^{r} \left(f\left(\frac{(n-i+1)r-j+1}{nr}\right) - f\left(\frac{(n-i+1)r-j+2}{nr}\right)\right)\right] x_i \tag{3.100}$$

Developing the expression within squared brackets, we obtain

$$W_f(\mathbf{x}^r) = \sum_{i=1}^{n} \left[\begin{array}{c} f\left(\frac{(n-i+1)r}{nr}\right) - f\left(\frac{(n-i+1)r-1}{nr}\right) \\ +f\left(\frac{(n-i+1)r-1}{nr}\right) - f\left(\frac{(n-i+1)r-2}{nr}\right) \\ \vdots \\ +f\left(\frac{(n-i+1)r-(r-1)}{nr}\right) - f\left(\frac{(n-i)r}{nr}\right) \end{array} \right] x_i \tag{3.101}$$

which, upon simplifying, reduces to

$$W_f(\mathbf{x}^r) = \sum_{i=1}^{n} \left(f\left(\frac{(n-i+1)r}{nr}\right) - f\left(\frac{(n-i)r}{nr}\right)\right) x_i = W_f(\mathbf{x}) \tag{3.102}$$

so that the Yaari social welfare function verifies condition PPF. □

Without loss of generality, we can therefore restrict attention to distributions of fixed dimension n that are non-decreasingly arranged. Given any two distributions $\mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D)$, we have $\mathbf{p}^{\mathbf{x}} = \mathbf{p}^{\mathbf{y}} = \mathbf{p} := (p_1, \dots, p_n)$, where $p_i := 1/n$, for all $i = 1, 2, \dots, n$.

An almost impossible result

We claim that the utilitarian model is not flexible enough to distinguish the views captured by the differentials and deprivation quasi-orderings we introduced above. Chateauneuf (1996) has already argued that the utilitarian social welfare function does not permit distinction between the differentials quasi-orderings and the Lorenz quasi-ordering. His conclusion is based on the fact that consistency of the utilitarian social welfare function with the differentials quasi-ordering implies that the utility function be concave. More generally, we have the following result:

Proposition 5 *Let $n > 2$ and $J \in \{AD, ADP, ASF, L\}$. Then, the following two statements are equivalent:*

(a) *For all* $\mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D)$ *such that* $\mu(\mathbf{x}) = \mu(\mathbf{y}) : \mathbf{x} \geq_J \mathbf{y} \Longrightarrow W_U(\mathbf{x}) \geq W_U(\mathbf{y})$.

(b) *U is concave.*

Proof Consider the following four statements:

(a-1) For all $\mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D)$ such that $\mu(\mathbf{x}) = \mu(\mathbf{y}) : \mathbf{x} \geq_{AD} \mathbf{y} \Longrightarrow W_U(\mathbf{x}) \geq W_U(\mathbf{y})$.

(a-2) For all $\mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D)$ such that $\mu(\mathbf{x}) = \mu(\mathbf{y}) : \mathbf{x} \geq_{ADP} \mathbf{y} \Longrightarrow W_U(\mathbf{x}) \geq W_U(\mathbf{y})$.

(a-3) For all $\mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D)$ such that $\mu(\mathbf{x}) = \mu(\mathbf{y}) : \mathbf{x} \geq_{ASF} \mathbf{y} \Longrightarrow W_U(\mathbf{x}) \geq W_U(\mathbf{y})$.

(a-4) For all $\mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D)$ such that $\mu(\mathbf{x}) = \mu(\mathbf{y}) : \mathbf{x} \geq_{L} \mathbf{y} \Longrightarrow W_U(\mathbf{x}) \geq W_U(\mathbf{y})$.

The proof consists in establishing the four chains of implications: (b) $\Longrightarrow$ (a-4), (a-4) $\Longrightarrow$ (a-3) $\Longrightarrow$ (a-1), (a-4) $\Longrightarrow$ (a-2) $\Longrightarrow$ (a-1), and (a-1) $\Longrightarrow$ (b);

(b) $\Longrightarrow$ (a-4) This is a well-known result in the theory of inequality measurement (see, for example, Marshall and Olkin 1979, B.1);

(a-4) $\Longrightarrow$ (a-3). We argue *a contrario* and show that $\neg$(a-3) $\Longrightarrow$ $\neg$(a-4). Suppose that:

$$\exists\, \mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D) \text{ with } \mu(\mathbf{x}) = \mu(\mathbf{y}) \mid \mathbf{x} \geq_{ASF} \mathbf{y} \wedge \neg\left[W_U(\mathbf{x}) \geq W_U(\mathbf{y})\right] \tag{3.103}$$

Since $\geq_{ASF} \subset \geq_L$, this implies that

$$\exists\, \mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D) \text{ with } \mu(\mathbf{x}) = \mu(\mathbf{y}) \mid \mathbf{x} \geq_{L} \mathbf{y} \wedge \neg\left[W_U(\mathbf{x}) \geq W_U(\mathbf{y})\right] \tag{3.104}$$

(a-4) $\Longrightarrow$ (a-2) Similar to the proof that (a-4) $\Longrightarrow$ (a-3);

(a-3) $\Longrightarrow$ (a-1) Similar to the proof that (a-4) $\Longrightarrow$ (a-3);

(a-2) $\Longrightarrow$ (a-1) Similar to the proof that (a-4) $\Longrightarrow$ (a-3);

(a-1) $\Longrightarrow$ (b) We argue *a contrario* and show that $\neg$(b) $\Longrightarrow$ $\neg$(a-1). Suppose that there exists $u, v \in D$ $(u < v)$ such that

$$2U\left(\frac{u+v}{2}\right) < U(u) + U(v) \tag{3.105}$$

Consider next distributions $\mathbf{x} = ((u+v)/2, (u+v)/2, \ldots, (u+v)/2, (u+v)/2)$ and $\mathbf{y} = (u, (u+v)/2, \ldots, (u+v)/2, v)$. By construction $\mu(\mathbf{x}) = \mu(\mathbf{y})$ and

$$x_1 - y_1 = \frac{v-u}{2} > x_2 - y_2 = 0 = \cdots = 0 = x_{n-1} - y_{n-1} > -\frac{v-u}{2} = x_n - y_n \tag{3.106}$$

so that $\mathbf{x} >_{AD} \mathbf{y}$. Using (3.105), we obtain

$$W_U(\mathbf{x}) - W_U(\mathbf{y}) = 2U\left(\frac{u+v}{2}\right) - [U(u) + U(v)] < 0 \tag{3.107}$$

and we conclude that condition (a-1) is violated, which makes the proof complete. □

The consistency of the utilitarian social welfare function with the different inequality views captured by our three quasi-orderings leads to the same restriction as the one implied by Lorenz-consistency: the utility function has to be concave. Thus, the utilitarian model does not permit distinction between the views embedded in the Lorenz quasi-ordering and its three competitors: the differentials, the deprivation and the satisfaction quasi-orderings.[14] On the contrary, an ethical observer endowed with the Yaari social welfare function will be able to make a difference between these alternative views, as we will demonstrate in a short while.

Consistency of the Yaari model with different inequality views

Before we state our main results, we need first to introduce some definitions concerning the weighting function. Given a function $g : \mathbb{R} \longrightarrow \mathbb{R}$ and an interval $V \subseteq \mathbb{R}$, we will say that g is *convex over* V if

$$\forall\, u, v \in V, \forall\, \lambda \in (0,1) \;:\; g((1-\lambda)u + \lambda v) \leq (1-\lambda)g(u) + \lambda g(v) \tag{3.108}$$

Given $V := (\underline{v}, \overline{v}) \subseteq \mathbb{R}$ and $\xi \in V$, we will say that g is *star-shaped from above at* ξ if

$$\forall\, u, v \in (\underline{v}, \xi) \cup (\xi, \overline{v}) \;:\; u < v \Longrightarrow \frac{g(u) - g(\xi)}{u - \xi} \leq \frac{g(v) - g(\xi)}{v - \xi} \tag{3.109}$$

(see e.g. Landsberger and Meilijson 1990). Each of the four following classes of weighting functions will play a crucial role in subsequent developments:

$$\mathcal{F}_1 := \{f \in \mathcal{F} \mid f(Q) \leq Q, \forall Q \in (0,1)\} \tag{3.110a}$$

$$\mathcal{F}_2 := \{f \in \mathcal{F} \mid f \text{ is star-shaped from above at } 0\} \tag{3.110b}$$

$$\mathcal{F}_3 := \{f \in \mathcal{F} \mid f \text{ is star-shaped from above at } 1\} \tag{3.110c}$$

$$\mathcal{F}_4 := \{f \in \mathcal{F} \mid f \text{ is convex over } (0,1)\} \tag{3.110d}$$

It is a straightforward exercise to check that the classes of weighting functions defined above are nested in the way indicated below.

Remark 6 (a) $\mathcal{F}_4 \subset \mathcal{F}_2 \subset \mathcal{F}_1$; (b) $\mathcal{F}_4 \subset \mathcal{F}_3 \subset \mathcal{F}_1$; (c) $\mathcal{F}_2 \neq \mathcal{F}_3$.

We are interested in identifying the restrictions to be placed on the weighting function $f \in \mathcal{F}$ for the Yaari social welfare function to be consistent with our absolute quasi-orderings. Actually, we are able to provide more general results that establish the links between the different equalizing transformations we introduced on pp. 40–6, the subclasses of the Yaari social welfare functions and the inequality quasi-orderings.

Considering first the inequality view captured by the absolute differentials quasi-ordering, we obtain:

Theorem 1 *Let $n > 2$ and consider two distributions $\mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D)$ such that $\mu(\mathbf{x}) = \mu(\mathbf{y})$. Then, the following three statements are equivalent:*

(a) $\mathbf{x}$ *is obtained from* $\mathbf{y}$ *by means of a finite sequence of* T_1*-transformations.*
(b) $W_f(\mathbf{x}) \geq W_f(\mathbf{y})$, *for all* $f \in \mathcal{F}_1$.
(c) $\mathbf{x} \geq_{AD} \mathbf{y}$.

Proof Since we have already shown that statements (a) and (c) are equivalent (see Proposition 2), it suffices to check that statements (b) and (c) are equivalent. Letting $\Delta W_f := W_f(\mathbf{x}) - W_f(\mathbf{y})$ and using the fact that by assumption $\mu(\mathbf{x}) = \mu(\mathbf{y})$, it can be shown that:

$$\Delta W_f = [f(Q_1) - Q_1](x_1 - y_1) + \sum_{h=2}^{n} [f(Q_h) - Q_h] \times \left[AD\left(\frac{h-1}{n}, \frac{h}{n}; \mathbf{x}\right) - AD\left(\frac{h-1}{n}, \frac{h}{n}; \mathbf{y}\right)\right] \tag{3.111}$$

(c) $\Longrightarrow$ (b) Suppose that $f \in \mathcal{F}_1$, which implies that $f(Q_i) - Q_i \leq 0$, for all $i = 2, 3, \ldots, n$. Upon substituting into (3.111) and using the fact that by definition $f(Q_1) - Q_1 = 0$, we deduce that $\mathbf{x} \geq_{AD} \mathbf{y}$ is sufficient for $W_f(\mathbf{x}) \geq W_f(\mathbf{y})$.

(b) $\Longrightarrow$ (c) Let $\phi^h(Q) := \alpha_h^1 + \beta_h^1 Q$, for all $Q \in (0,1)$, where

$$\alpha_h^1 := -\frac{Q_h - Q_{h+1}}{Q_{h-1} - Q_h} Q_{h-1} < 0 \quad \text{and} \quad \beta_h^1 := \frac{Q_{h-1} - Q_{h+1}}{Q_{h-1} - Q_h} > 1 \tag{3.112}$$

for $h = 2, 3, \ldots, n$. Consider, then, the piecewise linear function $f^h : (0,1) \longrightarrow (0,1)$ defined by

$$f^h(Q) := \begin{cases} Q, & \text{for } 0 \leq Q < Q_{h+2} \\ Q_{h-1}, & \text{for } Q_{h+2} \leq Q < Q_{h+1} \\ \phi^h(Q), & \text{for } Q_{h+1} \leq Q < Q_h \\ Q, & \text{for } Q_{h+1} \leq Q \leq Q_1 \end{cases} \tag{3.113}$$

for $h = 2, 3, \ldots, n$. Clearly $f^h \in \mathcal{F}_1$, for $h = 1, 2, \ldots, n$. Assuming that condition (b) holds and upon substitution into (3.111), we obtain

$$W_f(\mathrm{x}) - W_f(\mathrm{y}) = (Q_{h+1} - Q_h)\left[AD\left(\frac{h-1}{n}, \frac{h}{n}; \mathrm{x}\right) - AD\left(\frac{h-1}{n}, \frac{h}{n}; \mathrm{y}\right)\right] \geq 0 \tag{3.114}$$

for all $h = 2, 3, \ldots, n$. Since by definition $Q_{h+1} - Q_h < 0$, for all $h = 2, 3, \ldots, n$, we conclude that $x_h - x_{h-1} \leq y_h - y_{h-1}$, for all $h = 2, 3, \ldots, n$, hence $\mathrm{x} \geq_{AD} \mathrm{y}$. □

The conditions that ensure inequality reduction in terms of absolute income differentials are rather weak: the weighting function must lie below the main diagonal in the $(0,1) \times (0,1)$ space. Therefore, the class of Yaari social welfare functions that are consistent with the views expressed by the differentials quasi-ordering is quite large.

The next result identifies the restrictions to be placed on the weighting function that guarantee that the Yaari social welfare function is consistent with the absolute deprivation quasi-ordering.

Theorem 2 *Let $n > 2$ and consider two distributions $\mathrm{x}, \mathrm{y} \in \mathcal{Y}_n(D)$ such that $\mu(\mathrm{x}) = \mu(\mathrm{y})$. Then, the following three statements are equivalent:*

(a) *x is obtained from y by means of a finite sequence of T_2-transformations.*
(b) *$W_f(\mathrm{x}) \geq W_f(\mathrm{y})$, for all $f \in \mathcal{F}_2$.*
(c) *$\mathrm{x} \geq_{ADP} \mathrm{y}$.*

Proof Since we have already shown that statements (a) and (c) are equivalent (see Proposition 3), it suffices to check that statements (b) and (c) are equivalent. Letting $\Delta W_f := W_f(\mathbf{x}) - W_f(\mathbf{y})$, it can be shown that

$$\Delta W_f = \frac{f(Q_1)}{Q_1}[\mu(\mathbf{x}) - \mu(\mathbf{y})] + \sum_{h=1}^{n-1}\left[\frac{f(Q_{h+1})}{Q_{h+1}} - \frac{f(Q_h)}{Q_h}\right] \times \left[ADP\left(\frac{h}{n};\mathbf{x}\right) - ADP\left(\frac{h}{n};\mathbf{y}\right)\right] \tag{3.115}$$

(c) $\Longrightarrow$ (b) Suppose that $f \in \mathcal{F}_2$ which implies that $f(Q_{h+1})/Q_{h+1} \leq f(Q_h)/Q_h$, for all $h = 1, 2, \ldots, n-1$. Using the fact that by assumption $\mu(\mathbf{x}) = \mu(\mathbf{y})$, it follows from (3.115) that $\mathbf{x} \geq_{ADP} \mathbf{y}$ is a sufficient condition for $W_f(\mathbf{x}) \geq W_f(\mathbf{y})$.

(b) $\Longrightarrow$ (c) Let $\psi^h(Q) := \alpha_h^2 + \beta_h^2 Q$, for all $Q \in (0,1)$, where

$$\alpha_h^2 := -\frac{Q_h Q_{h+1}}{Q_h - Q_{h+1}} < 0 \quad \text{and} \quad \beta_h^2 := \frac{Q_h}{Q_h - Q_{h+1}} > 1 \tag{3.116}$$

for $h = 1, 2, \ldots, n-1$. Consider, then, the piecewise linear function $f^h : (0,1) \longrightarrow (0,1)$ defined by:

$$f^h(Q) := \begin{cases} 0, & \text{for } 0 \leq Q < Q_{h+1} \\ \psi^h(Q), & \text{for } Q_{h+1} < Q < Q_h \\ Q, & \text{for } Q_h \leq Q \leq 1 \end{cases} \tag{3.117}$$

Clearly, $f^h \in \mathcal{F}_2$, for $h = 1, 2, \ldots, n-1$. Assuming that condition (b) holds and upon substitution into (3.115), we obtain

$$W_f(\mathbf{x}) - W_f(\mathbf{y}) = -\left[ADP\left(\frac{h}{n};\mathbf{x}\right) - ADP\left(\frac{h}{n};\mathbf{y}\right)\right] \geq 0 \tag{3.118}$$

for $h = 1, 2, \ldots, n-1$, from which we conclude that $\mathbf{x} \geq_{ADP} \mathbf{y}$. □

The conditions that ensure inequality reduction in terms of absolute deprivation quasi-ordering are weaker than convexity: the weighting function must be star-shaped at the origin on the interval $(0,1)$.

Concerning next the restrictions to be placed on the weighting function that guarantee that the Yaari social welfare function is consistent with the absolute satisfaction quasi-ordering, we obtain:

Theorem 3 *Let $n > 2$ and consider two distributions $\mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D)$ such that $\mu(\mathbf{x}) = \mu(\mathbf{y})$. Then, the following three statements are equivalent:*

(a) $\mathbf{x}$ *is obtained from* $\mathbf{y}$ *by means of a finite sequence of* T_3*-transformations.*
(b) $W_f(\mathbf{x}) \geq W_f(\mathbf{y})$, *for all* $f \in \mathcal{F}_3$.
(c) $\mathbf{x} \geq_{ASF} \mathbf{y}$.

Proof Since we have already shown that statements (a) and (c) are equivalent (see Proposition 4), it suffices to check that statements (b) and (c) are equivalent. Letting $\Delta W_f := W_f(\mathrm{x}) - W_f(\mathrm{y})$ and after tedious computation, we obtain

$$\Delta W_f = \sum_{h=2}^{n} \left(\frac{1 - f(Q_{h+1})}{1 - Q_{h+1}} - \frac{1 - f(Q_h)}{1 - Q_h} \right) \left[ASF\left(\frac{h}{n}; \mathrm{x}\right) - ASF\left(\frac{h}{n}; \mathrm{y}\right) \right] - \frac{1 - f(Q_{n+1})}{1 - Q_{n+1}} [\mu(\mathrm{x}) - \mu(\mathrm{y})] \tag{3.119}$$

(c) $\Longrightarrow$ (b) Suppose that $f \in \mathcal{F}_3$ so that $(1 - f(Q_{h+1}))/(1 - Q_{h+1}) \leq (1 - f(Q_h))/(1 - Q_h)$, for all $h = 2, 3, \ldots, n$. Then, since by assumption $\mu(\mathrm{x}) = \mu(\mathrm{y})$, we conclude that $\mathrm{x} \geq_{ASF} \mathrm{y}$ guarantees that $W_f(\mathrm{x}) \geq W_f(\mathrm{y})$.

(b) $\Longrightarrow$ (c) Let $\varphi^h(Q) := \alpha_h^3 + \beta_h^3 Q$, for all $Q \in (0, 1)$, where

$$\alpha_h^3 := -\frac{Q_h Q_{h+1}}{Q_h - Q_{h+1}} < 0 \quad \text{and} \quad \beta_h^3 := \frac{Q_h}{Q_h - Q_{h+1}} > 1 \tag{3.120}$$

for $h = 2, 3, \ldots, n$. Consider, then, the piecewise linear function $f^h : (0, 1) \longrightarrow (0, 1)$ defined by:

$$f^h(Q) := \begin{cases} Q, & \text{for } 0 \leq Q < Q_{h+1} \\ Q_{h+1}, & \text{for } Q_{h+1} \leq Q < Q_h \\ \varphi^h(Q), & \text{for } Q_h \leq Q \leq 1 \end{cases} \tag{3.121}$$

Clearly $f^h \in \mathcal{F}_3$, for $h = 2, 3, \ldots, n$. Assuming that condition (b) holds and upon substitution into (3.119), we obtain

$$W_f(\mathrm{x}) - W_f(\mathrm{y}) = \left(1 - \frac{1 - Q_{h+1}}{1 - Q_h}\right) \left[ASF\left(\frac{h}{n}; \mathrm{x}\right) - ASF\left(\frac{h}{n}; \mathrm{y}\right) \right] \geq 0 \tag{3.122}$$

for $h = 2, 3, \ldots, n$. Since by definition $Q_{h+1} < Q_h$, for all $h = 1, 2, \ldots, n$, we deduce from (3.122) that $\mathrm{x} \geq_{ASF} \mathrm{y}$. □

The conditions that ensure inequality reduction in terms of the absolute satisfaction quasi-ordering are weaker than convexity: the weighting function must be star-shaped at 1 over (0, 1).

Finally, for the sake of completeness, we recall the conditions to be met by the weighting function for the Yaari social welfare function to be consistent with the Lorenz quasi-ordering.

Theorem 4 *Let $n > 2$ and consider two distributions $\mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D)$ such that $\mu(\mathbf{x}) = \mu(\mathbf{y})$. Then, the following three statements are equivalent:*

(a) $\mathbf{x}$ *is obtained from* $\mathbf{y}$ *by means of a finite sequence of progressive transfers.*
(b) $W_f(\mathbf{x}) \geq W_f(\mathbf{y})$, *for all* $f \in \mathcal{F}_4$.
(c) $\mathbf{x} \geq_L \mathbf{y}$.

Proof Since it is well-known that statements (a) and (c) are equivalent (see Proposition 1), it suffices to check that statements (b) and (c) are equivalent. Letting $\Delta W_f := W_f(\mathbf{x}) - W_f(\mathbf{y})$, we obtain

$$\Delta W_f = \sum_{h=1}^{n} \left[\frac{f(Q_h) - f(Q_{h+1})}{Q_h - Q_{h+1}} - \frac{f(Q_{h+1}) - f(Q_{h+2})}{Q_{h+1} - Q_{h+2}} \right] \times \sum_{i=1}^{h} \left[L\left(\frac{i}{n}; \mathbf{x}\right) - L\left(\frac{i}{n}; \mathbf{y}\right) \right] + \left[\frac{f(Q_n) - f(Q_{n+1})}{Q_n - Q_{n+1}} \right] [\mu(\mathbf{x}) - \mu(\mathbf{y})] \tag{3.123}$$

(c) $\Longrightarrow$ (b) Suppose that $f \in \mathcal{F}_4$; that is, f is convex in Q over $(0, 1)$, which implies that

$$\frac{f(Q_1) - f(Q_2)}{Q_1 - Q_2} \leq \frac{f(Q_3) - f(Q_4)}{Q_3 - Q_4} \tag{3.124}$$

for all $Q_1, Q_2, Q_3, Q_4 \in (0, 1)$ such that $Q_4 < Q_3 \leq Q_1$ and $Q_4 \leq Q_2 < Q_1$ (see, for example, Marshall and Olkin 1979, 16.b.3.a). Then $\mathbf{x} \geq_L \mathbf{y}$ guarantees that $W_f(\mathbf{x}) \geq W_f(\mathbf{y})$.

(b) $\Longrightarrow$ (c) Let $\chi^h(Q) := \alpha_h^4 + \beta_h^4 Q$, for all $Q \in (0, 1)$, where

$$\alpha_h^4 := -\frac{Q_{h+1}}{1 - Q_{h+1}} \leq 0 \quad \text{and} \quad \beta_h^4 := \frac{1}{1 - Q_{h+1}} \geq 1 \tag{3.125}$$

for $h = 2, 3, \ldots, n$. Consider then the piecewise linear function $f^h : (0, 1) \longrightarrow (0, 1)$ defined by

$$f^h(Q) := \begin{cases} 0, & \text{for } 0 \leq Q < Q_{h+1} \\ \chi^h(Q), & \text{for } Q_{h+1} \leq Q \leq 1 \end{cases} \tag{3.126}$$

Clearly, $f^h \in \mathcal{F}_4$, for $h = 2, 3, \ldots, n$. Assuming that condition (b) holds and upon substitution into (3.123), we obtain

$$W_f(\mathbf{x}) - W_f(\mathbf{y}) = \frac{1}{1 - Q_{h+1}} \left[L\left(\frac{h}{n}; \mathbf{x}\right) - L\left(\frac{h}{n}; \mathbf{y}\right) \right] \geq 0 \tag{3.127}$$

for $h = 1, 2, \ldots, n-1$, from which we conclude that $\mathbf{x} \geq_L \mathbf{y}$. □

The convexity of the weighting function is therefore necessary and sufficient for welfare as measured by the Yaari social welfare function to increase as the Lorenz curve moves upwards.

Contrary to the utilitarian model, which does not allow distinction between the inequality views considered in this chapter, the Yaari social welfare function permits separation of these different approaches to inequality. This is achieved by means of the weighting function, which captures the planner's concern for inequality. Under the equal mean condition, Theorems 1 to 4 identify the restrictions to be placed on the weighting function that guarantee that welfare does not decrease when inequality as measured by our four quasi-orderings goes down. The propositions also identify the appropriate sequences of transformations that are needed in order to convert the dominated distribution into the dominant one for the differentials, deprivation and satisfaction quasi-orderings. These transformations are more complicated than – and are generally distinct from – the traditional progressive transfers.

Extensions when distributions have different means

So far, we have restricted our attention to the comparison of distributions with equal means, in which case the notions of inequality and welfare are related in a one-to-one manner. Here, we turn to the more general case where the distributions under comparison do not necessarily have the same mean. Then, the relationship between inequality and welfare is no longer unambiguous and we will focus here on inequality.

Since the social welfare functions we consider both satisfy the principle of population (see Remark 5), we can restrict attention to income distributions of the same dimension without loss of generality. Given the social welfare function $W : \mathcal{Y}_n(D) \longrightarrow \mathbb{R}$, we let Ξ represent the *social evaluation function* implicitly defined by

$$W(\underbrace{x_1, \ldots, x_n}_{n}) = W(\underbrace{\Xi(\mathbf{x}), \ldots, \Xi(\mathbf{x})}_{n}), \quad \forall\, \mathbf{x} \in \mathcal{Y}_n(D) \tag{3.128}$$

where $\Xi(\mathbf{x})$ is the *equally distributed equivalent income* corresponding to distribution $\mathbf{x}$. Solving (3.128) when W is the utilitarian social welfare function, we have

$$\Xi_U(\mathbf{x}) = U^{-1}\left(\sum_{i=1}^{n} p_i\ U(x_i)\right), \quad \forall\ \mathbf{x} \in \mathcal{Y}_n(D) \tag{3.129}$$

while, in the case of the Yaari social welfare function, we obtain

$$\Xi_f(\mathbf{x}) = \sum_{i=1}^{n} [f(Q_i) - f(Q_{i+1})]\, x_i, \quad \forall\ \mathbf{x} \in \mathcal{Y}_n(D) \tag{3.130}$$

The *absolute inequality index*, which measures the average income loss due to inequality, is defined by

$$I_U^A(\mathbf{x}) := \mu(\mathbf{x}) - \Xi_U(\mathbf{x}) \text{ and } I_f^A(\mathbf{x}) := \mu(\mathbf{x}) - \Xi_f(\mathbf{x}),\ \forall\ \mathbf{x} \in \mathcal{Y}_n(D) \tag{3.131}$$

in the case of the utilitarian and Yaari social welfare functions, respectively. We denote as $\tilde{\mathbf{x}} := (\tilde{x}_1, \ldots, \tilde{x}_n)$ the *mean-reduced* distribution of $\mathbf{x} \in \mathcal{Y}_n(D)$ obtained by letting $\tilde{x}_i := x_i - \mu(\mathbf{x})$, for all $i = 1, 2, \ldots, n$. Given distribution $\mathbf{x} \in \mathcal{Y}_n(D)$, we let $AL(p; \mathbf{x}) := L(p; \tilde{\mathbf{x}})$, for all $p \in (0, 1)$.

Definition 9 Given two income distributions $\mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D)$, we will say that $\mathbf{x}$ *absolute Lorenz dominates* $\mathbf{y}$, which we write $\mathbf{x} \geq_{AL} \mathbf{y}$, if and only if

$$AL(p; \mathbf{x}) \geq AL(p; \mathbf{y}), \quad \forall\ p \in (0, 1) \tag{3.132}$$

(see Moyes 1987).

The two following conditions related to the notion of absolute inequality introduced by Kolm (1976) are important for our purpose.

Translation invariance for inequality indices (TII) The inequality index $I \in \mathbb{I}(D)$ satisfies translation invariance – equivalently is translatable of degree zero – if, for all $\mathbf{x} \in \mathcal{Y}_n(D)$ and all $\gamma > 0$ such that $\mathbf{x} + \gamma \mathbf{1_n} \in \mathcal{Y}_n(D)$, we have $I(\mathbf{x} + \gamma \mathbf{1_n}) = I(\mathbf{x}) + \gamma$.

Translation invariance for inequality quasi-orderings (TIO) The quasi-ordering $\geq_J$ over $\mathcal{Y}_n(D)$ satisfies translation invariance if, for all $\mathbf{x} \in \mathcal{Y}_n(D)$ and all $\gamma > 0$ such that $\mathbf{x} + \gamma \mathbf{1_n} \in \mathcal{Y}_n(D)$, we have $\mathbf{x} + \gamma \mathbf{1_n} \sim_J \mathbf{x}$.

The following remark, which is immediate, will be helpful later on.

Remark 7 *Let $J \in \{AD, ADP, ASF, AL\}$. Then, the inequality quasi-ordering $\geq_J$ verifies TIO.*

As the next result indicates, substituting the (absolute) inequality index for the social welfare function in the utilitarian case does not change anything.

Building upon arguments from Ebert (1988), we obtain:

Proposition 6 *Let $n > 2$ and $J \in \{AD, ADP, ASF, AL\}$. Then, the following two statements are equivalent:*

(a) *For all $\mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D)$: $\mathbf{x} \geq_J \mathbf{y} \Longrightarrow I_U^A(\mathbf{x}) \leq I_U^A(\mathbf{y})$.*

(b) *Either $U(y) = \frac{1}{\eta}\exp(\eta y)$ when $\eta < 0$, or $U(y) = y$ when $\eta = 0$, for all $y \in D$.*

Whatever the inequality views one considers, we end up with the Kolm–Pollak family of inequality indices and CARA utility functions (see Kolm 1976). Proposition 6 confirms, in the case of situations with possibly differing mean incomes, what we learnt from Proposition 2: the utilitarian model does not allow one to distinguish between the views associated with the differentials, the deprivation, the satisfaction and the Lorenz (absolute) quasi-orderings.

On the contrary, Yaari's approach allows separatation of these different views and one obtains the same restrictions on the weighting function as those identified in Theorems 1 to 4. Using the fact that I_f is translatable of degree zero,[15] we have

$$\forall\, \mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D) : I_f^A(\mathbf{x}) \leq I_f^A(\mathbf{x}) \Longleftrightarrow W_f\left(\tilde{\mathbf{x}}\right) \leq W_f\left(\tilde{\mathbf{y}}\right) \tag{3.133}$$

This implies that, for all $J \in \{AD, ADP, ASF, AL\}$, the two statements below are equivalent:

$$\forall\, \tilde{\mathbf{x}}, \tilde{\mathbf{y}} \in \mathcal{Y}_n(D) : \tilde{\mathbf{x}} \geq_J \tilde{\mathbf{y}} \Longrightarrow W_f(\tilde{\mathbf{x}}) \leq W_f(\tilde{\mathbf{y}}) \tag{3.134}$$

$$\forall\, \mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D) : \mathbf{x} \geq_J \mathbf{y} \Longrightarrow I_f^A(\mathbf{x}) \leq I_f^A(\mathbf{y}) \tag{3.135}$$

The argument is made complete invoking the translation invariance of the quasi-orderings we have considered (see Remark 7).

Conclusion

We have argued in this chapter that the utilitarian model, which frames most of the theory of welfare and inequality measurement, may be considered inappropriate when one is interested in inequality views that are more in accordance with society's values than that expressed by the Lorenz quasi-ordering. Considering three such concepts of inequality – captured by the differentials, the deprivation and the satisfaction quasi-orderings respectively – we have shown that the Yaari model allows one to distinguish between these views. We have furthermore identified the classes of Yaari social welfare functions consistent with these three views. These appear to be subclasses of the general class of Lorenz consistent Yaari social welfare functions. It is possible to consider other (absolute) inequality views – equivalently quasi-orderings – and investigate their implications for the properties of the social welfare function. We would rather point at three other directions in which our analysis could be extended.

Firstly, given the predominance of the relative approach in the literature it would be interesting to see if it is possible to find analogous results when one considers the relative versions of the criteria examined here. The relative versions of the quasi-orderings introduced in this chapter can easily be derived. However, up to now we have not been able to identify the restrictions to be imposed on the weighting function to guarantee that social welfare increases as the result of more equally distributed incomes in this case (see, however, Chateauneuf 1996).

Secondly, the rank-dependent expected utility model introduced by Quiggin (1993), which comprises as particular cases the two approaches examined in this chapter, provides a natural avenue for generalizing our approach. Indeed, it offers more flexibility as the chosen value judgements can be reflected by either the utility function or the weighting function.

Thirdly, the next step surely is to characterize, by means of additional conditions, particular elements of the different classes we have identified. Indeed, one may raise doubts about the ability of the differentials quasi-ordering – and to a lesser extent, the deprivation and satisfaction quasi-orderings – to generate conclusive verdicts in practice. Although it must be stressed that these criteria provide guidance in some cases, such as taxation design (see, for example, Chakravarty and Moyes 2003), it is equally true that their ability to rank arbitrary real-world distributions is limited. They must therefore be considered a first round approach, which should be supplemented by the use of particular indices in the general classes we have identified.

Notes

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1. Although parts of this general result appeared in different places in Hardy *et al.* (1952), it was Berge (1963) who collected these scattered statements and provided a self-contained proof of what is know now as the Hardy–Littlewood–Polya theorem. This result has been rediscovered in the field of inequality measurement independently by Kolm (1969), Atkinson (1970) and Fields and Fei (1978) among others (see also Dasgupta *et al.* 1973, Sen 1973, and Foster 1985).
2. Most scholars take for granted that individual deprivation is simply the sum – possibly normalized in a suitable way – of the income gaps between the individual's income and the incomes of all individuals richer than her. An axiomatic

characterization of the absolute deprivation profile is provided by Ebert and Moyes (2000).

3. See Blackorby *et al.* (1999) for a recent survey of the literature on the ethical approach to inequality measurement.
4. The rank-dependent expected utility model is flexible enough to accommodate most of the inequality views one encounters in the literature.
5. It is premature to conclude that these different concepts of inequality cannot be distinguished by the welfarist social welfare function since the utilitarian approach does not exhaust all the possibilities. However, a similar conclusion holds for the *maximin* and *leximin* rules, which both record a welfare increase when inequality as measured by the differentials, deprivation and satisfaction quasi-orderings decreases.
6. Yaari's (1987) model was introduced in the fields of choice under risk and then applied to the measurement of inequality in Yaari (1988). Related approaches have been proposed in the inequality literature by Ebert (1988) and Weymark (1981).
7. Throughout the chapter, we adopt the terminology proposed by Sen (1970, ch. 1*), but we recognize that there are other possibilities.
8. The principle of population justifies the use of the cumulative distribution functions and/or the quantile functions for making welfare and inequality comparisons.
9. In the necessity part of the proof, where it is shown that, if $\mathbf{x}$ Lorenz dominates $\mathbf{y}$, then $\mathbf{x}$ can be obtained from $\mathbf{y}$ by means of a sequence of progressive transfers, the way these transfers are combined is important.
10. Consider the function g defined over the set $\{(u,v) \mid u \leq v\}$ such that: (i) $g(u,u) = \xi$, for all u and some ξ; (ii) $g(u+\Delta, v) < g(u,v)$, for all u, v and $\Delta > 0$ such that $u + \Delta \leq v$; (iii) $g(u, v-\Delta) < g(u,v)$, for all u, v and $\Delta > 0$ such that $u \leq v - \Delta$; (iv) $g(u+\Delta, v-\epsilon) < g(u,v)$, for all u, v and $\Delta, \epsilon > 0$ such that $u + \Delta \leq v - \epsilon$. Particular cases are $g(u,v) = v - u$ (absolute differentials) and $g(u,v) = v/u$ (relative differentials).
11. This might also be of some help to the experiment designer interested in confronting the differentials, satisfaction and deprivation quasi-orderings to the public's attitudes.
12. To make things more precise, consider two distributions $\mathbf{x}, \mathbf{y} \in \mathcal{Y}_n(D)$ such that $\mu(\mathbf{x}) = \mu(\mathbf{y})$. Following Chakravarty (1997), let us say that distribution $\mathbf{x}$ is obtained from distribution $\mathbf{y}$ by means of a *fair transformation* if $(n-k)\,(x_k - y_k) \geq \sum_{j=k+1}^{n} (x_j - y_j)$, for all $k = 1, 2, \ldots, n-1$. It is immediate that, if $\mathbf{x} \geq_{ADP} \mathbf{y}$, then a *single* fair transformation is needed to convert $\mathbf{y}$ into $\mathbf{x}$. A fair transformation is not an elementary transformation in the sense that it is possible to find transformations (i) that all imply dominance according to the absolute deprivation quasi-ordering, and (ii) whose combination results in a fair transformation.
13. Actually for our purpose it is not necessary that $f(Q) \geq 0$, for all $Q \in (0,1)$, and all our results hold as well for f not positive everywhere on the interval $(0,1)$.
14. Although Proposition 5 demonstrates that the utilitarian social welfare function is not flexible enough to allow the ethical planner to draw a distinction between the different inequality views we have introduced, it is fair to note that it by no means proves that the welfarist approach suffers from the same drawback. Clearly, a similar conclusion holds for the *maximin* and *leximin* rules, which both record a welfare increase when inequality as measured by the differentials, deprivation and satisfaction quasi-orderings decreases. Quite interestingly, it can be shown that Proposition 5 still holds when one substitutes the generalized

Lorenz ranking of the distributions of utilities for the utilitarian social welfare function.

15. The fact that I_f is translatable of degree zero is equivalent to the fact that Ξ_f is unit-translatable; that is, $\Xi_f\,(\mathbf{x} + \gamma \mathbf{1_n}) = \Xi_f(\mathbf{x}) + \gamma$, for all $\mathbf{x} \in \mathcal{Y}_n(D)$ and all $\gamma > 0$ such that $\mathbf{x} + \gamma \mathbf{1_n} \in \mathcal{Y}_n(D)$.

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4

Relative Deprivation, Personal Income Satisfaction and Average Well-being under Different Income Distributions: An Experimental Investigation

Christian Seidl, Stefan Traub and Andrea Morone

Introduction

In this chapter, we make an experimental investigation of the role of context dependence in the individual assessment of incomes. Suppose, for example, you had to assign 14 equally spaced incomes to seven categories ranging from 'very bad' to 'excellent'. As the incomes are equally spaced, it is very likely that you would assign two neighbouring incomes in increasing order to the respective categories. However, this picture would change dramatically if the very same incomes (stimuli) were embedded in sets of additional background incomes, which serve to create different income distributions. The background context would cause you to rate the same income stimulus higher if there were only a few higher incomes in the respective income distribution. The same income stimulus would be rated lower if there were many incomes greater than the considered income in the respective income distribution. Thus, income categorization and, *a fortiori*, income satisfaction, depend on the background context.

When subjects are asked to categorize incomes, they seem to step into the shoes of the income recipients and categorize the respective incomes with respect to *relative deprivation*. Although context dependence of categorization was widely investigated in psychology, it has, to the best of our knowledge, never been systematically studied with respect to the satisfaction with and the categorization of incomes. This is perhaps due to the prevalence of positively skewed income distributions in virtually all societies. However, it is tempting to examine the effects of relative deprivation of other income distributions and compare the results. For a given aggregate income in an economy, this implies that different patterns of income distributions engender different welfare effects. This chapter considers the context effects of five different shapes of income distribution.

We first briefly review the literature on context dependence and then present a short survey on range-frequency theory. We go on to describe the experiment, discuss its results and offer our conclusions. The instructions and the stimulus material of the experiment have been relegated to the Appendix.

Context dependence: a brief literature review

The context of an event has been shown to influence the perception of the event in a variety of disciplines. For example, in psychology research Parducci (1968: 84) observed that acts of wrongdoing are rated more leniently in a context of rather nasty behaviour than in a context of mild misbehaviour. Experimental research by Birnbaum (1973, 1974b), too, evidenced that subjects tend to judge persons by their worst bad deed.

Birnbaum *et al.* (1971) presented subjects with lines of different lengths. They found that the effects of any particular line upon the judgement of average length varied inversely with the length of the other lines within the same set. Birnbaum (1974a) investigated subjects' perceptions of the magnitude of numerals. He observed that the categorization of numerals depended decisively on the shape of their distributional arrangement. Birnbaum (1992) found that certainty equivalents of binary lotteries are rated higher when associated with negatively skewed than with positively skewed distributions of proposals.

Parducci (1982) observed that subjects' categorization of squares of different sizes depended decisively on the skewness of the distribution according to which the differently sized squares were presented. In a similar experiment, Mellers and Birnbaum (1982) found that the members of identical sets of squares were assigned higher categories of darkness (expressed as the number of dots contained in a square) when their presentation was embedded in a positively skewed dot distribution of other squares as compared to a negatively skewed dot distribution.

Notice that these findings, although related to them, go beyond mere anchoring effects[1] and simple context effects.[2] They establish a relationship between the shape of the distribution of the presented stimuli and subjects' judgements on a categorical scale. Strong contextual effects exist for category ratings. Parducci (1982: 90) has characterized such effects as a constituent of human behaviour:

> I would have little interest in subjects' expressions of value experiences if these did not change with context. A particular income that might have seemed magnificent at an early stage in one's career would seem totally inadequate at a later stage. If a response scale did not reflect this change, it would miss the all important decline in experienced value.

Closer inspection shows that categorization of stimuli depends not only on the shape of the distribution of the stimuli but also on their range. It differs also for closed sets of categories and open ended categories.

While Luce and Galanter (1963: 268) deplored the lack of a sophisticated theory of category judgements which defines a scale of sensation that is invariant under experimental manipulations, Parducci and associates, upon having noticed that subjects' evaluation and categorization of objects depended on their background context, set to work to develop such a theory, to wit, *range-frequency theory*. It was developed from Parducci's (1965) *limen model* and has proved to reveal important insights into category rating (Parducci *et al.* 1960; Parducci 1968, 1974, 1982; Birnbaum *et al.* 1971; Parducci and Perret 1971; Birnbaum 1973, 1974a, 1974b, 1992; Mellers 1982, 1986; Mellers and Birnbaum 1982). This theory takes into account that the *distribution of stimuli* in which they are embedded matters for the evaluation and categorization of the very same objects.

Range-frequency theory: a short survey

Range-frequency theory captures the dependence of category assignments on the distribution and the range of stimuli. It comprises equations for the range value, the frequency value, for judgement, and for the category assignment (Parducci 1982: 94–5).

The *range value* R_i of stimulus S_i depends on the value of this stimulus and on stimulus range

$$R_i := \frac{S_i - \min_j\{S_j\}}{\max_j\{S_j\} - \min_j\{S_j\}} \tag{4.1}$$

The *frequency value* F_i of stimulus S_i depends on the rank of this stimulus, r_i, and on the ranks of the largest and smallest stimulus values, N and 1.

$$F_i := \frac{r_i - 1}{N - 1} \tag{4.2}$$

The *judgement* of stimulus i, J_i, is modelled as a weighted mean of the range value and the frequency value

$$J_i := wR_i + (1 - w)F_i, \quad 0 \leq w \leq 1 \tag{4.3}$$

The *category assignment* of stimulus i, C_i, is then the simple transformation

$$C_i := bJ_i + a \tag{4.4}$$

where b denotes the range of possible categories and a the rank assigned to the lowest category, 1 in most cases. Thus, category assignment assumes that categories are equally spaced; adjoining categories differ precisely by 1.

For $w = 0$ only the frequency value matters; that is, the same number of stimuli is assigned to each category in increasing order. For instance, if there were seven categories, then the seventh lowest ranked stimuli would be assigned to the lowest category, and so on.

For $w = 1$ only the range value matters; that is, the range of the stimuli is equally split. Stimuli are assigned to categories according to the limens of the equally wide intervals of the range of the stimuli. This means that if S_i is placed in another context with the same minimum value but a higher maximum value of the stimuli, then S_i tends to fall back in judgement and categorization. On the other hand, if the minimum of the stimuli decreases while their maximum remains unchanged, S_i tends to advance in judgement and categorization.

Range-frequency theory considers categorization to be a weighted average of these two components. Thus, it posits that categorization is linear both in stimulus value (range component) and in stimulus rank (frequency component). Arranging stimuli on the abscissa and categories on the ordinate should produce a nonlinear graph if $w < 1$ and if the distribution of stimuli were not uniform. Nonlinearity of this graph is caused solely by the assumption of linearity of categorization in stimulus rank (frequency component). Psychologists sometimes estimated $w = 0.45$ (Parducci *et al.* 1960: 74) or $w = 0.475$ (Birnbaum 1974a: 92); sometimes they simply adopted $w = 0.5$ (for example, Parducci and Perrett 1971: 429).

Tests of range-frequency theory use sundry distributions of stimuli; for example, uniform, symmetrical unimodal, symmetrical bimodal, positively skewed and negatively skewed distributions. The respective distributions are generated either by appropriate spacing and/or appropriate frequency of stimuli (see, for example, Parducci 1965, 1974; Parducci and Perrett 1971), or by embedding a set of (usually equally spaced) stimuli into a superset of adventitious stimuli (see, for example, Mellers 1982, 1986; Mellers and Birnbaum 1982; Parducci 1982) which shape the intended distribution.

For all distributions of stimuli, the judgement function becomes steeper where the stimuli are more densely packed. Thus, if the subsets of equally spaced stimuli (which are common to all distributions) are arranged on the abscissa and the mean categorial value on the ordinate, symmetric unimodal distributions produce an S-shaped curve, bimodal distributions produce an ogival shaped curve, positively skewed distributions produce a concave curve, and negatively skewed distributions a convex curve, where the curve of positively skewed distributions lies above the curve of negatively skewed distributions. The distance between curves is greater the fewer categories are admitted. Moreover, subjects tend to exhaust the available categories. If the set of stimuli is truncated, all categories are nevertheless occupied, although relatively more tenuously.

Range-frequency theory has been successfully employed by Mellers (1982, 1986) for the investigation of equity judgements such as equitable salaries or equitable taxation as functions of merit. Mellers winnowed out the 'Aristotelian' subjects; that is, those whose responses conformed with proportionality. For the rest, she placed merit ratings on the abscissa and mean salaries on the ordinate, and received precisely the pattern described in the preceding paragraph (Mellers 1982: 259–61; 1986: 82–6).

In an attempt to rescue his linear equity model (Harris 1976, 1980), Harris (1993) transformed Mellers' merit stimuli to yearly salaries, used these as stimuli, and observed a linear relationship between his stimuli and the equitable salaries. However, when using Mellers' merit design proper as stimuli, he found Mellers' results confirmed. Thus, he concluded that stimulus dimension, too, matters for subjects' behavioural conformity with range-frequency theory. Note, however, that Harris' treatment contains an element of *equitable redistribution* of a given salary structure, which is different from a primordial assignment of salaries according to merit.

Experiment

Aims and scope

This chapter pursues four aims. Firstly, we examine whether *background context* matters. In other words, we canvass how the categorization of the same set of stimuli systematically depends on the background context. Indeed, categorization of incomes using different distributions of stimuli has never been studied thoroughly. Mellers and Harris examined the judgement of equitable salaries, not income categorization based on different income distributions.

Secondly, we investigate *relative deprivation* by way of income categorization.[3] When subjects categorize incomes, they cannot wholly avoid stepping into the shoes of the income recipients whose incomes they are asked to judge. Thus, they feel relative deprivation of an income position if many incomes are encountered which are greater than this income (likewise, they may feel 'relative elation' if the particular income appears among the higher income strata within the income distribution).

Thirdly, we investigate whether *range-frequency theory* is a valid description of the categorization of incomes. Moreover, we focus on the proper weights of the range and frequency components, an issue that has been understudied in earlier research. After deriving the weights of the range and frequency components, we will investigate which income distribution generates most happiness both in terms of personal income satisfaction and aggregate well-being.

Finally, we investigate the reverse side of income categorization; to wit, the *production of the limens of income categories*. In particular, we check whether the structure of the limens matches income categorization.[4] If the limens of income categories depend on the distribution of the presented stimuli, then utility functions of income estimated from such data cannot but reflect the respective pattern.[5]

Experimental design

The experiment was computerized and subjects were told a cover story of the income distribution on a planet called Utopia, inhabited by small green individuals with the UFO as the local currency (see the Appendix). This extraterrestrial story was employed to distort as much as possible any connotation with the extant positively skewed income distributions and, thereby, provide an unbiased test of context dependence of categorization. For this purpose, we chose a support of 100 and 1000 UFO for all income distributions and used Italian subjects who were at the time of the experiment accustomed to a completely different dimension of currency units.

For our experiment, we used five distributions, which were truncated to secure the above finite support: uniform, normal, bimodal (mixture of two normal distributions), positively skewed (lognormal), and negatively skewed (negative lognormal). To generate the experimental design, we used the parameters stated in the second and third columns of Table 4.1. In a first step, we computed the respective truncated distribution functions, divided their range (the unit interval) by 43 and computed the projection of these equally spaced values on the support (the 100–1000 interval), which produced the mathematical bases of our stimuli.

In order to be able to compare a subset of identical stimuli across the five experimental designs, we formed a sequence of 14 equally spaced values,[6] which were embedded in 28 adventitious income values which provided the background context of the respective experimental distributions. To accomplish that, we replaced the nearest values in the mathematical bases of the distributions by the values of the equally spaced subsets of stimuli, which

Table 4.1 Descriptive statistics of experimental distributions

	Parameters for generation		**Moments of experimental distributions**			
Distribution	μ	σ	**Mean**	**Std dev.**	**Skewn.**	**Kurtosis**
Uniform	(550)	(260)	550	258	.000	−1.200
Normal	550	230	549	198	.000	−.544
Bimodal[a]	325, 775	100	550	241	.000	−1.479
Positively skewed[b]	6	1	408	230	.738	−.349
Negatively skewed[c]	6	1	692	230	−.735	−.348

Notes: 1 All distributions truncated at 100 on the left and at 1000 on the right.
2 [a]Mixture of two normal densities;
[b]Lognormal density;
[c]Lognormal density with $x^{-} = 1100 - x$.

formed our experimental design. The right side of Table 4.1 provides mean, standard deviation, skewness and kurtosis of the experimental stimulus distributions. To check whether our manipulation to create the experimental stimulus distributions changed the character of the mathematical distributions, we applied a Wilcoxon signed ranks test, which did not reject the null hypothesis of identity.[7]

Procedure

As a warm-up introduction, subjects were first shown 25 values taken from the mathematical distributions. Then the 42 values of the experimental design were presented to the subjects in a random order, first as a synopsis, and then one at a time. Subjects were asked to assign them to one of the categories *excellent, good, sufficient, barely sufficient, insufficient, bad, very bad.* After that, all stimuli were again shown together with the subject's categorization. Subjects were asked to confirm or change their categorization assignment. Thereafter, subjects were asked to provide limens of the seven income categories.

The experiment was administered from 24 April to 5 May 2001 at the Laboratorio Informatico, Department of Economics, University of Bari, Italy. Two hundred and fifty subjects participated in this experiment, 50 for each of the five distributions. Subjects were only admitted to a single participation. Each subject received a lump sum reimbursement of 15,000 Italian Lire (about €7.5). Subjects spent between six and 43 minutes in completing the experiment (mean: 16.1 minutes, standard deviation: 6.2).[8]

Results

Comparing subjects' primary and revised category assignments, we found them to be not significantly different. This allowed us to use only the revised assignments of categories for our analyses.

Background context matters

Table 4.2 contains the mean (μ) and median (M) assignments of the 14 common stimuli to the seven categories, coded from 1 (very bad) to 7 (excellent). The table shows that the subjects actually exhausted the categories irrespective of the distribution of stimuli because the categories coincide for the tails (135 UFO and 965 UFO, respectively).

For our experiment, testing on background context effects is equivalent to testing on whether the five sets of observations have the same underlying distribution D for a given stimulus income. That is, the null hypothesis for stimulus i, $i = \{135, 199, \ldots, 965\}$, is given by

$$\mathcal{H}_0^i \; : \; D_{un}^i(z) = D_{no}^i(z) = D_{bi}^i(z) = D_{ps}^i(z) = D_{ns}^i(z) \quad \forall\, z \in \mathbb{R}$$

Table 4.2 Test on background context effects: Kruskal–Wallis (KW) test

	Uniform		**Normal**		**Bimodal**		**Pos.-sk.**		**Neg.-sk.**		**KW test**	
Stim.	μ	M	μ	M	μ	M	μ	M	μ	M	χ^2	p
135	1.06	1	1.04	1	1.10	1	1.00	1	1.06	1	5.334	.255
199	1.38	1	1.18	1	1.36	1	1.48	1	1.14	1	13.399	.009
263	2.02	2	1.98	2	2.00	2	2.24	2	1.76	2	23.966	.000
327	2.42	2	2.40	2	2.56	3	2.78	3	2.28	2	21.353	.000
390	2.84	3	2.80	3	2.76	3	3.10	3	2.46	2	32.927	.000
454	3.48	4	3.26	3	3.44	3	3.58	4	3.24	3	8.624	.071
518	3.80	4	3.96	4	4.04	4	4.16	4	3.82	4	15.288	.004
582	4.12	4	4.16	4	4.16	4	4.42	4	3.86	4	27.036	.000
646	4.46	5	4.76	5	4.74	5	4.88	5	4.24	4	45.872	.000
710	5.24	5	5.10	5	5.40	5	5.46	5	4.78	5	31.661	.000
773	5.46	5.5	5.56	6	5.68	6	5.80	6	4.96	5	40.838	.000
837	6.04	6	6.02	6	6.32	6	6.18	6	5.92	6	18.646	.001
901	6.76	7	6.54	7	7.00	7	6.92	7	6.86	7	16.377	.003
965	6.96	7	6.98	7	7.00	7	7.00	7	6.98	7	4.065	.397

Note: μ = mean, M = median, $n = 50 \times 5$, $df = 4$ for all tests.

where un=uniform, no=normal, bi=bimodal, ps=positively skewed, ns=negatively skewed. Since neither normality nor cardinality of the observations hold, we use the (non-parametric) Kruskal–Wallis (KW) test in order to test on background context effects. The results (χ^2 values and significance levels p) of this test for each of the 14 common stimuli are given in the last two columns of the table.

For the interior common stimuli (199–901 UFO), we observe considerable background context effects: The respective Kruskal–Wallis tests are significant at the 1 per cent level, except for the 454 UFO stimulus which is significant at the 10 per cent level. That is, for 12 of 14 tests performed, we have to reject the null hypothesis that the five different sets of observations came from the same distribution. Assuming stochastic independence of the 14 observations (per subject) under the null hypothesis,[9] a supplementary binomial test would strongly reject the null hypothesis stating that this results from pure chance ($p = .006$).

This subsection demonstrates that background context matters for income categorization. Our test was global in the sense that it did not allow pairwise comparisons. In the following, we are concerned with a directional hypothesis.

Relative deprivation

The figures in Table 4.2 show a clear tendency. The positively skewed distribution, by and large, exhibits the highest mean assignments, followed by the bimodal distribution, and the uniform and the normal distributions. Under the negatively skewed distribution, subjects' categorization of incomes turns out worst. Take, for example, the 773 UFO category. The difference between the positively skewed and the negatively skewed distributions amounts to no less than 0.84 categories.

Consequently, we hypothesize that identical income stimuli are perceived to belong to higher evaluation categories if the background context shifts more income mass to lower income brackets. If the background context exhibits more income mass concentrated among higher income strata, then the evaluation of identical income stimuli is downgraded (*relative deprivation*). In order to test on relative deprivation, we compute Spearman's rank correlations between the subjects' categorizations of a stimulus and the number of incomes larger than that stimulus as a measure of relative deprivation. Note that if alternative measures of relative deprivation, such as the sum of incomes exceeding the stimulus and so on, are applied, results do not change qualitatively.

Table 4.3 contains the relevant data and the results of the test. The null hypothesis is

$$\mathcal{H}_0^k \ : \ X^k \text{ and } Y^k \text{ are independent}$$

where X^k and Y^k denote the distributions of the categorizations of stimulus k and the corresponding number of incomes larger than the stimulus (see Table 4.3), respectively. As can be seen in Table 4.3, the null hypothesis of independence is rejected for 11 of 14 tests at the 5 per cent significance level and for 12 of 14 tests at the 10 per cent significance level. Furthermore, all significant correlations exhibit the right (negative) sign. Again, a binomial test would strongly confirm that this does not result from pure chance ($p = .006$).

Hence, we conclude that relative deprivation is an important factor in the evaluation of incomes. The more incomes exist that exceed the income to be evaluated – that is, the greater the relative deprivation associated with this income is – the worse is this income's categorization for the respective background.

Range-frequency theory

In order to test the empirical performance of range-frequency theory in the context of income categorization, we generalize the judgement equation (4.3) to

$$J_i^k = \alpha^k + w_R^k R_i^k + w_F^k F_i^k + u_i^k \tag{4.5}$$

Table 4.3 Test on relative deprivation

	No. of incomes larger than the stimulus					Rank correlation[a]	
Stim.	**un**	**no**	**bi**	**ps**	**ns**	r_s	p
135	40	41	41	39	41	.100	.116
199	37	40	40	33	40	−.183	.004
263	34	38	36	27	39	−.277	.000
327	31	35	31	22	38	−.274	.000
390	28	32	26	18	36	−.298	.000
454	25	28	23	15	34	−.171	.007
518	22	23	21	12	32	−.188	.003
582	19	18	20	9	29	−.274	.000
646	16	13	18	7	26	−.359	.000
710	13	9	15	5	23	−.262	.000
773	10	5	10	3	18	−.353	.000
837	7	3	5	2	14	−.122	.055
901	4	1	1	1	8	−.183	.004
965	1	0	0	0	2	−.044	.491

Notes: 1 un = uniform, no = normal, bi = bimodal, ps = positively skewed, ns = negatively skewed. $n = 250$ for all tests.
2 [a]Spearman's rank correlation between the number of incomes larger than the stimulus and the categorization of that stimulus.

where the judgement of stimulus i under income distribution k is given by solving the category assignment equation (4.4) for J_i^k

$$J_i^k = \frac{1}{6}(C_i^k - 1) \tag{4.6}$$

α^k denotes an intercept term, w_R^k and w_F^k denote the weights of the range and the frequency components, respectively, and u_i^k is an error term.

Using equation (4.5), we can test three postulates of range-frequency theory. The first postulate requires the intercept term α^k to equal zero (*neutrality*); that is, we will test

$$\mathcal{H}_0^k : \alpha^k = 0 \quad \text{versus} \quad \mathcal{H}_1^k : \alpha^k \neq 0$$

The second postulate requires the weights to add up to 1 (*additivity*)

$$\mathcal{H}_0^k : w_F^k + w_R^k = 1 \quad \text{versus} \quad \mathcal{H}_1^k : w_F^k + w_R^k \neq 1$$

and the third postulate demands that the weights must be *non-negative* (if additivity holds, this means that they must not exceed one)

$$\mathcal{H}_0^k : w_F^k, w_R^k \geq 0 \quad \text{versus} \quad \mathcal{H}_1^k : w_F^k \leq 0, \quad \text{or} \quad w_R^k \leq 0$$

Furthermore, we can also test on whether different distributions of income stimuli generate different sets of weights (*background context*); that is, we test

$$\mathcal{H}_0^k : w_R^k = w_R^\ell,\ w_F^k = w_F^\ell \quad \text{versus} \quad \mathcal{H}_1^k : w_R^k \neq w_R^\ell,\ w_F^k \neq w_F^\ell \quad \text{for} \quad k \neq \ell$$

In order to test additivity, we estimate a restricted equation

$$J_i^k - F_i^k = \alpha^k + w_R^k(R_i^k - F_i^k) + u_i^k \tag{4.7}$$

in addition to (4.5) and compute the respective F tests. Background context dependence is tested by means of a pooled sample and dummy variables. Note that we did not run any regressions for the uniform distribution since range and frequency values coincide (the numerator of the range equation (4.1) becomes exactly F_i times the denominator of (4.7)).

Table 4.4 OLS estimation of weights

Coefficients			**95% CI w_R**	**Model summary**		**Test on**
α	w_R	w_F	**95% CI w_F**	F^a	R^2	**additivity^b**
			Normal distribution			
**.010	**.939	(.061)	[.853, 1.025]	460.986	.398	
.003	*.044*		—	.000		
−.001	**1.034	−.012	[.895, 1.173]	5814.075	.943	6664.298
.007	*.071*	*.061*	[−.131, .108]	.000		.000
			Bimodal distribution			
**.019	**.765	(.235)	[.612, .918]	96.151	.121	
.003	*.078*		—	.000		
.004	**.823	**.206	[.665, .981]	5357.242	.939	9346.656
.006	*.080*	*.078*	[.052, .359]	.000		.000
			Positively skewed distribution			
−.001	**.850	(.150)	[.777, .923]	523.337	.428	
.006	*.037*		—	.000		
** −.028	**.855	**.187	[.783, .927]	5658.250	.942	6176.862
.009	*.037*	*.038*	[.112, .262]	.000		.000
			Negatively skewed distribution			
*.015	**.783	(.217)	[.692, .874]	284.062	.289	
.008	*.046*		—	.000		
−.002	**.791	**.256	[.701, .882]	3717.255	.914	5065.407
.009	*.046*	*.047*	[.163, .348]	.000		.000

Notes: 1 $n = 700$. *$p \leq .10$, **$p \leq .05$; tested against 0. Above: restricted model; below: unrestricted model. Standard errors in italics.

2 [a]First row: F value, second row: significance level.

3 [b]F value and significance level.

Table 4.4 contains the estimates of the weights using OLS. For every distribution of income stimuli, the table compares the restricted (above) with the unrestricted regression (below). The model summary shows a much better fit of the unrestricted model. Hence, the F test (last column) strongly rejects the null hypothesis of additivity; that is, the restriction $w_F = (1 - w_R)$ does not hold. In all four cases, the sum of the estimated coefficients for R_i and F_i slightly exceeds 1.[10] Hence, we focus our attention on the unrestricted model in the following.

With the exception of the positively skewed distribution, the intercept terms are insignificant as maintained by the neutrality hypothesis. The intercept term of the positively skewed distribution exhibits a negative sign. This means that a positively skewed distribution of stimuli biases subjects' categorizations of incomes downwards. That is, although relative deprivation is lowest and, thus, income categorizations are highest under the positively skewed income distribution, a (relatively small) premium is attached to the judgement function of the positively skewed income distribution. This result is possibly due to an endowment effect (Tversky and Griffin 1991: 117) caused by the relatively low mean income of the positively skewed income distribution.

Except for the normal distribution, the estimated weights of the range and the frequency components are inside the unit interval; that is, the non-negativity hypothesis cannot be rejected. The weight of the range component amounts to about 0.8; that is, distinctly more weight is given to the range component than to the frequency component. With regard to the normal distribution, we observe a weight of the range component larger than 1. Computing the t value for the null hypothesis $w_R - 1 = 0$ shows, however, that w_R does not differ significantly from one. On the other hand, the frequency component does not matter at all for the categorization of incomes.

Eventually, we ran an (unrestricted) pooled regression with the positively skewed distribution as the benchmark case and dummies for the differential intercepts and slopes of the other distributions in order to test on background context. The adjusted R^2 of this regression is .935 ($F = 3645$, $p \leq .01$). As compared to the positively skewed income distribution, the intercept terms of the normal, the bimodal, and the negatively skewed distributions are significantly larger (the t values are between 2.090 and 2.863; $p \leq .05$), which confirms that the neutrality hypothesis is rejected only for the positively skewed income distribution, whose mean income is lowest. Moreover, the pooled regression confirms that the range component is given a significantly greater and the frequency component a significantly smaller weight, respectively, under the normal distribution (the t values are 2.081 and -2.576, respectively; $p \leq .05$). That is, the shape of the normal distribution seems to induce subjects to categorize the stimulus incomes by range alone. The differences between the weights of the bimodal, the positively skewed, and the negatively skewed distributions are not significant (bimodal versus positively skewed: $t = -.345$, $p = .723$ (range), $t = .203$, $p = .839$ (frequency); negatively versus positively skewed: $t = -1.132$, $p = .258$ (range), $t = 1.181$, $p = .238$ (frequency)). For these three income distributions, the structural part of income categorization in terms of the weights entering the judgement function is equal and independent of the shape of the income distribution to be judged. In other words, under the bimodal, the positively skewed, and the negatively skewed income distribution background context matters for the categorization of incomes but not for the judgement function itself.

Summarizing, we find, first, the neutrality hypothesis of range-frequency theory not confirmed for the positively skewed income distribution. However, in the case of the other income distributions we could not reject neutrality. Second, additivity is not supported by our results for all income distributions considered. The component weights are slightly super-additive. The estimates demonstrate that, third, the weights are within the interval $[0, 1]$ and, fourth, not significantly different for the negatively skewed, the positively skewed, and the bimodal income distributions but, fifth, far off from values around $w = .5$ estimated (and sometimes merely assumed) by psychologists (for example, Parducci *et al.* 1960: 74; Parducci and Perrett 1971: 429; Birnbaum 1974a: 92). Instead, the weight of the frequency component is much smaller, being close to .2.

Two reasons can account for the low weight of the frequency component. Firstly, Harris' (1993) conjecture seems to have some justification. Using incomes instead of ratings could have moved subjects' behaviour closer to the linear model. However, relying on real monetary values, Mellers (1986) observed pronounced curvatures of the judgement functions in her work on equitable taxes. Also Parducci *et al.* (1960) and Birnbaum (1974a) found distinct curvatures of the judgement functions of experiments on a size categorization of numerals that ranged within the interval from 108 to 992 (similar to the support of the income distributions used for our experiment).

Secondly, recall that Mellers (1982, 1986) winnowed out the subjects with Aristotelian equity values (which endorsed proportionality for distributive justice). In effect, this means elimination of all subjects who behaved in conformity with the range component only. This had somewhat increased the influence of the frequency component.

Income satisfaction versus well-being: a paradox

Whereas psychologists construct the graphs of the judgement functions or the category assignment functions for the common stimuli only, using the mean category assignments as exhibited in Table 4.2, we construct the graphs of the judgement functions for all 42 stimulus values using the estimates of the unrestricted weights of the range and the frequency components. The respective graphs are shown in Figure 4.1.

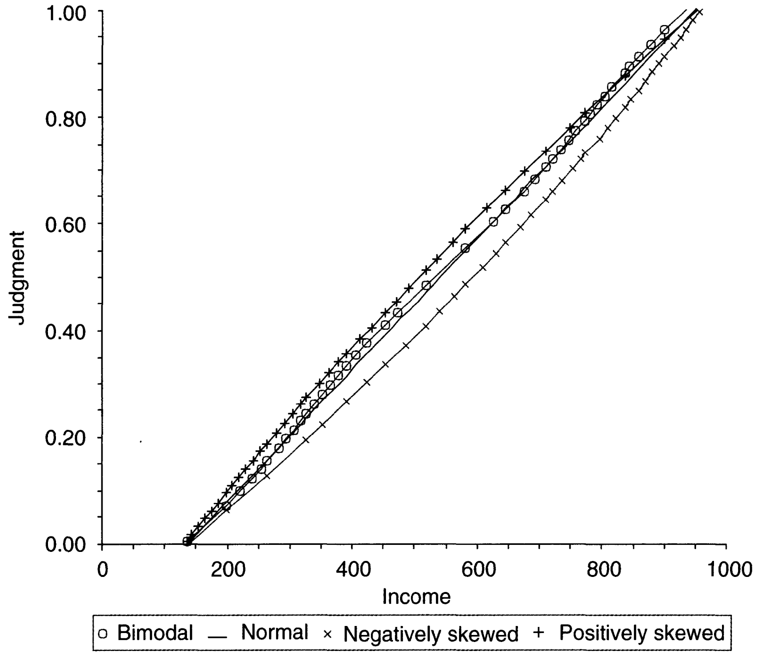

Figure 4.1 Graphs of judgement functions

This figure confirms the message conveyed by the entries in Table 4.2. The graph of the judgement function of the positively skewed income distribution exhibits a concave shape and dominates all other judgement functions up to incomes of about 800 UFO. The graph of the judgement function of the negatively skewed income distribution exhibits a convex shape and is dominated by all other judgement functions over the whole interval of stimulus incomes. For the judgement functions of the normal and the bimodal income distributions we observe linear and S-shaped graphs, respectively. The latter two intersect several times, and lie for most incomes between the graphs of the judgements functions of the positively skewed and the negatively skewed income distributions. For incomes above about 800 UFO, the graph of the judgement function of the bimodal income distributions dominates all other income distributions. Thus, a positively skewed income distribution generates the highest income satisfaction for small and moderate incomes. Concerning the top incomes, the highest income satisfaction is conveyed by a bimodal income distribution. Under a negatively skewed income distribution, personal income satisfaction turns out to be lowest. These observations are perfectly in line with our previous result that a positively skewed income distribution generates less relative deprivation than a negatively skewed one.

Notice that income satisfaction is inverse to the means of the distributions. Mean income is highest for the negatively skewed distribution, yet income satisfaction is lowest. For the positively skewed distribution, the mean income is lowest, yet income satisfaction is highest. The mean income of the other three distributions does not show significant differences and lies in between, as does, by and large, income satisfaction. Does this imply that the positively skewed income distributions, which prevail in the real world, are able to elicit the highest income satisfaction from a given aggregate income? Our experiment even suggests that the negatively skewed distribution elicits the minimum individual income satisfaction from the maximum total income.

However, greater individual income satisfaction does not necessarily imply a higher level of well-being or social welfare within the respective society. Instead, we should aggregate the individual welfare of the income recipients. Applying a Harsanyi-type social welfare function, we sum up individual income satisfaction below and divide the partial sums by the number of income recipients whose incomes do not exceed y_i. Formally, we have

$$\bar{W}(y_i) = \frac{1}{i}\sum_{j=1}^{i} J_j(y_j) \tag{4.8}$$

Accordingly, the graph of $\bar{W}$ shows average social welfare for all income recipients disposing of an income of y_i or less. Figure 4.2 graphically depicts the average well-being of the society under different income distributions.

Figure 4.2 shows that average well-being is highest under a bimodal income distribution for those income recipients who do not dispose of more than about 400 UFO. If we also consider better incomes between 400 and 800 UFO, then the Utopians are best off with a normal income distribution. Eventually, if we take the top earners into account as well, the negatively skewed income distribution generates greatest average well-being.

Comparing the graphs of the judgement functions and average well-being of the positively skewed and the negatively skewed income distributions, we get into a paradoxical situation. Under a positively skewed income distribution every single income recipient, even the top earners, experiences higher individual income satisfaction than under a negatively skewed income distribution; yet, for each stratified subset of subjects, average well-being under a negatively skewed income distribution exceeds average well-being under a positively skewed income distribution.

This is akin to an observation made by Camacho-Cuena *et al.* (2005). When subjects had to assess income distributions as a whole from under a veil of ignorance, they seem to pay attention to all possible incomes to which they may be attributed within an income distribution. This affects their ratings of income distributions. Even for income distributions with identical means, negatively skewed distributions are rated distinctly higher than positively

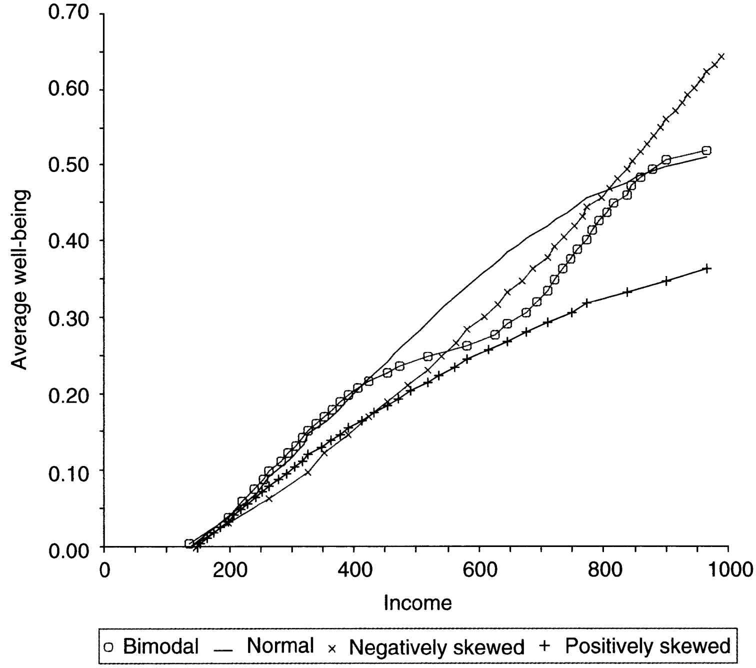

Figure 4.2 Average well-being under different income distributions

skewed distributions, possibly because they offer the better chance to end up at a comparatively satisfactory income level.[11]

Pattern of limens

After subjects had categorized the 14 stimulus incomes, they were told that, for the purpose of future use in Utopia's statistical office, they should state limens for the seven income categories. Moreover, they were told that the limens should properly reflect the income distribution prevailing in Utopia. Note that subjects were not forced to express consistent behaviour in the sense that the upper limen of of a category had to be equal to the lower limen of the following category.[12]

We observed only one category overlap,[13] but several empty intervals between category limens.[14] What might have prompted subjects to behave in this way? They are too many to explain their behaviour simply by error, even more so as these subjects made their responses not only without overlaps between limens, but also with empty intervals between them. Therefore, it seems as if these subjects answered our question under the proviso of making entirely unambiguous statements such as: 'An income between 405 and 506 UFO is certainly insufficient. But for incomes between 331 and 404 UFO I am not entirely sure whether they are still bad or already insufficient. Likewise, for incomes between 507 and 598 UFO, I am not entirely sure whether they are still insufficient. Therefore, to be on the safe side, I only make statements for those areas for which I am entirely confident.'[15]

Table 4.5 lists the means (μ) and medians (M) of the lower and upper limens of the seven income categories for the five income distributions. In analogy to Table 4.2, the test on background context effects with respect to income categorization, the Kruskal–Wallis test (see the last two columns of Table 4.5), shows that background context matters. For nine of the 12 tests conducted, the null hypothesis of the five sets of observations coming from the same distributions has to be rejected ($p \leq .10$).

Comparing Table 4.5 with Table 4.2 and confining ourselves to the middle limens (from 'bad' to 'sufficient'), we see that the negatively skewed distribution of stimuli, which exhibits the lowest category assignments and, therefore, income satisfaction, in Table 4.2, exhibits the highest limens in Table 4.5. It is followed rather indiscriminately by the uniform and the symmetric distributions which rank third in Table 4.2, and then by the bimodal distribution, which occupies rank two in Table 4.2. The positively skewed distribution of stimuli, which exhibits the highest category assignments in Table 4.2, shows the lowest limens in Table 4.5. Thus, the ordering of the limens corresponds, by and large, with the category assignments; subjects behaved consistently in both approaches. This reflects again the influence of the background context on the perception of income limens for the calibration of categories. If the background context exhibits more income mass concentrated among the higher income brackets, subjects become more exacting, which shifts the limens of income categorization in the direction of higher incomes. However, if more income mass is concentrated among the lower income brackets, subjects become more humble as to income categorization; that is, categorial limens are shifted in the direction of lower incomes. Background context of stimuli matters also with respect to the perception of limens of income categorization.

This shows that limen setting reflects *relative deprivation*. Limens are higher the more incomes are greater than the limen incomes. On the other hand, inspection of Table 4.5 reveals that the *total income level*, too, matters. A modest endowment effect is, therefore, also at work. Higher income levels are capable of compensating for enduring a greater number of better off income recipients, which constitutes the second influence on limen setting.

Conclusion

This chapter uses the data gained from an income categorization experiment to investigate background context effects, relative deprivation, range-frequency theory to explain background context effects, individual income

Table 4.5 Limens under different income distributions: Kruskal–Wallis (KW) Test

		Uniform		Normal		Bimodal		Pos.-sk.		Neg.-sk.		KW test	
Category	**Limen**	μ	M	μ	M	μ	M	μ	M	μ	M	χ^2	p
Very bad	Low	(100)	(100)	(100)	(100)	(100)	(100)	(100)	(100)	(100)	(100)	—	—
	Up	200	199	204	200	190	200	188	195	205	200	4.230	.376
Bad	Low	219	200	213	200	201	200	196	200	212	200	3.535	.473
	Up	338	310	339	350	335	310	311	300	352	355	12.129	.016
Insufficient	Low	343	332	343	350	335	311	313	301	363	400	14.251	.007
	Up	468	450	468	473	452	450	430	427	472	500	13.031	.011
Barely sufficient	Low	473	456	470	473	457	455	431	428	482	500	14.552	.006
	Up	599	600	603	600	586	600	565	599	609	605	13.021	.011
Sufficient	Low	607	601	604	600	588	600	570	600	619	646	14.878	.005
	Up	726	750	733	750	704	700	700	700	750	770	21.998	.000
Good	Low	736	750	733	750	707	701	707	705	755	781	21.117	.000
	Up	859	864	856	880	841	850	853	864	862	895	5.834	.212
Excellent	Low	876	898	867	893	851	850	865	877	880	900	8.062	.089
	Up	(965)	(965)	(965)	(965)	(965)	(965)	(965)	(965)	(965)	(965)	—	—

Notes: μ = mean, M = median. $n = 50 \times 5$, $df = 4$ for all tests.

satisfaction versus aggregate well-being, and the dual patterns of income categorization and limen setting.

Five groups of 50 subjects were asked to assign 14 common income stimuli to seven income categories. These common stimuli were embedded in 28 adventitious stimuli to form five different income distributions, uniform, normal, bimodal, positively skewed, and negatively skewed. Each distribution was presented to a group of subjects.

Firstly, we found that background context matters. Using a Kruskal–Wallis test, we had to reject the hypothesis that the five different sets of observations of income categorization came from the same distribution. This means that the background of the 28 adventitious income stimuli had influenced income categorization.

Secondly, we investigated the direction of background context effects, which led us to discover that relative deprivation shapes the pattern of background context. Spearman's rank correlations between income categorization and the number of incomes ahead of the respective stimuli shows a significantly positive relationship. Thus, identical income stimuli are perceived to belong to higher evaluation categories if the background context shifts more income mass to lower income brackets. Conversely, if the background context exhibits more income mass concentrated among higher income strata, then the evaluation of identical income stimuli is downgraded.

Thirdly, background context effects have been explained by means of range-frequency theory, which posits that the categorization of a stimulus is a weighted mean of this stimulus' range and frequency component. We found that neutrality is not confirmed for the positively skewed distribution, which reflects the working of a modest endowment effect. Furthermore, the weights are slightly super-additive and non-negative. The frequency component is ruled out for the normal distribution. For the negatively skewed the positively skewed and the bimodal income distributions, the weight of the frequency component is about .2 and not significantly different for the negatively skewed, the positively skewed, and the bimodal income distributions. This result is remarkable, because for the negatively skewed, the positively skewed and the bimodal income distributions background context matters for the categorization of incomes, but not for the judgement function itself.

Fourthly, we observed a paradox between individual income satisfaction and aggregate well-being. Whereas the judgement functions show that individual income satisfaction is highest for the positively skewed income distribution and lowest for the negatively skewed income distribution, a Harsanyi-type social welfare function demonstrates that average aggregate well-being is higher for all stratified subsets of subjects for the negatively skewed income distribution than for the positively skewed income distribution. This paradox results from the weighting of income satisfaction with the frequency of the involved subjects.

Finally, we found that limen setting of income categories provides a picture that is perfectly consistent with income categorization. This demonstrates that response mode effects are absent for experiments on income categorization on the one hand, and limen setting on the other.

Acknowledgements

This research was supported by a grant from the European Commission under contract ERBFMRXCT 980248. We are indebted to Maria Vittoria Levati for having written the computer program of our experiment and for her suggestions concerning our experimental treatment, as well as to the participants of the Third Annual Meeting of LivinTaX in Madrid for an encouraging discussion. We thank two anonymous referees and Mark McGillivray for helpful comments as well as Sibylle Franz for improving our English writing style. The usual disclaimer applies. The data of this experiment are available from Stefan Traub, e-mail: traub@economics.uni-kiel.de.

Appendix

Instructions and stimulus material

Income evaluation in Utopia[16]

Suppose you live in the future and participate in a space shuttle flight to the planet Utopia, which is inhabited by small green individuals. The local currency in Utopia is the UFO.

Suppose further that each small green individual bears on his or her chest a visible identification card, which (among other information) also shows his or her income. Utopia's constitution states that the lowest allowable income is 100 UFO, while the upper income ceiling is 1000 UFO: nobody must earn less than 100 UFO, and nobody must earn more than 1000 UFO. Consider that 100 UFO is beyond the survival income level and that more income is always preferable.

After your landing on Utopia, you walk around in Utopia's capital, called Haley, and observe the income of several subjects.

> *Then 25 values taken from the true mathematical distribution of the respective group were shown in a random order to allow subjects to become acquainted with the experimental procedure.*

After your short trip through Haley, you meet Utopia's prime minister, who had invited you to consult him with respect to an important issue. As you are an economist (a species completely unknown in Utopia), the prime minister asks you to make an evaluation of the incomes earned in Utopia. He wants you to categorize the incomes earned in Utopia into seven categories, viz.:

1 Excellent;
2 Good;
3 Sufficient;
4 Barely sufficient;
5 Insufficient;
6 Bad;
7 Very bad.

In order to perform this job properly, the prime minister presents you with a list containing a random sample of the incomes of 42 income recipients. You are assured that this sample is a perfect representation of the income distribution in Utopia.

In the following you can see the 42 entries of this list:

> *Now 42 values of the respective experimental distribution were shown in random order. First, the whole set of values was shown on the monitor and thereafter all entries were shown one at a time (in the very same order) and subjects were asked to assign them to one of the above categories. After all values had been assigned to categories, subjects were shown all values together with their categorization and could either confirm or change their categorization. Both the prior and posterior categorizations were recorded.*

The prime minister is quite happy with your categorization of incomes, which enables him to gain insights into the social stratification of Utopia. For future use of Utopia's statistical office, he asks you to state also limens for the seven income categories (notice that there is no inflation in Utopia). For this purpose, he gives you a questionnaire and asks you to fill it in. Your limens should properly reflect the income distribution prevailing in Utopia.

A green individual's income is				
very bad	if it is	less than		UFO
bad	if it is	between	and	UFO
insufficient	if it is	between	and	UFO
barely sufficient	if it is	between	and	UFO
sufficient	if it is	between	and	UFO
good	if it is	between	and	UFO
excellent	if it is	higher than		UFO

After this, your task is done. The prime minister thanks you and awards you the Utopian Order of the Garter in return for your services to his planet.

Notes

1. Anchoring has been studied by Hunt and Volkmann (1937), Rogers (1941), McGarvey (1942/43), Helson (1947). For more recent work compare, for example, Tversky (1974: 154); Tversky and Kahneman (1974: 1128); Quattrone *et al.* (1984); Northcraft and Neale (1987); Green *et al.* (1995); Jacowitz and Kahneman (1995).
2. See, for example, Glaeser and Sacerdote (2000) for criminal sentences that increase if the victim is considered as 'more valuable' or if the offender is considered as 'less valuable'.
3. Relative deprivation was introduced by Stouffer *et al.* (1949), and further elaborated by Runciman (1966). Similar ideas were developed by philosopher Temkin (1986, 1993). Temkin suggests that inequality aversion results from the complaints of income recipients in the low income echelons akin to relative deprivation. In an experimental investigation of the Temkin theory, Devooght (2002) found particular support for the dependence of complaints on the weighted sum of the gaps of incomes in excess of mean income and of mean income.

4. It seems that only Birnbaum (1974a) had paid attention to the reverse side of income categorization. Instead of asking subjects for the limens of income categories, he asked his subjects for their judgements of their *typical numbers* for each category.
5. For instance, the Leyden school has ventured to estimate utility functions or individual welfare functions of income from data of limens of income categories. See, for example, van Praag (1968, 1971), van Praag and Kapteyn (1973), van Herwaarden *et al.* (1977), Kapteyn and van Herwaarden (1980), van Herwaarden and Kapteyn (1981). For a criticism of the Leyden approach, see Seidl (1994). This chapter offers another explanation of the lognormal hypothesis of the Leyden utility function of income; to wit, that it is a reflection of income categorization stemming from everyday experience with positively skewed income distributions. In a seminal study, Birnbaum (1974a) reconciled range-frequency theory with the existence of a psychophysical function that is indeed *invariant* with respect to background context effects. In the realm of income, this function is but a utility function of income. In this view, the lognormal utility function emerges as a manifestation of a unique utility function of income that owes its particular shape to the positively skewed appearance of empirical income distributions.
6. We started at 135 UFO, and formed the sequence using a distance of 64 (in two cases 63) UFO.
7. We neither report the mathematical bases and the experimental values of our stimuli nor the details of the Wilcoxon test here in order to save space. The respective tables are available from the authors on request.
8. Given that the average time subjects needed to complete the experiment was only about a quarter of an hour, the show up fee of €7.5 should have overcompensated subjects for their time and effort. We decided not to employ a performance related incentive scheme in order to pay subjects because, in the case of our experimental design, such a scheme would have been rather complicated and, therefore, possibly harmful rather than eliciting preferences.
9. This assumption is, of course, not unproblematic.
10. This contradicts a result obtained by Parducci *et al.* (1960: 75).
11. For ample experimental evidence see Camacho-Cuena *et al.* (2005), who observed also a preference reversal phenomenon (see Seidl 2002) between the rating and the evaluation of income distributions. For the context of this chapter, categorization of incomes is more akin to rating than to evaluation. Beckman *et al.* (2002) observed less opposition to Pareto improving moves of income distributions when subjects make their judgements under a veil of ignorance. For known positions, opposition to extra income is highest for income gains of persons in a higher income echelon, less for persons in a lower income echelon, and least for own extra income. This finding matches with our results for individual income satisfaction in this work.
12. This is in contrast to the surveys of the Leyden school, where subjects could indicate only one of these two figures.
13. This one instance seems to be an error because this same subject exhibited empty intervals for the other categories.
14. Among our 50 subjects per distribution, we observed 12 subjects with empty intervals for the uniform distribution, 10 for the symmetric distribution, 11 for the bimodal distribution, 9 for the positively skewed distribution, and 14 for the negatively skewed distribution.
15. Obviously Birnbaum (1974a) had anticipated such an attitude. Wisely, he asked his subjects only for their 'typical numbers' for each category. Indeed, if in everyday

life one asks subjects for sufficient incomes, one often gets a representative income level as an answer rather than an income interval.

16. The emphasized text illustrates what the subject was shown on the monitor.

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5

Contradictory Trends in Global Income Inequality: A Tale of Two Biases

Steve Dowrick and Muhammad Akmal

Introduction

An apparent increase in inequality in international income distribution over the 1980s, was highlighted by Korzeniewicz and Moran (1997) and by the United Nations Development Programme in the *Human Development Report* (UNDP 1999). Whilst the former is reluctant to draw strong conclusions about underlying causes, the UNDP report argues for international policies to mitigate rising inequality caused by economic globalization.

For some purposes, such as assessing a nation's capacity to repay foreign debt or its bargaining power in international trade negotiations, the foreign exchange (FX) income comparison may well be appropriate. For the purpose of measuring inequality in living standards, however, we need to take account of the real purchasing power of national currencies, which typically differs from the purchasing power implied by the exchange rate.

A number of studies[1] have shown world inequality to be falling when income comparisons are made using the purchasing power parities of the Penn World Table (PWT5.6).[2] Firebaugh (1999) reports that both the Theil and Gini indexes of intercountry inequality decline between 1965 and 1989, confirming the findings of Schulz (1998) who also reports a decrease in the variance of log income. These results are confirmed by Melchior *et al.* (2000), by Sala-í-Martin (2002b) and by our own calculations using PWT data, which are illustrated in Figure 5.1. We follow previous studies in using population-weighted measures of inequality, conceiving inequality as a function of income gaps between people or households. If we focus on the national economy as the unit of observation, we find that PWT incomes have been diverging.

It is well established that international currency markets tend to undervalue the domestic purchasing power of currencies of low productivity/low income countries. This is the common experience of international travellers who find that their money will go much further in India or Indonesia than it will in Western Europe. The phenomenon has been analyzed by

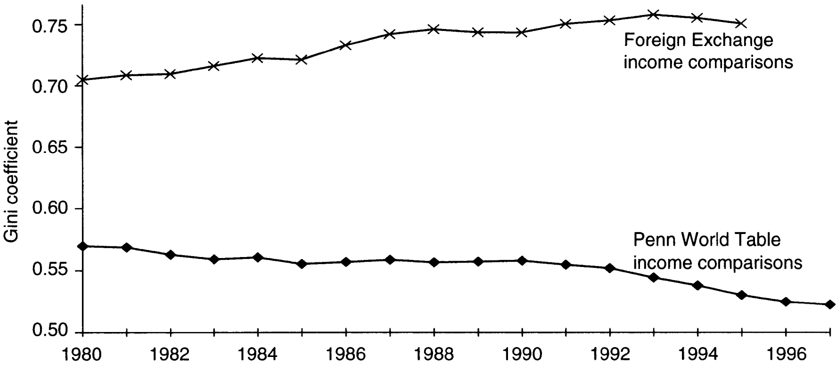


Figure 5.1 Inter-country income inequality, 1980–97

Notes: 1 'Foreign Exchange income' is GDP at market prices (current US$) divided by population. Dollar figures for GDP are converted from domestic currencies using single year official exchange rates.

2 'Penn World Table income' is real GDP per capita in constant dollars (international prices, base year 1985) derived from PWT5.6. Missing data are calculated by Global Development Network from 1985, GDP per capita and GDP per capita growth rates using Global Development Finance and World Development Indicators.

3 Gini coefficients are calculated on per capita GDP for 115 countries using population weights.

Sources: World Bank (2001), Global Development Network database: Macro Time Series.

Balassa (1964), Samuelson (1964) and Bhagwati (1984). Real wages are low in countries with low labour productivity, so non-traded labour-intensive services are cheap relative to capital-intensive traded goods. Market exchange rates are more likely to equate prices across countries in the traded sector rather than in the non-traded sector of the economy. Consequently, markets tend to undervalue the domestic purchasing power of the currencies of poor countries. An important implication of this 'traded sector bias' is that FX income comparisons, because they understate the real incomes of poorer economies, will overstate the degree of international inequality.

Alternatives to FX income comparisons rely on the estimation of purchasing power parities. A massive research effort, based on detailed price surveys in many countries under the auspices of the International Comparison Program (ICP), has resulted in the publication of the Penn World Table (PWT) which provides ready access to measures of real GDP per capita at constant international prices for over one hundred countries. These data are commonly referred to as PPP (purchasing power parity) measures of real income.

Many users of the PWT data are unaware, however, that attempts to measure purchasing power are problematic. The PWT is based on the Geary–Khamis (GK) method of construction of 'average international prices' for a benchmark year. The GDP of each country in each year is valued at these fixed prices. However, constant price valuations introduce systematic bias by ignoring consumers' ability to substitute towards goods and services that are locally cheap. The progenitors of the Penn World Table describe the problem of substitution bias thus:

> The issue arises out of a familiar problem in price and quantity index number construction.... Valuation at other than own prices tends to inflate the aggregate value of the bundle of goods because no allowance is made for the substitutions in quantities toward the goods that are relatively cheap. ... The practical importance of this issue ... may loom large in comparisons between countries that have widely divergent price and quantity structures. (Kravis *et al.* 1982: 7)

Use of the PWT estimates of international incomes, whilst avoiding the traded sector bias in the FX income data, introduces substitution bias in its place. We suspect that this issue may be very significant when it comes to assessing the level of and trends in world inequality, because we know that the price and quantity structures in the world's poorest economies are very different from those typically obtaining in the richer industrialized economies, and we expect that price differentials vary over time.

Take, for example, the local currency prices for domestic services and for passenger cars, comparing an archetypal labour-intensive service with an archetypal traded good, in a sample of countries from the 1980 ICP survey. These prices are listed in the first two rows of Table 5.1. Local prices are for the real quantity that could be purchased for one GK international dollar.

The last row of Table 5.1 shows the price of domestic services as a ratio of the price of cars. We find that this price ratio varies by a factor of fifty

Table 5.1 Relative prices of traded goods and non-traded services for six countries, 1980

	Canada	**Germany**	**Brazil**	**Korea**	**Indonesia**	**India**
Per capita GDP at GK international prices	*$11573*	*$10390*	*$3758*	*$2393*	*$1101*	*$605*
Local currency prices						
Domestic services	1.44	3.23	7.68	116.2	70.4	0.45
Passenger cars	1.00	1.97	22.13	612.3	669.4	14.56
Price ratio						
Services/cars	1.44	1.63	0.35	0.19	0.11	0.03

Note: Local currency prices are defined as the number of units of a country's currency required to purchase the same amounts of goods and services as one US dollar would purchase in the USA.

Source: World Bank (1993), Tables 4 and 5.

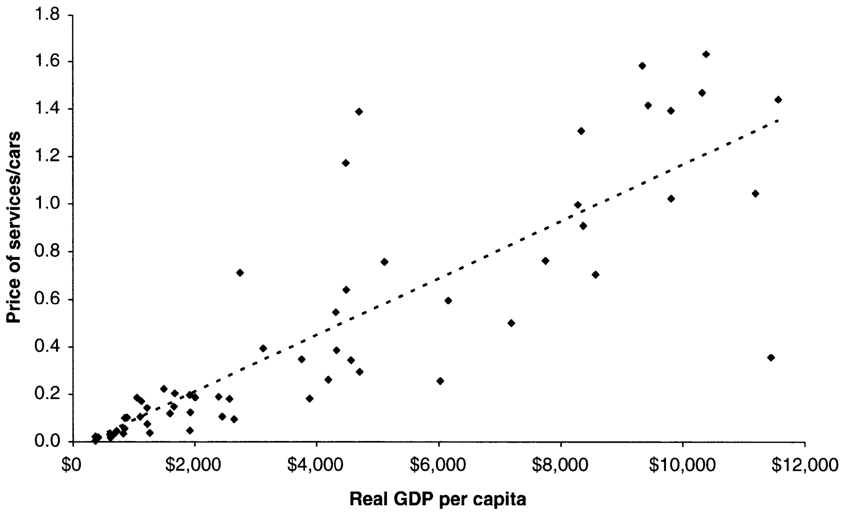


Figure 5.2 Price of domestic services relative to cars, 1980

Notes: 1 Real GDP per capita is measured in GK international dollars.

2 The price ratio is between the categories 'domestic services' and 'passenger cars'.

Source: World Bank (1993).

between Germany and India. In Figure 5.2 we show a plot of this price ratio against per capita GDP for all of the 60 countries in the 1980 ICP survey, confirming that there is a very strong tendency for non-traded services to be much cheaper in low income countries than in rich countries.

Given this massive variation in relative prices, the problem of substitution bias is likely to be substantial. In the next section, we develop a simple model that captures the biases in both the FX and GK measures of inequality. We find that whilst the Balassa–Samuelson effect causes FX comparisons to exaggerate inequality, the opposite bias is engendered by the GK method of measuring purchasing power parities. This suggests an explanation for the contradictory observations of trends in income inequality. If true inequality stays approximately constant, but measurement bias increases due to prices becoming less similar over time, then the FX method will show rising inequality whilst the GK method will indicate falling inequality.

We test this hypothesis in the subsequent sections. First, we investigate whether or not national price structures have diverged over the past few decades – a necessary condition for our hypothesis to hold. Second, we use the multilateral true index methodology of Dowrick and Quiggin (1997) to yield true PPP income comparisons that are free of both substitution bias and traded sector bias to test the conjecture that true inequality has been approximately constant.

A model of sectoral and substitution bias in international income comparisons

In order better to understand the sources of both FX and substitution bias, we construct a simple model of two trading economies. Each produces a non-traded labour-intensive service, S. Country 1 has comparative advantage in producing an intermediate good, A, which we might think of as an unprocessed agricultural or mineral product. Both countries manufacture a final tradable good, M, using labour and the intermediate good. The production technologies exhibit constant returns to scale and are identical across countries, except that country-specific private knowledge determines labour productivity in manufacturing. We define country 2 as the high productivity country.

To keep the model simple we assume Cobb–Douglas production functions in manufacturing, we treat labour as the only factor of production, we disregard transport costs for trading the intermediate and manufactured goods and we assume competitive pricing behaviour in product and labour markets, including free trade. We assume that all goods and services must be produced, traded and consumed within the one time period. The production side of the economy in country i can be summarized as follows:

$$S^i = L^i_s; \quad A^1 = L^1_a; \quad M^i = (\lambda^i L^i_m)^\alpha \ (A^i_m)^{1-\alpha} \quad 0 < \alpha < 1 \tag{5.1}$$

where $Z^i = (S^i, A^i, M^i)$ represent the domestic output of non-traded services, the traded intermediate product and the traded manufactured product respectively in country i; L^i_z represents the amount of labour employed in each sector; A^i_m is the amount of intermediate input used in manufacturing; and λ^i is the productivity of labour in country i's manufacturing sector.

Comparative advantage dictates that country 1 will export the intermediate good and import manufactures. We assume that the productivity differential and relative population size are such that it is feasible for country 1 to produce all of the intermediate good demanded in both countries.

Given constant returns to scale and competitive pricing, we can solve for the domestic price of the manufactured good in country i, P^i_m, in terms of the input prices for labour and the intermediate good, w^i and P^i_a (see Appendix for details):

$$P^i_m = \mu(w^i/\lambda^i)^\alpha (P^i_a)^{1-\alpha}; \quad \text{where}\ \mu = \alpha^{-\alpha}(1-\alpha)^{\alpha-1} \geq 1 \tag{5.2}$$

We normalize prices and productivity by setting the wage and productivity level in country 1 to unity. We can then use $\lambda(> 1)$ without a superscript to represent manufacturing labour productivity in country 2. This allows us to derive the price vector for country 1 as

$$\mathbf{P}^1 \equiv (P^1_s, P^1_a, P^1_m) = (1, 1, \mu) \tag{5.3}$$

The exchange rate is E units of currency 2 per unit of currency 1. The price of the imported intermediate good in country 2 is E. This determines the price of the manufactured good, using (5.2) as

$$P_m^2 = \mu \left(\frac{w^2}{\lambda} \right)^{\alpha} E^{1-\alpha} \tag{5.4}$$

Free trade in the manufactured good equalizes prices across countries, requiring $P_m^2 = E.P_m^1 = \mu E$. These conditions fully determine the wage in country 2. Setting the right hand side of (5.4) equal to μE yields

$$w^2 = \lambda E \tag{5.5}$$

That is to say, productivity adjusted factor prices are equalized across the traded sectors.

By assumption, there are no differences across countries in the productivity of labour in the production of non-traded services. The price of services is simply the wage. It follows that services are relatively expensive in the high productivity, high wage country where the price vector, in units of country 1's currency, is

$$\mathbf{P}^2 \equiv E(\lambda, 1, \mu) \tag{5.6}$$

We analyze demand and welfare by assuming common Cobb–Douglas preferences for the representative consumer–producer who is supplying a unit of labour inelastically

$$U^i(s^i, m^i) = (s^i)^{\beta}(m^i)^{1-\beta} \quad 0 < \beta < 1 \tag{5.7}$$

where *s* and *m* refer to per capita consumption of services and manufactured goods.

The budget share of services is β in each country. Given that per capita income in each country equals the wage, the per capita consumption bundles are

$$\mathbf{q}^1 \equiv [s^1, m^1] = \left[\beta, \frac{1-\beta}{\mu}\right]; \quad \mathbf{q}^2 \equiv [s^2, m^2] = \left[\beta, \frac{\lambda(1-\beta)}{\mu}\right] \tag{5.8}$$

Per capita consumption of services is identical in the two countries, despite the fact that services are more expensive in country 2, because the income effect of higher manufacturing productivity offsets the price effect. This exact offsetting is an artefact of the Cobb–Douglas production and utility functions, but it is not crucial to our results. There is higher per capita consumption of manufactures in the higher productivity country. Thus, we can refer to the higher productivity country as the high income or richer country.

Evaluating the common utility function (5.7) at $\mathbf{q}^1$ and $\mathbf{q}^2$ gives the welfare ranking $U^2 > U^1$. In general, the utility ratio, U^2/U^1, is greater than unity but otherwise indeterminate because the utility function is ordinal rather than cardinal. Cardinality is achieved by the money metric Allen welfare index:

$$A_r^{2:1} \equiv \frac{e[U(q^2), p^r]}{e[U(q^1), p^r]} \tag{5.9}$$

which compares the minimum expenditures required to achieve utilities U^2 and U^1 at some reference price vector, $\mathbf{p}^r$. In our case, where the assumed preference function is homothetic, the Allen index is independent of the reference price vector. The true per capita income ratio between country 2 and country 1 is the unique value

$$A^{2:1} = \frac{U(q^2)}{U(q^1)} = \lambda^{1-\beta} > 1 \tag{5.10}$$

These findings are summarized in the following proposition which is a restatement of the Balassa–Samuelson results:

Proposition 1 *With free trade in intermediate and manufactured goods and competitive pricing, the country with higher productivity in manufacturing will exhibit the following features:*

(i) *per capita utility is higher;*
(ii) *non-traded labour intensive services are more expensive relative to traded goods.*

Foreign exchange rate comparisons of incomes

In this model, per capita National Income and Gross Domestic Product are identical and, measured in local currencies, are simply equal to the wage. So the GDP or income ratio that is obtained from exchange rate comparison is simply the ratio of the wage levels expressed in a common currency. Noting that the wage in country 1 is normalized to unity and substituting in (5.5) for wages in country 2, we derive the FX income ratio

$$FX^{2:1} = \frac{w^2}{Ew^1} = \lambda \tag{5.11}$$

Proposition 2 Non-traded sector bias in FX comparisons

(i) *Market exchange rates overstate true international income differentials;*
(ii) *The magnitude of the bias is an increasing function of: (a) the underlying productivity differential between the countries; and (b) the domestic expenditure share of the non-traded sector.*

Proof From (5.10) and (5.11), given $0 < \beta < 1$ and $\lambda > 1$,

$$\frac{FX^{2:1}}{A^{2:1}} = \lambda^{\beta} > 1 \tag{5.12}$$

The first part of this proposition simply restates the more general result found in Samuelson (1974). The overstatement of true income differentials is due to 'traded sector bias'. The market exchange rate equates purchasing power of currencies over the traded good but overstates the purchasing power of the currency over non-traded services in the rich country where non-traded labour intensive services are relatively expensive. It follows that the use of market exchange rates to compare incomes exaggerates the degree of international inequality. Our parameterization of the model leads to the unsurprising conclusion that the extent of this bias is increasing in both the productivity differential λ (which causes the real wage differential) and the size of the non-traded sector β (where the real wage differential affects the relative price). □

Geary–Khamis comparisons of incomes

We turn now to the measurement of the international income ratio by the Geary–Khamis (GK) method, which is used to construct the PWT. This method values each country's GDP bundle at 'international prices'. The international price of manufactures, relative to services, is constructed as a quantity weighted average of the relative prices of all the countries in the GK system. For the purposes of our model, we have considered only two countries but we can allow for other countries with a range of productivity levels in the GK system.

We represent the GK final consumption price vector as:

$$\boldsymbol{p}^{GK}(g) \equiv \left[p_S^{GK}, p_M^{GK}\right] = [g, \mu] \tag{5.13}$$

Referring back to equation (5.6), we see that the relative price of manufactures, μ/g, corresponds to the relative price that would be found in a country where the manufacturing productivity parameter is g. If the GK system is dominated by countries more productive than country 2, g will be greater than λ. If the rest of the world is less productive than country 1, g will be less than unity.

The GK measure of real GDP per capita for country i is the per capita consumption bundle evaluated at international prices: $(\mathbf{q}^i)'\mathbf{P}^{GK}$. Evaluating the consumption bundles given in (5.8) at prices $[g, \mu]$, the GK income ratio between countries 1 and 2 is

$$GK^{2:1}(g) \equiv \frac{GK^2(g)}{GK^1(g)} = \frac{\beta g + (1-\beta)\lambda}{\beta g + (1-\beta)} \tag{5.14}$$

Whether this under- or overstates the true income ratio depends on the value of g. We summarize the relationship in our third proposition.

Proposition 3 Substitution bias in Geary–Khamis comparisons

(i) *A bilateral international comparison of per capita income that values expenditure at constant prices will understate the true income differential if the constant price vector corresponds to that of the high productivity country, or the prices of an even richer country;*
(ii) *A constant price comparison will overstate the true income differential if the constant price vector corresponds to that of the low productivity country, or the prices of an even poorer country;*
(iii) *The bias is greater, the less similar is the reference price vector with respect to the comparison country prices;*
(iv) *Where (i) or (ii) holds, the magnitude of the bias is an increasing function of the underlying productivity differential between the two comparison countries.*

Proof See Appendix for details. The ratio of the constant price (GK) income ratio to the true income ratio is R(g):

$$R(g) \equiv \frac{GK^{2:1}(g)}{A^{2:1}} = \frac{\beta g + (1-\beta)\lambda}{[\beta g + (1-\beta)]\lambda^{1-\beta}} \tag{5.15}$$

$R(g)$ measures the proportional bias in the GK index, with $R = 1$ representing no bias. Evaluating (5.15) gives $R(1) > 1$ and $R(\lambda) < 1$ (for $\lambda > 1$ and $0 < \beta < 1$, as assumed). Differentiating (5.15), $R_g < 0$. Hence (i) and (ii). □

R is less than 1 for all $g > \lambda$. As g rises above λ (i.e., as the reference price vector becomes less similar to prices in countries 1 and 2) R falls. R is greater than 1 for all $g < 1$. As g falls below unity (i.e., as the reference price vector becomes less similar to prices in countries 1 and 2), R rises. Hence (iii). □

R is decreasing/increasing in the productivity differential, λ, as $\lambda < g$ or $\lambda > g$. Hence (iv). □

Proposition 3 clarifies the nature of substitution bias in fixed price comparisons. It is well known that the use of country 1's prices is likely to exaggerate country 2's welfare, since goods that are in high demand in 2, because of their relative cheapness, will be overvalued at 1's prices. In other words, the Laspeyres quantity index is usually larger then the Paasche index – and must be so if the underlying preferences are common and homothetic. It follows that valuing demand at country 1's prices will tend to overstate true inequality, if 1 is poorer than 2, and vice versa. This implication of substitution bias in the measurement of inequality is sometimes referred to as the *Gerschenkron Effect*, after Gerschenkron (1951).

Nuxoll (1994) has shown that the Gerschenkron effect will apply when the income ratio between country 1 and country 2 is measured at the prices of some third country, if relative prices and quantities are inversely correlated across all three countries. Proposition 3 formalizes Nuxoll's result in the context of an explicit model where prices, quantities and the true income ratio are endogenously determined by tastes and technology.

It follows from Proposition 3 that the direction and magnitude of bias in GK bilateral income ratios depends on whether the GK price vector corresponds most closely to the relative price structures of high-income (high-productivity) countries – in which case most bilateral ratios will be underestimated – or whether the GK price vector corresponds most closely to the relative price structures of low-income (low-productivity) countries – in which case most bilateral ratios will be overestimated. The former situation is most likely to apply given that the GK method weights each country's price vector by its share in total GDP, implying that more weight is given, *ceteris paribus*, to the price vectors of the richer countries.

The predicted magnitude of both fixed price substitution bias and FX bias is illustrated in Figure 5.3 for the case where non-traded services comprise half of total expenditure; that is, $\beta = 0.5$. The horizontal axis measures λ, the productivity differential between the manufacturing sectors of the two

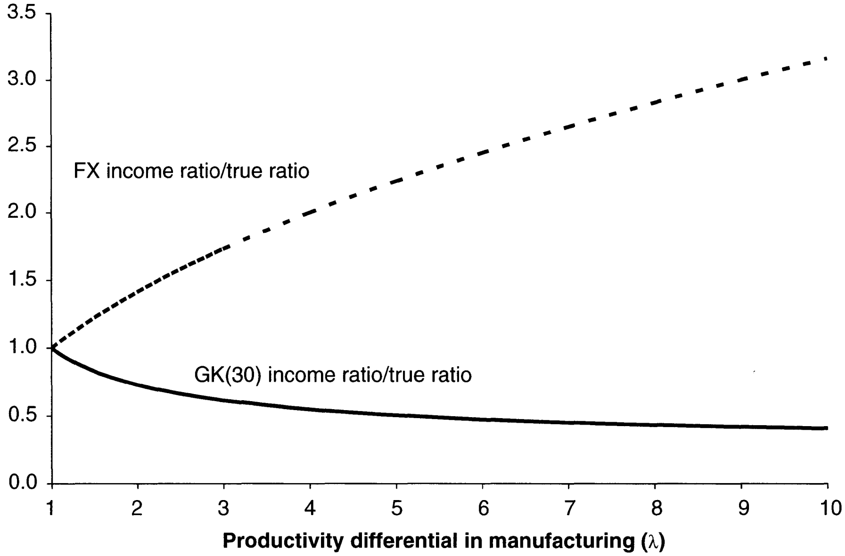


Figure 5.3 Predicted exchange rate bias and substitution bias

Note: Predicted biases are calculated according to equations (5.12) and (5.15) with the parameters set as $\beta = 0.5$ and $g = 30$.

countries, which is illustrated for values between 1 and 10. The uppermost line shows the extent of positive bias in the foreign exchange comparison: $FX^{2:1}/A^{2:1}$. If the productivity differential is large (for example, if $\lambda = 10$), FX comparisons overstate the true income ratio more than threefold. The lower line shows bias in GK income comparisons, $GK^{2:1}/A^{2:1}$, using constant prices that correspond to those of a high productivity country with a productivity parameter, g, of 30. In this case, the GK method understates the true income ratio. This bias can be substantial; for example, it reaches a ratio of 0.4 when $\lambda = 10$.

The biases illustrated here refer to the measurement of the income ratio between a pair of countries. We expect to find similar bias in measures of multilateral inequality because most measures are constructed from bilateral ratios.

Can we explain increasing FX inequality and decreasing PWT inequality?

We have demonstrated that there are potentially serious biases in the two most commonly used methods of comparing international incomes. Moreover, it is likely that the biases may work in opposite directions, with the FX income measure overstating the true level of inequality whilst the GK measure understates inequality. This occurs when the GK method is evaluating GDP at a set of 'international' prices that correspond to the price structure of a high productivity economy.

Nuxoll (1994) analyzes bilateral income ratios and reports on page 1431 that the international prices underlying the 1980 ICP measures of real GDP correspond to those of 'some moderately prosperous' country such as Hungary or Yugoslavia, with per capita GDP of around $5,000. We apply a similar analysis to a multilateral measure, the variance of log income. We use the 1980 ICP data to construct measures of the variance of constant price log GDP per capita across the 60 countries, using first the prices of the USA and then the prices of each of the other 59 countries. The results are shown in Figure 5.4, with the variance on the vertical axis and the log of GK GDP per capita for the reference country on the horizontal axis. As predicted by our Proposition 3, the higher is the income of the reference country, the lower is the measured variance of log income across countries. The measured variances range from 0.85 to nearly 1.6. The empirical relationship is very strong with the linear regression line explaining more than 80 per cent of the variation in measured inequality.

In Figure 5.4 we have also illustrated the variance of 1980 log incomes from PWT5.6 for the same countries, represented by the solid horizontal line at 1.02. This coincides with the regression line at a log income of 9.1, corresponding to income of $\exp(9.1) \approx \$9{,}000$. It appears that the GK constant international price vector underlying this version of the PWT is most closely

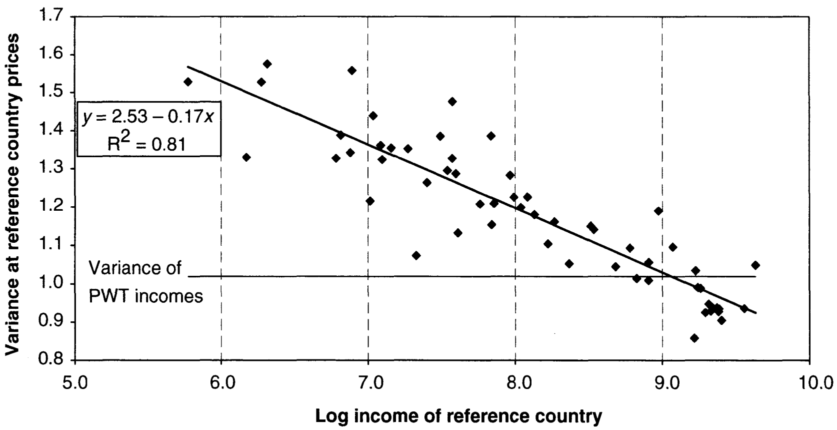


Figure 5.4 Variance of 1980 log real GDP per capita at reference country prices

Notes: 1 'Income' is real GDP at GK international prices.

2 The 'variance at reference country prices' is the variance of log GDP per capita measured for each country at the prices of the reference country.

Source: World Bank (1993) Tables 4 and 5 for 1980 ICP data.

represented by the price structure of a relatively rich country such as Hong Kong, Japan or the UK with 1980 per capita income around $9,000. This is not surprising, since the rich OECD economies, with average income levels above $13,000, accounted for more than half of world GDP, exerting a dominant influence on the construction of the GK international price vector. More than two thirds of the countries have income levels lower than $9,000, suggesting that most of the GK income ratios are likely to be biased downwards, understating the true level of inequality – as demonstrated by Dowrick and Quiggin (1997) and by Hill (2000).

We have shown that the degrees of bias in both FX and GK income measures are likely to increase as the true level of inequality increases. This suggests one possible explanation for the conflicting messages coming from the analysis of FX and GK income inequality. If true inequality is rising over time, both the upward FX bias and the downward GK bias will increase. It follows that the rate of increase in inequality will be exaggerated by FX measures and understated by the GK measure.

On its own, this hypothesis is not enough to explain why FX inequality has been trending up and GK inequality trending down. In the earlier simple model, where there are only two consumption goods, both the FX and GK measures move in the same direction when true inequality changes. In reality, however, there is a multiplicity of goods and relative prices which are influenced to varying degrees by trade restrictions, by international differences in productivity and factor endowments, by monopoly pricing, by government regulation, and so on. For instance, Falvey and Gemmell (1996) report that international differences in the price of services are explained by differences in factor endowments as well as differences in total factor productivity and Dowrick and Quiggin (1997) find that the degree of substitution bias in fixed price income comparisons is increasing in the dissimilarity between countries' price vectors. It follows that the degree of bias in both FX and GK measures is likely to increase not only when underlying productivity differentials increase but also when national price structures diverge for other reasons. Thus, we propose an explanation for the riddle of GK inequality falling whilst FX inequality rises:

Conjecture

(i) True inequality changed little between 1980 and 1993; but
(ii) national price structures became less similar;

causing the FX measure of world inequality to rise and the GK measure of inequality to fall.

Firebaugh (1999) discounts this possibility, citing the finding of Dowrick and Quiggin (1997) that price structures had become more similar over the 1980s. Those findings were, however, only for the OECD countries where price convergence had been promoted by European economic integration. There is no presumption of price convergence across the rest of the world or between OECD countries and the rest.

We investigate trends in price similarity across the world using data from the Penn World Table 5.6 on the relative prices of private consumption, government consumption and investment for over one hundred countries. We use two measures of price similarity. First is the Kravis *et al.* (1982: hereafter KHS) definition of bilateral price similarity as the cosine of the angle between a pair of price vectors. For country i, we define its price similarity, $\mathbf{PS}^{\mathrm{GK}}$, in relation to the GK price vector, $\mathbf{P}^{\mathrm{GK}}$, as

$$PS_i^{GK} = \frac{\sum_{j=1}^{J} w_j P_j^i P_j^{GK}}{\sqrt{\sum_{j=1}^{J} w_j (P_j^i)^2 \sum_{j=1}^{J} w_j (P_j^{GK})^2}} \tag{5.16}$$

with j indexing the categories of expenditure and w_j representing the share of total expenditure by all countries on that category.

Second, we use the Diewert (2002) bilateral index of relative price dissimilarity (the unweighted asymptotically linear index), $D(\mathrm{x}, \mathbf{g})$, which is defined as

$$D(x,g) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{\mu g_i}{x_i} + \frac{x_i}{\mu g_i} - 2\right)$$
$$\mu = \prod_{i=1}^{n}\left(\frac{x_i}{g_i}\right)^{\frac{1}{n}} \tag{5.17}$$

where x represents the price vector of country x and g represents the GK price vector, and $i = 1 \ldots n$ indexes the commodities that make up the GDP bundle. Note that μ is defined as the geometric mean of x relative to g, and the dissimilarity index is the sum of the normalized price ratios and their reciprocals, adjusted to make the index equal zero when the prices in country x are an exact multiple of the GK prices.

Figure 5.5 displays the time trends between 1980 and 1991 for the crosscountry means of the Diewert and KHS indices. The time trends are almost mirror images of each other because the former index measures dissimilarity rather than similarity. It is evident that between 1980 and 1991 price structures became markedly less similar/more dissimilar,[3] confirming the second part of our conjecture.

To investigate the other part of our conjecture, we need measures of international incomes that are free of both substitution bias and traded sector bias.

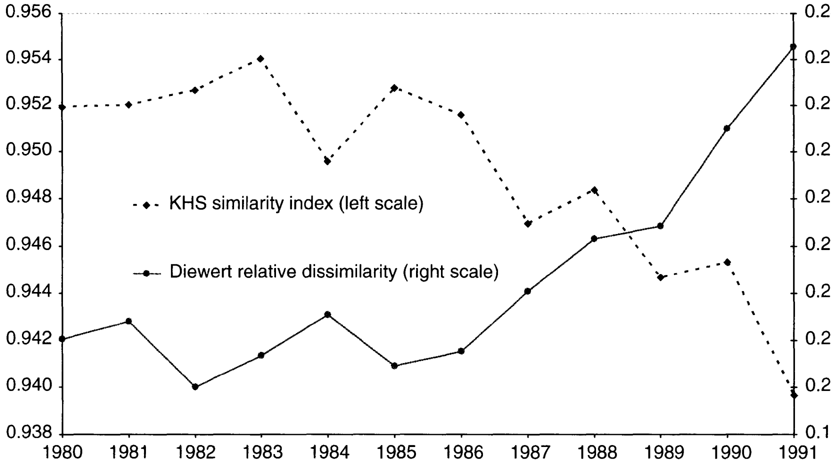


Figure 5.5 Price similarity and dissimilarity indexes across 131 countries, 1980–91

Note: The index number formulae are equations (5.16) and (5.17). There are 131 country observations 1980–89, 102 for 1990 and 100 for 1991.

Source: Prices of private consumption, government consumption and investment are from PWT5.6.

Dowrick and Quiggin (1997) develop a method that achieves this, based on the economic index number approach of Afriat (1984). This method involves searching for multilateral index numbers satisfying the condition that the bilateral ratios lie within the Paasche and Laspeyres bounds, making use of the algorithm described by Varian (1983). If such an index exists, the underlying price and quantity observations are consistent with optimizing choice by a representative consumer with homothetic preferences and the index numbers can be interpreted as an Allen welfare index. We apply this method to the 1980 and 1993 ICP benchmark studies.[4]

We find that for a majority of countries (52 out of the 60 countries in the 1980 benchmark study, and 49 out of the 53 countries in the 1993 study) the ICP observations on prices and quantities do satisfy the Afriat test for common homothetic preferences. For this set of observations the Allen money metric utility comparisons, $E(u(\mathbf{q}_i), \mathbf{p}_r)/E(u(\mathbf{q}_j), \mathbf{p}_r)$, are independent of the choice of $\mathbf{p}_r$. We then follow Dowrick and Quiggin (1997) in constructing the Ideal Afriat Index, which gives utility consistent income ratios[5] that are free of substitution bias, reflecting true purchasing power. We refer to this measure as Afriat income or, following Afriat (1984) and Dowrick and Quiggin (1997), as 'true' income.

We note the finding of Ackland *et al.* (2004) that the Elteto–Köves–Szulc (EKS) index is close to being a true Afriat index when applied to the 1993 ICP data. We confirm this finding for the 1980 ICP data where the variance of log income measured by the EKS index is 1.18, only slightly higher than the variance of log income measured by the ideal Afriat index. Indeed, a linear regression of the former index on the latter yields the result: ln EKS = 1.009 ln Afriat, $R^2 = 0.9989$. These results suggest that the subsequent data analysis would be altered very little if we were to substitute the EKS index for the Afriat index. We prefer to use the Afriat index because it is consistent with generalized homothetic preferences, as in our theoretical model, and because we are able to test whether the data are consistent with such preferences.

Estimating Afriat GDP for non-benchmark countries

We are able to calculate true GDP per capita for all of the countries included in the 1980 and 1993 ICP surveys, using the minimum of the Laspeyres valuations for those countries outside the homothetic sets as recommended by Dowrick and Quiggin (1997). However, in order to get genuinely representative estimates of world inequality, it is necessary to increase population coverage, requiring the extension of PPP estimates to non-benchmark countries. This is especially important because China is not covered in either of the ICP surveys and India is only in the 1980 benchmark.

In order to predict true incomes for non-benchmark countries, we use a procedure developed by KHS[6] who use ICP benchmark data to estimate a regression model with GK income as the dependent variable and FX income as an explanatory variable.

Our model suggests that FX overstates true income for poorer countries. Substituting (5.11) into (5.10), and normalizing true income in country 1 to unity, implies that true income per capita in country i, A^i, is

$$A^i = (FX^i)^{1-\beta} \Rightarrow \ln A^i = (1-\beta)\ln FX^i; 0 < \beta < 1 \tag{5.18}$$

We use this log-linear relationship as the basis of a regression model, augmenting it with a variable, OPEN, capturing the exposure of the country to foreign trade. Deviations between FX and A are driven by price differences between the traded and non-traded sectors of the economy – so where the traded sector constitutes a larger proportion of GDP, we expect the A/FX ratio to be closer to unity. We regress log A on log FX and OPEN for each of the two benchmark years separately, testing for functional form by including the square of log FX and the interaction of OPEN with log FX. Then, data for the two years is combined to obtain pooled estimates.

The regression results are reported in Table 5.2. Following tests for heteroscedasticity, the 1980 equation and the pooled model were estimated using weighted least squares whereas the 1993 equation was estimated by OLS.

The logarithm of FX income is highly significant in all equations. As expected, the coefficient is less than unity – varying between 0.6 and 0.8 according to the sample – implying that the true income distribution is less dispersed than the distribution of FX income.

The square of log FX adds significant explanatory power to the regression only in the 1980 sample. The openness variable is statistically significant in the 1993 regression, along with the interaction term, implying that there is less of a gap between FX income and true income in economies that are more exposed to world trade. The explanatory power, as defined by $\bar{R}^2$, is close to 97 per cent for the 1993 sample and the standard error of the regression is around 16 per cent which is very similar to the standard errors for the KHS regression reported in the final column of Table 5.2 and the regressions underlying the construction of PWT version 5.6 as reported in the December 1994 update to Appendix B of Summers and Heston (1991). The 1993 data are displayed in Figure 5.6 along with the predicted value of the regression. We see that a linear relationship between log A and log FX, as predicted by (5.18), fits the data reasonably well, with only minor additional explanatory power coming from the additional variables.

It is not clear why the OPEN variable is not significant in the 1980 sample, when we have a strong prior that the level of openness should be important in reducing the gap between FX and PPP incomes. One possible explanation is that we have misspecified the relationship between FX income and true income, perhaps through omission of variables or through incorrect functional form. This poses a problem when it comes to predicting true incomes for the non-benchmark countries in each year. Should we use the particular

Table 5.2 Estimating the relationship between Afriat or GK income and FX income

A Regression results

	1980	**1993**	**Pooled data**	**GK comparison**[3]
Estimation by	**Weighted least squares**[4]	**OLS**	**Weighted least squares**[4]	**OLS**
Intercept	0.075	−0.137	−0.044	*−0.086*
	(1.36)	(2.75)	(0.70)	*(0.87)*
D93			−0.121**	
			(2.25)	
Log(FX)	0.795***	0.632***	0.742***	*0.493****
	(27.1)	(16.3)	(16.3)	*(4.3)*
[Log(FX)]^2	−0.045*			*−0.047**
	(1.89)			*(1.84)*
OPEN		0.200***	0.163*	
		(2.71)	(1.77)	
		(2.71)	(1.77)	
OPEN × Log(FX)		−0.104*	−0.100	
		(1.79)	(1.53)	
Sample size	60	48	108	*34*
Standard error of regression	0.251	0.159	0.270	*0.157*
$\bar{R}^2$	0.936	0.968	0.915	*0.967*

Notes: 1 The dependent variable is log Afriat GDP per capita in columns 1–3, and the GK measure in column 4.

2 t-statistics are given in parenthesis; ***, ** and * indicate significance at the 1, 5 and 10 per cent levels respectively.

3 The regression summarized in column 4 is for 1975 GK measures of real GDP; it also contained statistically insignificant openness variables.

4 To take account of heteroscedasticity, the WLS are the predicted values of the absolute values of the residuals from the OLS regressions regressed on log FX.

Sources: Authors' estimation columns 1–3. Kravis *et al.* (1982: 335) for column 4.

B Descriptive statistics

	Mean	**Std dev.**	**Min.**	**Max.**
1980				
log income (Afriat)	0.00	1.31	−2.65	2.07
log income (FX)	0.00	1.09	−2.36	1.62
OPEN	0.62	0.32	0.12	1.81
1993				
log income (Afriat)	0.00	1.52	−3.42	1.83
log income (FX)	0.00	0.89	−2.20	1.20
OPEN	0.80	0.60	0.16	3.49

Note: 1 Both FX and Afriat incomes have been normalized to a geometric mean of unity.

Sources: World Bank (2000) for 1993 ICP data, World Bank (1993) for 1980 ICP data and World Bank (2001) for FX data.

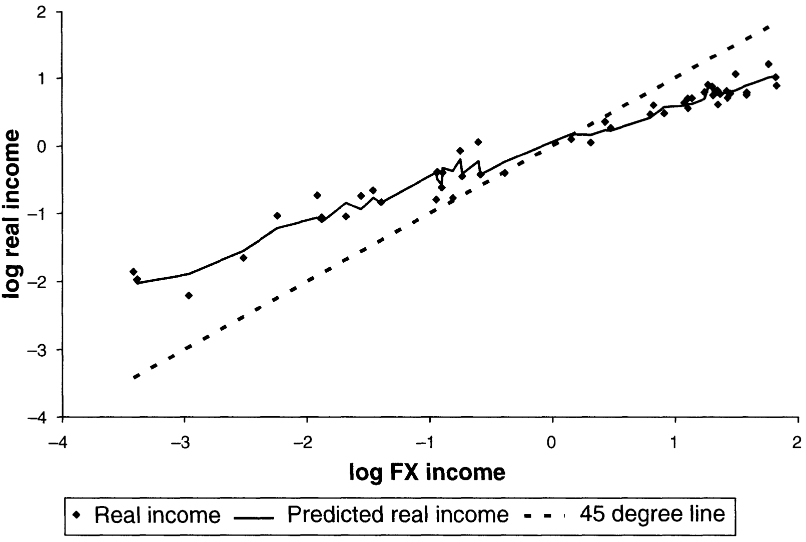


Figure 5.6 Real Afriat income vs FX income, 1993, actual and predicted values

Notes: 1 'Predicted real income' is the predicted value of Afriat GDP per capita from the second regression reported in Table 5.2A.

2 All of the inequality measures are population weighted as defined in equations (5.19) and (5.20).

Sources: World Bank (2001) for FX income (GDP per capita at current exchange rates); World Bank (2000) for 1993 ICP data and authors' calculations of real income (Afriat index of true GDP per capita).

coefficients for that year, or should we use the coefficients from the pooled regression? The former method may yield more accurate predictions for each particular year, but it creates further problems. The object of the exercise is to compare levels of true income inequality between 1980 and 1993. If we are using two different models to predict true incomes for non-benchmark countries, we cannot be sure whether any changes in estimated inequality are due to changes in the real income distribution or to the different methods used for predictions. Because we are primarily interested in the intertemporal comparison, we focus on the results from the pooled regressions. We recognize that the possible misspecification of the exact relationship reduces the accuracy of the incomes predicted for non-benchmark countries, noting that similar problems apply to the PWT estimates of real income.

Different measures of inequality

We use four alternative measures of inequality – Gini (G), Theil (T), the squared coefficient of variation (CV^2) and the variance of logarithmic income (L). Firebaugh (1999) shows each measure can be represented by a distance function of the form:

$$I_m = \sum_{i=1}^{N} p_i f_m(y_i); \quad m = G, T, CV^2, L \tag{5.19}$$

where p_i is the share of the ith country's population in the total population of all N countries and y_i is the ratio between the income per capita of the ith country and the average income (per capita) across N countries. We use population weights to reflect our judgement that a proportional change in per capita income in a populous country such as China or India has more significance for global inequality than a similar change in a low population country such as Luxembourg or Iceland.

Firebaugh (1999) gives the specific functional forms that distinguish the four indexes:

$$G = \sum_{i=1}^{N} p_i y_i (q_i - Q_i); \quad T = \sum_{i=1}^{N} p_i y_i \log y_i;$$

$$CV^2 = \sum_{i=1}^{N} p_i (y_i - 1)^2; \quad L = \sum_{i=1}^{N} p_i \{\log y_i - E[\log y_i]\}^2 \tag{5.20}$$

where E is mean value operator, log is the natural logarithm, q_i is the proportion of total population in the N countries that is poorer than country i and Q_i is the proportion of population richer than country i.

Intra-country inequality

Some studies of world income distribution have analyzed income inequality across countries while ignoring the intra-country component: Theil (1979, 1996), Theil and Seale (1994) and Firebaugh (1999). In other studies, however, the within-country dimension has been taken into consideration, giving a more accurate picture of inequality across all households in the world – as in Berry *et al.* (1983), Grosh and Nafziger (1986), Chotikapanich *et al.* (1997), Schulz (1998), Milanovic (2002), Sala-í-Martin (2002a), Sala-í-Martin (2002b) and Bourguignon and Morrisson (2002).

Before considering within-country inequality we begin by analyzing the between-country component of global inequality for each of the three income measures: FX, Afriat and PWT. The inequality indexes cover 115 countries with 86 per cent of world population in 1997. The PPP (Afriat) based indexes are calculated for 1980 and 1993 only.

Intra-country inequality measures have been taken from Deininger and Squire (1996) who have put together a data set containing quintile income shares and Gini coefficients classified by country, year, income type (gross or net), coverage (national or sub-national), form of recipient unit (person or household) and, importantly, by data quality. The data set includes 682 observations of the highest quality. Relying almost exclusively on the highest quality data, mostly on quintile distribution and occasionally on Gini, the four inequality indexes are computed for 1980 and 1993 for 47 countries.

For many countries Deininger and Squire do not report quintile distribution data of reliable quality for the two years. In order to increase the number of countries, the distribution data for the closest year, while constraining the departure to at most three years from the year of interest – 1980 or 1993, are chosen as a proxy. This increases the country coverage to 67, covering nearly 70 per cent of the world population in both years.

In a few cases, only the Gini coefficient was available. In these cases, we approximate the underlying quintile distribution using the single parameter functional form of the Lorenz curve suggested by Chotikapanich (1993).[7] The mean per capita income for different quintiles are obtained by multiplying the relevant quintile income share with the country's per capita income and then dividing by 0.2, the population share per quintile. We treat each country quintile, with its average income and appropriate population weight, as a single observation in calculating global inequality.[8]

Inequality results

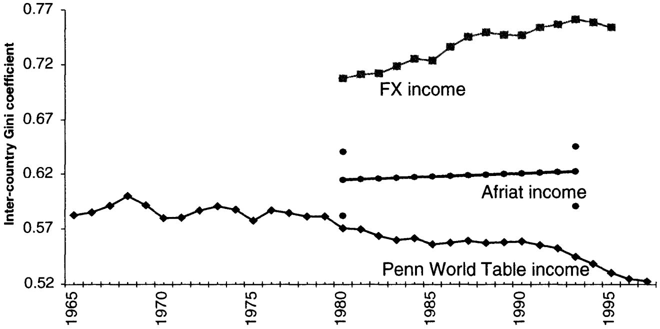
The upper panel in Figure 5.7 repeats the illustration of FX and PWT Gini coefficients across 115 countries, but adds in our estimates of inequality of true Afriat incomes for 1980 and 1993. Our predictions of bias are confirmed. The FX measure overstates the true level of inequality whilst the PWT measure understates it.

With respect to changes over time in the inequality of true incomes, there is a very slight rise in the inter-country Gini coefficient, from 0.615 in 1980 to 0.623 in 1993. This finding is not, however, robust to the inequality index employed. The lower panels of Figure 5.7 display the other indexes of inequality – Theil, CV^2 and the variance of log income. We see a slight reduction in the variance of log true income, from 1.551 to 1.522, between 1980 and 1993, whilst the other two measures of inequality register a slight increase.

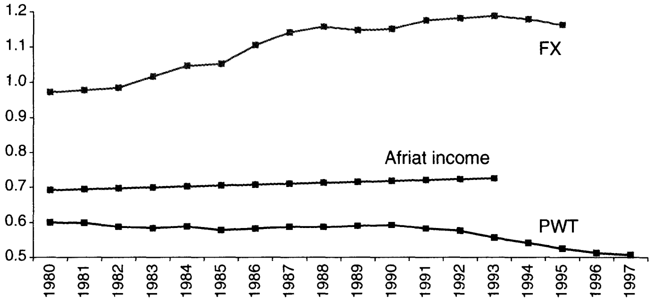
Estimation error for countries not included in the ICP benchmark surveys

Because China was not included in either the 1980 or 1993 ICP price surveys, both the PWT and our own Afriat measures of real income rely on out of sample regression forecasts. Given that China accounts for over one fifth of world population, it is crucial to analyze the robustness of measured inequality with respect to the prediction errors associated with Chinese real income. The standard errors of the regressions are at least fifteen per cent within the sample of benchmark countries – and we expect the error to be even greater

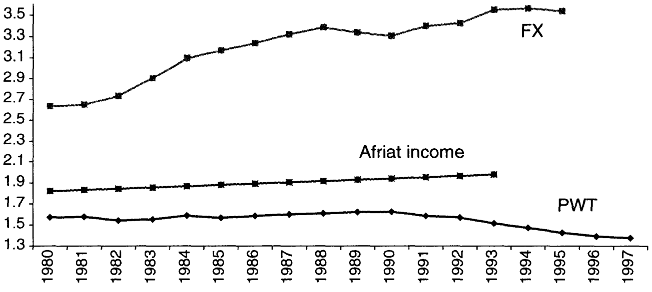
(a) Gini coefficient



(b) Theil index



(c) Squared coefficient of variation



(d) Varience of log income

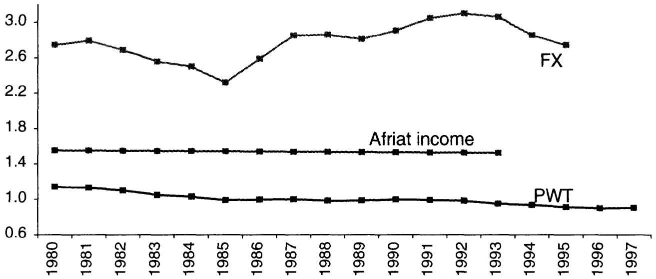


Figure 5.7 Inter-country inequality across 115 countries: three income measures and four inequality measures

Note: Afriat incomes are measured only in 1980 and 1993. The other series are annual.

Table 5.3 Estimated GDP per capita China (as a percentage of US income)

	1980			1993		
	FX	**Afriat**	**PWT**	**FX**	**Afriat**	**PWT**
Lower prediction		*2.6*	*4.7*		*2.5*	*6.7*
Baseline	**1.7**	**4.7**	**6.4**	**1.5**	**4.5**	**9.2**
Higher prediction		*8.5*	*8.7*		*8.1*	*12.6*

Notes: 1 The upper and lower predictions for Afriat income are calculated using forecast values from Regression 3 in Table 5.2 and adding or subtracting two standard errors of the regression.

2 The standard error of regression for the PWT income is taken from Summers and Heston (1991) who report seven alternative regression equations which are used for prediction depending on the availability of data on different countries. The median standard error of the regression, 0.15, is used to perform the sensitivity analysis.

Table 5.4 Sensitivity of inter-country Gini coefficients to estimates of Chinese income

	PWT income			Afriat income		
	Low estimate	Baseline	High estimate	Low estimate	Baseline	High estimate
1980	0.552	0.571	0.592	0.582	0.615	0.641
1993	0.526	0.545	0.566	0.591	0.623	0.646

Note: The low and high estimates of the Gini coefficients are calculated by replacing the baseline estimated income level for China with the high and low income estimates, respectively, from Table 5.3.

when making predictions outside the sample. To check robustness, we calculate upper and lower bounds for real GDP per capita in China by adding or subtracting two standard errors from the baseline predicting regressions. The results are displayed in Table 5.3. In the case of the PWT income estimates, the upper and lower estimates suggest that per capita income in China in 1980 was between 5 and 9 per cent of US income. For the Afriat income estimates, the bounds are 3 and 8 per cent.

The four inequality indexes are recalculated for 1980 and 1993 using the upper and lower estimates of Chinese real GDP for both the Afriat and the PWT income measures. Results are reported in Table 5.4. The low and high values of the Gini differ significantly from their baseline counterparts. The 'confidence intervals' for true income inequality are displayed as dots in the top panel of Figure 5.7.

In another experiment, we dropped China from our sample and computed inequality indexes for the rest of the 114 countries in the sample. We know that, as a country with relatively low income and relatively high growth, China is lowering global inequality over the time period. As expected, using either the FX or Afriat definitions of income, inequality increases more sharply when China is excluded. Using the PWT income estimates, the decline in world inequality disappears altogether when China is excluded.

We are not, of course, suggesting that China should be excluded from measures of inter-country inequality. But it is important to emphasize that all international comparisons of average real income for China are based on estimates, not on direct price measurement by the ICP. In the absence of such direct measurement, any estimates of recent trends in international inequality are subject to substantial uncertainty.

Global inequality

Adding the dimension of within-nation inequality to our measures of international inequality allows us to analyze global inequality. We augment our measures of inter-country inequality, utilizing intra-country income distribution by quintiles, to derive estimates of global inequality for the 67 countries, including both China and India, for which sufficient data are available. Table 5.5 presents estimates of the global Gini coefficient for 1980 and 1993. We again find that global income inequality has been rising if we use the FX income definition, whilst the PWT measure of income records a fall. Our estimates of true (Afriat) income suggest a slight rise in global inequality.

In Table 5.5, the figures in parentheses give the ratio of the inter-country Gini to the global Gini calculated on quintile incomes for each country. This ratio varies between 85 and 89 per cent for the PWT and Afriat measures of income and between 91 and 92 per cent for the FX income-based Gini. Intra-country inequality contributes relatively little, no more than 15 per cent, to the global Gini coefficient – in line with the findings of Berry *et al.* (1983), Korzeniewicz and Moran (1997), Li *et al.* (1998) and Milanovic (2002).[9]

Table 5.5 Global income inequality: Gini coefficients by country quintiles across 67 countries

	PWT income	Afriat income	FX income
1980	0.659	0.698	0.779
	(86.7)	(88.0)	(90.9)
1993	0.636	0.711	0.824
	(85.6)	(87.7)	(92.4)
Change 1980–93 (%)	−3.4	1.7	5.8

Notes: 1 The average income of each country quintile is treated as a single observation. The Gini coefficient is calculated using country quintile population weights.
2 The numbers in parentheses are the inter-country Gini coefficients expressed as a percentage of the global coefficient.

Given that intra-country inequality is only a minor component of world inequality, it is not surprising to find that the global Gini coefficient displays a similar time trend to the inter-country Gini. The inclusion of within-country inequality tends to rescale the indexes upwards, without greatly altering rates of change over recent years.

The corresponding results for the other three inequality indexes – Theil (T), coefficient of variation squared (V^2) and variance of log income (L) – are presented in Table 5.6. We find that for each of these indexes the contribution of within-country inequality is proportionally greater than was the case with the Gini index. Nevertheless, the trends identified for the global Gini index, are essentially robust to the choice of inequality index. Global inequality falls between 1980 and 1993 for PWT incomes, but it increases for true Afriat income and FX income, according to all four indexes.

We are concerned, however, that the use of grouped income data may substantially understate the contribution of intra-country inequality to global inequality. As an experiment, we generated ten thousand log-normally distributed incomes and computed the variance of log income when the income is grouped into various fractiles (shown in Figure 5.8). We find that the variance of log income grouped into quintiles is 90 per cent of the variance of the full sample. We conclude, therefore, that the quintile income shares that we and other researchers have used are likely to come close to capturing the full contribution of intra-country inequality to world inequality. This conclusion is reinforced by the finding of Sala-í-Martin (2002a) that estimating a

Table 5.6 Alternative measures of global income inequality by country quintiles across 67 countries

	PWT	**Afriat**	**FX**
Theil index			
1980	0.84 (70.9)	0.96 (71.5)	1.25 (77.3)
1993	0.79 (70.4)	1.01 (71.4)	1.50 (79.0)
Change 1980–93 (%)	−6.7	+5.0	+19.5
Squared CV			
1980	2.86 (55.0)	3.34 (54.5)	4.33 (60.7)
1993	2.73 (55.4)	3.63 (54.5)	5.73 (61.9)
Change 1980–93 (%)	−4.4	+8.7	+32.1
Variance of log			
1980	1.74 (65.3)	2.21 (70.1)	3.67 (74.7)
1993	1.51 (62.8)	2.40 (63.4)	4.23 (72.2)
Change 1980–93 (%)	−13.3	+8.5	+15.4

Note: Values in parenthesis are the between-country index as a percentage of the global index.

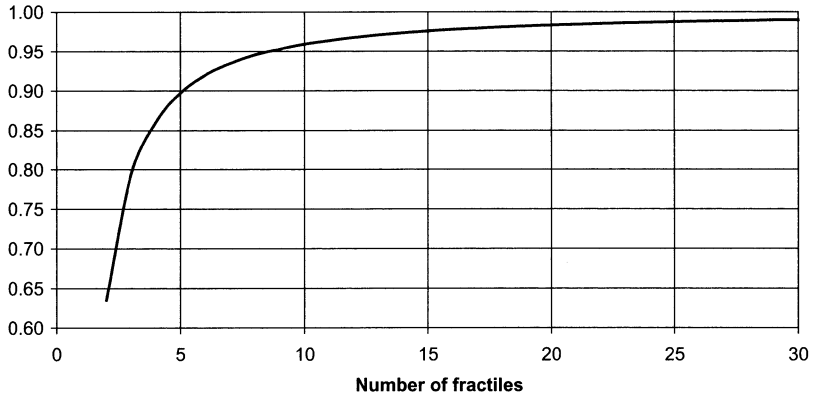


Figure 5.8 Variance of fractile income/variance of population

Note: Variances are calculated on 10,000 randomly generated log normal incomes which have been ordered and averaged over fractiles.

Gaussian kernel density function has minimal impact on measures of global inequality.

Sala-í-Martin (2002b) criticizes our sample selection, based on an earlier version of this chapter,[10] on the grounds that 'the selection of countries that do not have Gini data is not random. In particular, these are countries that are poor and that have diverged. Excluding these countries from the analysis tends to bias the results towards finding reductions in world income inequality' (2002b: 9). This point simply reinforces our conclusion that, once we correct for the bias in the PWT measures of purchasing power parity, there is no convincing evidence of a fall in world income inequality over the period in question. Moreover, there is no such selection bias in our measures of the inter-country component which dominates quintile-based measures of global inequality.

Conclusion

A preliminary point that emerges from our theoretical and empirical analysis is that researchers who want to compare real income levels across countries need to be wary of the label 'purchasing power parity'. There is no unique concept of purchasing power, and there are substantial differences in the methods underlying the construction of widely used data sets that attach this label to their income measures. Our preference is to define purchasing power in terms of the capacity of a representative consumer to attain the same levels of utility when confronted with the particular price structure of each country. Using Afriat's non-parametric tests, we are able to construct true income comparisons and to demonstrate that the frequently used PWT data substantially understate the true level of income inequality due to substitution bias. We also note that the EKS index number approach, favoured by the OECD in its calculations of PPPs, does not suffer from such bias.

Regarding trends in global inequality, we replicate previous findings that the fixed price method of calculating purchasing power parity incomes, which is used to construct the PWT, leads to measures of inequality that tend to fall over the 1980s and 1990s, whilst market exchange rate comparisons of income suggest that inequality was rising. These observations raise a puzzle: with falling inequality in productivity and real income tending to reduce the sectoral bias of FX income comparisons, we should expect FX measures of income inequality to have been falling even faster than inequality measured at PPP. But the exact opposite has occurred, the gap between FX and PWT inequality has increased.

Our explanation for these contradictory trends in measures of global inequality rests on two hypotheses for which we have found empirical support. The first hypothesis is that both the FX method and the PWT's fixed-price method of comparing incomes across countries are biased, the former method tending to overstate inequality and the latter method tending to understate it. These are the predictions of the trade model that we developed. Our model exhibits the standard Balassa–Samuelson effect whereby cross-country differences in productivity in the traded sector lead to lower relative prices in the non-traded sectors of low productivity countries, leading to an exaggeration of true income differentials when national incomes are compared at the market rate of FX. The novel result of our model is that these inter-sectoral price differentials impart a downward bias to fixed price measures of income differentials when the price vector is similar to that of high productivity countries. Our empirical analysis suggests that the fixed international price vector underlying the valuations of the PWT does indeed correspond to the price structures of relatively rich economies.

The second hypothesis is that national price structures became increasingly dissimilar over recent decades. The biases in both PWT and FX measures of inequality are driven by differences in relative prices across countries – if relative prices were identical in all countries, there would be no bias in either method of valuing real incomes. Our model demonstrates that the magnitude of the bias is increasing in the size of the sectoral price differentials. As price structures become less similar, there is increasing downward bias in the PWT estimates of income inequality and increasing upward bias in FX estimates.

Over a period when world trade has been increasing, it may seem unlikely that national price structures would have become less similar – but this is exactly what we find when we examine trends in indexes of crosscountry price similarity and dissimilarity. Such a result is not necessarily contrary to the Balassa–Samuelson model where the bias in FX valuations of income arises out of the underlying productivity differentials and the relative sizes of the tradable and non-tradable sectors. An increase in actual trade will not affect the bias if the relative sizes of the domestic production sectors are unchanged, but an increase in the productivity differential will increase the bias. Moreover, a wide range of domestic supply and demand factors, as well as changes in government tax and subsidy policies, can be expected to influence domestic price structures differently in different countries. Further investigation of these issues is clearly warranted.

Here, we have an explanation for the radical differences in measured inequality trends. True inequality was stable or increasing slightly – as suggested by our measures of true Afriat income – over the 1980s and 1990s whilst price structures became increasingly dissimilar.

Whilst this explanation is plausible, we cannot be sure that it is true. There are substantial errors involved in estimating real incomes for countries that have not been included in the ICP benchmark surveys, errors that apply equally to our estimates of Afriat incomes and to the PWT income estimates. Moreover, until the publication of the 2003 ICP survey, which for the first time includes both India and China, all methods of calculating purchasing power parities had to resort to imprecise estimates of real income for more than one third of the world's population.

Appendix: mathematical proofs

Derivation of (5.2)

Output in sector m of country i is given by the production function

$$M^i = (\lambda L^i_m)^\alpha (A^i_m)^{1-\alpha} \tag{A.5.21}$$

Assuming competitive markets, the wage w^i and the price of the intermediate input, p^{ia}, are equated with the value of their marginal products

$$p^{im}\alpha\lambda^\alpha (L^{im})^{\alpha-1}(A^{im})^{1-\alpha} = w^i \Rightarrow \left(\frac{L^{im}}{A^{im}}\right)^{1-\alpha} = \frac{\alpha\lambda^\alpha p^{im}}{w^i} \tag{A.5.22}$$

$$p^{im}(1-\alpha)(\lambda L^{im})^\alpha (A^{im})^{-\alpha} = P^{iA} \Rightarrow \left(\frac{L^{im}}{A^{im}}\right)^{\alpha} = \frac{p^{iA}}{(1-\alpha)\lambda^\alpha p^{im}} \tag{A.5.23}$$

Equating relative factor demands from (A2) and (A3) gives result (2):

$$p^{im} = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}}\left(\frac{w^i}{\lambda}\right)^\alpha (P^{iA})^{1-\alpha} \tag{A.5.24}$$

Proof of Proposition 3

The GK income ratio, expressed as a proportion of the true income ratio, is $R(g)$

$$R(g) = \frac{\beta g + (1-\beta)\lambda}{[\beta g + (1-\beta)]\lambda^{1-\beta}} \tag{A.5.25}$$

Differentiating with respect to g, representing the denominator of (A.5.25) as D, gives

$$D^2R'(g) = \beta(1-\beta)(1-\lambda)\lambda^{\beta-1} \qquad \text{(A.5.26)}$$
$$\Rightarrow R'(g) < 0 \quad \forall\lambda > 1, 0 < \beta < 1$$

We need to show that $R(1) > 1$ and that $R(\lambda) < 1$.

$$R(1) = \frac{\beta + (1-\beta)\lambda}{\lambda^{1-\beta}} > 1 \text{ iff } \beta + (1-\beta)\lambda - \lambda^{1-\beta} > 0 \qquad \text{(A.5.27)}$$

$$R(\lambda) = \frac{\lambda^{\beta}}{\beta\lambda + (1-\beta)} < 1 \text{ iff } (1-\beta) + \beta\lambda - \lambda^{\beta} > 0 \qquad \text{(A.5.28)}$$

The conditions in (A.5.27) and (A.5.28) are of the same form:

$$f(\lambda) \equiv d + (1-d)\lambda - \lambda^{1-d} > 0 \qquad \text{(A.5.29)}$$
$$\text{where } 0 < d < 1, \text{ since } d = \beta \text{ or } d = (1-\beta)$$

We evaluate the function $f(\lambda)$ as follows:

$$f(1) = d + 1 - d - 1 = 0$$
$$f'(\lambda) = (1-d) - (1-d)\lambda^{-d}$$
$$= (1-d)\left(1 - \frac{1}{\lambda^d}\right) > 0 \quad \forall\lambda > 1, \quad 0 < d < 1 \qquad \text{(A.5.30)}$$
$$\Rightarrow f(\lambda) > 0 \quad \forall\lambda > 1$$

This demonstrates that the conditions in (A.5.27) and (A.5.28) do hold. Hence (i) and (ii) of Proposition 3.

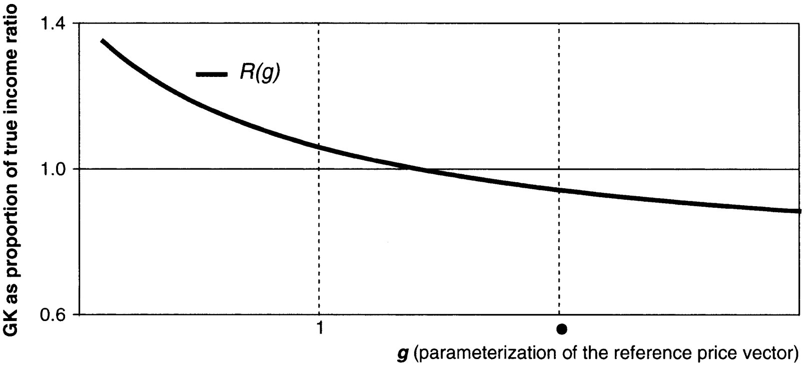


Figure 5A.1 Bias in the GK income ratio

Note: The function $R(g)$ is drawn from equation (A.5.25) for $\lambda = 2$ and $\beta = 0.5$.

Given $R(1) > 1$, $R(\lambda) < 1$ and $R'(g) < 0$, we can draw the function $R(g)$ for given λ as follows: the function $R(g)$ is illustrated in Figure 5A.1 for $\lambda = 2$ and $\beta = 0.5$. The degree of bias is the distance of R from unity. It is clear that this distance increases either as g is larger than λ, or as g becomes smaller than 1. Hence (iii) of Proposition 3.

Finally, for a given value of g, we can differentiate the expression in (A.5.25) with respect to λ:

$$\begin{aligned} &[\beta g + (1-\beta)]\frac{\partial R(\lambda;g)}{\partial \lambda} = \frac{\beta(1-\beta)(\lambda - g)}{\lambda^{1-\beta}} \\ &\Rightarrow \frac{\partial R(\lambda;g)}{\partial \lambda} >=< 0 \text{ as } \lambda >=< g \end{aligned} \tag{A.5.31}$$

If g is greater than λ as postulated in part (i), R is less than unity and decreasing in λ. Thus, the magnitude of the (downward) bias is increasing in λ. If g is less than unity, as postulated in (ii), then λ is greater than g and R is greater than unity and increasing in λ. Thus the magnitude of the (upward) bias is increasing in λ. Hence part (iv) of Proposition 3. □

Notes

The authors thank Stephan Klasen and two anonymous referees for their comments and suggestions. We are grateful for additional comments from participants at the UNU-WIDER Helsinki 1993 conference and from participants in seminars at Griffith University and the Australian National University.

1. An exception is Milanovic (2002) who reports rising inequality between 1988 and 1993. Although Milanovic adjusts for purchasing power parities, his income definition is different from all the other studies, which are based on concepts of national or domestic income. He uses household income derived from survey data. Bourguignon and Morrisson (2002) find that the direction of change in inter-country inequality between 1980 and 1992 depends in part on the choice of inequality measure. Their table 2 shows a small rise in the Theil index whilst both the mean logarithmic deviation and the standard deviation of log income decline. They use a different measure of purchasing power parity, relying on the estimates of Maddison (1995).
2. See Summers and Heston (1991).
3. The sample of countries is held constant at 131 for 1980 to 1989, dropping to 112 and 100 in 1990 and 1991 respectively due to missing price data in PWT5.6. The results are very similar if we restrict our sample to the 98 countries for which data are available for all 12 years.
4. The 1993 ICP data have been made available by the World Bank (2000) on a regional basis. We understand that the Bank has not published the global estimates. We merged the regional data sets, using common countries to scale the price and quantity data, to create our own global estimates. We have also used ICP data published by World Bank (1993) and time-series data published on the Internet (World Bank 2001).
5. The Afriat method yields multiple sets of utility ratios with defined upper and lower bounds. For the purposes of this chapter, we use the mid-points of the multilateral bounds, yielding the ideal Afriat index.

6. Our approach to estimating real incomes in non-benchmark countries is similar to that reported in Summers and Heston (1991: appendix B), except that they use various additional price surveys. An alternative approach used by Bergstrand (1991) uses the relative productivity levels in traded and non-traded sectors as an additional explanatory variable, but such data are not available for the non-benchmark countries.
7. Chotikapanich (1993) approximates the Lorenz curve by the following single parameter specification: $LC = (e^{kp} - 1)/(e^{k} - 1)$. The corresponding Gini coefficient is given by: $G = \lfloor (k - 2)e^{k} + (k + 2) \rfloor / k(e^{k} - 1)$, where p is the population share and k is a parameter, which is required to be greater than zero. In the first step, the Gini equation is solved for k which is then used to obtain the estimates of quintile income distribution.
8. These approximations, combined with our necessarily imperfect estimation of national incomes for non-benchmark countries and the more general problems of compiling data from secondary sources – see Atkinson and Brandolini (2001), mean that there must be a substantial degree of imprecision in our inequality estimates. The principal interest of this chapter, however, lies in comparing results from different methods of calculating purchasing power comparisons using the same secondary sources.
9. The relatively small contribution of within-country inequality is not surprising when we compare the quintile shares of income within and across countries in 1993. Within countries the ratio of income earned by the richest 20 per cent is, on average, eight times the income earned by the poorest 20 per cent of households. Across countries the inter-country quintile ratio of real per capita income is 25.
10. Whilst criticizing our sample selection procedures, Sala-í-Martin ignores the principal point of our earlier paper, which is that the PWT data on which he is relying bias the measurement of inequality.

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6

The Many Facets of Economic Mobility

Gary S. Fields

Introduction

The famed economist Joseph Schumpeter likened income distributions to the rooms in a hotel.[1] The rooms at the top are luxurious, those in the middle are ordinary, and those in the basement are substandard. On any given night, the occupants of the hotel experience quite unequal accommodations. Later, though, the same people are found to be in different rooms (or, equivalently, the same rooms are found to have different people in them). The difference in the quality of the hotel rooms at each point in time is what we call *inequality*. The constant quality of each room means that there is no *growth*, positive or negative. The movement of hotel guests among different quality rooms constitutes *mobility*, which is the topic of this chapter.

Schumpeter's hotel analogy raises some fundamental questions about what economic mobility is, how it relates to inequality and how both relate to economic growth. There is no question that the movement of guests among rooms is an aspect of mobility. But is that all there is to mobility? If the existing furnishings are moved from some rooms to others, is there mobility then? What if the hotel is refurbished so that some of the rooms are made nicer? Do the lucky residents of these rooms enjoy upward mobility? As for those whose rooms are not upgraded, do they suffer downward mobility because they are now in a relatively worse position?

The main point of this chapter is to show that the different indices used in the mobility literature are not measures of the same underlying conceptual entity.[2] In elementary statistics, students are taught that the mean and median are both measures of central tendency but they are different measures of central tendency; the variance and Gini coefficient are measures of dispersion but they are different measures of dispersion; and central tendency and dispersion are fundamentally different concepts from one another. In much the same way, this chapter maintains that the different mobility indices in common use are measuring fundamentally different mobility concepts from one another. It is in this sense that mobility really is multifaceted.

The term 'mobility' connotes precise ideas to various researchers, but it connotes *different* precise ideas to different researchers. Furthermore, these differences remain even after agreeing on a number of other aspects of the mobility under discussion. These other aspects, discussed in the following paragraphs, are whether the context is intergenerational or intragenerational, what is the indicator of social or economic status and whether the study is at the macro-mobility or micro-mobility level.

One issue is whether the aspect of mobility of interest is intergenerational or intragenerational. In the intergenerational context, the recipient unit is the family, specifically a parent and a child. In the intragenerational context, the recipient unit is the individual or family at two different dates. The analysis in this chapter applies equally to both.

Once the context has been decided upon, a second issue to be decided is, indicator of what among whom? The indicator could be income, consumption, labour market earnings, occupation, education, or any of a number of other indicators for a given recipient unit. The recipient unit may be an individual, a worker, a family, a per capita, or an adult equivalent. For brevity, the discussion below is phrased in terms of economic well-being (denoted 'income') among recipient units (denoted 'individuals') with the understanding that the analysis is equally applicable to any of the other status indicators and recipient units mentioned above.

Third, mobility research may be conducted at two levels, macro and micro. Macro mobility studies start with the question, 'How much economic mobility is there?' Answers are of the type; 'a per cent of the people stay in the same income quintile', 'b per cent of the people moved up at least \$1,000 while c per cent of the people moved down at least \$1,000', 'the mean absolute value of income change was \$d' and 'in a panel of length T, the mean number of years in poverty is t*'. The macro mobility studies often go beyond this question to ask, 'Is economic mobility higher here than there and what accounts for the difference?' Answers would be of the type; 'economic mobility has been rising over time', 'A has more upward mobility than B because economic growth was higher in A than in B' and 'incomes are more stable in C than in D because C has a better social safety net'. Micro mobility studies, on the other hand, start with the question, 'What are the correlates and determinants of the income changes of individual income recipients?' The answers to these questions would be of the type; 'unconditionally, income changes are higher for the better educated' and 'other things equal, higher initial income is associated with lower subsequent income growth'.

This chapter addresses macro mobility. The issues raised here are applicable to both economic and social mobility, to intergenerational and intragenerational mobility, and to mobility of individuals and of families. This work shows that mobility, growth and inequality are related but distinct concepts. Two simple examples are used to highlight the differences.

It is then demonstrated that in macro mobility studies, there are actually six fundamentally different concepts that are being measured. They are time-dependence, positional movement, income flux, directional income movement, movement of income shares, and mobility as an equalizer of longer-term incomes.

The chapter then turns to the question: Does it make a practical difference which income concept is measured? The most fundamental macro mobility questions are these: (1) Has mobility been rising or falling over time?; (2) Which group has more income mobility than another? Panel data drawn from the Panel Study of Income Dynamics in the United States and the Déclarations Annuelles de Données Sociales and the Echantillon Démographique Permanent in France are used to demonstrate that the answers to even these most fundamental questions depend on the mobility concept used. In both countries, mobility has been falling for some mobility concepts but not for others. In both countries, women have more mobility than men for some concepts but men have more mobility than women for others, and likewise the better educated have more mobility than others for some mobility concepts and less mobility for others.

The chapter closes with a brief conclusion. The major point is that before social scientists 'do a mobility study', it is necessary to be very clear about the mobility concept or concepts we wish to study. As this work shows, the choice can and does make a vital difference.

Mobility, inequality, and growth: two examples

In Schumpeter's hotel model, it is known which guest occupies which room on any given night. In the real world, such information can be obtained only from panel data, in which each individual is observed at two or more points in time. When such data are available and the guest-room pairings over a number of nights can be observed, the long-term equality of accommodations can be compared with the equality of accommodations on any given night. It is apparent in this example that the greater is the movement of guests among rooms of fixed quality, the greater is the long-term equality of accommodations.

Suppose such panel data had not been available but the data consisted only of comparable cross sections. Nothing could have been said about mobility or inequality in the longer run. It would only have been possible to have compared the inequality of accommodations at each point in time. The only lesson such a comparison could have produced is that inequality was unchanged.

Another example may also be considered. Suppose samples of two persons were to be drawn from an economy in a base year and a final year, and the incomes of each person in each of the two years were to be measured. Assume the data are drawn from comparable cross sections in the two years but that the individuals sampled are not the same in the two years (or if they are the same, the surveys do not record who is who). Let the distribution of income in the base year be $y_1 = (1,3)$ and in the final year $y_2 = (1,5)$. In a very straightforward sense, one would be justified in saying that the change in the income distribution from $y_1 = (1,3)$ to $y_2 = (1,5)$ entails economic growth, but the growth that takes place raises inequality.

What can be said about economic mobility from such anonymous data? Very simply: nothing. This is because the data are anonymous, and so the analyst does not know which income recipient is which.[3]

In this little example, there are only two underlying possibilities: either the two individuals occupied the same positions in each period or they swapped positions. Adopt the notational convention of arraying income recipients in some order in the base year distribution, keep these identified individuals in the same position in the final year, and denote the movement from a base year personalized vector to a final year personalized vector by $\rightarrow$. The two possible patterns of longitudinal income changes consistent with y_1 becoming y_2 may then be denoted:

(a) $(1, 3) \rightarrow (1, 5)$

and

(b) $(1, 3) \rightarrow (5, 1)$.

Do situations (a) and (b) have the same economic mobility as one another? Many observers would say that they do not.

More difficult, though, is the question, how specifically do the two mobility situations differ? To answer this question, one must have a clear idea of what is meant by economic mobility. Let us now turn now to six mobility concepts that have been used in the literature.

Six mobility concepts

The mobility literature is plagued by people using the same term 'economic mobility' (or 'social mobility') to mean different things. Six notions of mobility need to be distinguished. Briefly, they are: *time-dependence*, which measures the extent to which economic well-being in the past determines individuals' economic well-being at present; *positional movement*, which is what we measure when we look at individuals' changes in economic positions (ranks, centiles, deciles, or quintiles); *share movement*, which arises when individuals' shares of the total income change; *income flux*, which is what we gauge when we look at the size of the fluctuations in individuals' incomes but not their sign; *directional income movement*, which is what we measure when we determine how many people move up or down how many dollars; and *mobility as an equalizer of longer-term incomes*, which involves comparing the inequality of income at one point in time with the inequality of income over a longer period.

Let us now look at each of these in greater detail.

Time-independence

Time-dependence is the notion that incomes at present are determined by incomes in the past. Time-dependence is highest when income at present is entirely determined by past income. As the economy moves further away from this deterministic situation, it gets closer to the situation where current income is independent of past income. The notion of mobility as time-independence is that mobility is greatest when current and past income are unrelated to one another.

A common way of gauging time-dependence and -independence is by constructing a quantile mobility matrix. (A quantile mobility matrix classifies people in each year according to fixed categories such as five equal-sized quintiles or ten equal-sized deciles, with base year quantile determining the row and final year quantile determining the column. Each entry is the probability that, starting in a given row, the individual ends up in a given column.) If incomes were perfectly positively time-dependent, the quantile transition matrix would have all entries lying along the principal diagonal, and thus the transition matrix would be an identity matrix. For example:

$$P_1 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(A theoretical possibility, never observed in practice, is negative time-dependence, the limiting value of which is when final year quantile is inversely related to base year quantile. If such a case were to be observed, the transition matrix would be a mirror image of P_1.)

Suppose that instead of perfect time-dependence, incomes were time-independent. Again taking the example of classifying people into income quintiles, perfect time-independence would mean that 20 per cent of those in each base year income quintile would be found in each final year income quintile, producing the following quintile transition matrix:

$$P_2 = \begin{bmatrix} 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{bmatrix}$$

In order to be able to implement the notion of mobility as time-independence, one needs a way of measuring how close an actual transition matrix is to these theoretical possibilities. In the case of a quintile transition matrix, the number of people in cell i, j under time-independence is the P_2 matrix multiplied by an appropriate scaling factor such that the sum of the expected frequencies is the total sample size N:

$$P_3 = \begin{bmatrix} .04N & .04N & .04N & .04N & .04N \\ .04N & .04N & .04N & .04N & .04N \\ .04N & .04N & .04N & .04N & .04N \\ .04N & .04N & .04N & .04N & .04N \\ .04N & .04N & .04N & .04N & .04N \end{bmatrix}$$

These expected frequencies under time-independence EXP_{ij} may then be compared with the observed frequencies OBS_{ij} by calculating the standard (Pearson) chi-squared statistic:

$$\chi^2 = \sum_i \sum_j \frac{(OBS_{ij} - EXP_{ij})^2}{EXP_{ij}}$$

Note that the chi-squared statistic is highest the further the economy is from time-independence, and in this sense chi-squared measures *immobility*. In order to have a statistic that measures mobility, a measure is needed that increases as the economy gets closer to time-independence. One such measure is minus chi-squared; it is used below.

The chi-squared statistic is not the only measure of time-dependence. Standard statistical packages contain contingency table procedures that produce quite a number of independence statistics. For instance, in addition to producing a chi-squared value, Stata also generates the likelihood-ratio chi-squared, Cramer's V, gamma, and Kendall's tau-b. And, if the researcher has access to the micro data from which the quantile transition matrix has been constructed, s/he can also calculate the Pearson correlation coefficient or the Spearman rank correlation coefficient. Note that all of these indices take on higher values the more immobile the underlying situation is. Indices of this kind have been calculated by Friedman and Kuznets (1954), Schiller (1977), Atkinson *et al.* (1992), Hungerford (1993) and OECD (1996, 1997), among others. In order to make higher values correspond to greater mobility, some authors have proposed using 1 minus the correlation between incomes (e.g., Department of Employment 1973) or 1 minus the correlation between log-incomes (Hart 1981). The intergenerational earnings elasticity calculated for many countries around the world (Solon, 2002) also is a measure of time-dependence.

Positional movement

According to this notion of mobility, an individual is deemed to have experienced mobility if and only if s/he changes position in the income distribution. Although the most commonly used measures of economic position are individuals' quintiles or deciles in the income distribution, there is no reason why ventiles, centiles, or even ranks might not be used instead.

Positional-movement indices then gauge the extent to which positions change in a population or a sample. Many researchers have used the immobility ratio (namely, the fraction of cases lying along the principal diagonal), while others have used the mean number of quantiles moved (in absolute value), the mean upward jump, and the like (Boudon 1973; Lillard and Willis 1978; Gottschalk 1982; Atkinson *et al.* 1992; and OECD 1996, 1997). A sophisticated positional movement index was developed axiomatically by King (1983).

Observe that the notion of positional movement is thoroughly relative: a person can experience relative income mobility even if his/her own income does not change, provided that others' incomes change by enough that the person in question experiences a change in position. There is another way in which a person can experience relative income mobility even if his/her own income remains unchanged, and that is through share movement.

Share movement

Some mobility analysts, even thoroughgoing relativists, are concerned primarily about changes in income ratios rather than with changes in positions within the income distribution. Suppose one person's income rises by 50 per cent but everyone else's rises by 100 per cent. The analyst may feel that the first person has lost ground, because his/her income share has fallen. From the perspective of share movement, the first person may indeed be judged to have experienced downward mobility, not because that person's income has fallen (because in this example, it has not) but because that person's share of total income has fallen.

To gauge the extent of share-movement in a population, the mean share movement will not work as an index. This is because the income shares must sum to 100 per cent, and therefore the changes in shares must necessarily average out to zero. What would work as an index of share movement is the mean absolute value of share changes. This measure is used later.

Inadvertently, a measure of share movement is commonly calculated. It can readily be shown that the correlation between base year and final year incomes is the same as the correlation between base year and final year income shares. Thus, the correlation coefficient frequently calculated from micro data can be viewed as an (inverse) index of share movement.

Chakravarty *et al.* (1985) analyzed the issue of relative income mobility using ethical (welfarist) foundations. For them, an initial distribution of income exhibits complete relative immobility if and only if the income shares are the same for all individuals in all time periods. They then derived the following share movement index: $M_{CDW} = ((EY_a)/E(Y_1)) - 1$, where $E(Y_a)$ is an index of equality of average incomes and $E(Y_1)$ is an index of equality of incomes in period one. For them, the mobility index is positive (negative), and therefore the mobility process is desirable (undesirable), if and only if average incomes are more (less) equally distributed than initial incomes were.

Thus, Chakravarty *et al.* (1985) assign welfare significance only to the relative aspect of changes in incomes while ignoring whether incomes are rising or falling, a judgement that many would find objectionable.

What share-movement measures is flux; that is, how much variation there is between base year and final year. Here, the aspect of flux that is being measured is income shares, whereas on pp. 128–9, it was positions in the income distribution. For those observers who are interested in flux, but who are more concerned about incomes than shares or positions, the next class of measures may be appealing.

Income flux (also called 'instability' or 'non-directional income movement')

Consider two persons, one of whom experiences a $10,000 income gain and the other a $10,000 income loss. How much income movement has taken place? A respondent who answers that the total income movement is $20,000 total or $10,000 per capita has used an income flux measure, in the sense that the gains and losses are weighted similarly without regard to the direction of change. Precisely, this measure was devised and justified axiomatically by Fields and Ok (1996). Specifically, the first Fields–Ok per capita measure is

$$m_n^{(1)}(x,y) = \frac{1}{n}\sum_{j=1}^{n} |x_i - y_i|$$

that is, the mean absolute income change. This index makes the implicit assumption that a dollar gain or loss is the same regardless of the income level of the person experiencing it. To the contrary, one may want to consider a dollar change differently depending on how rich or poor the person was initially – specifically, by regarding a given dollar amount of change as counting for less the richer is the income recipient. A measure that was derived axiomatically and shown to possess this property is the second per capita measure proposed by Fields and Ok (1999b):

$$m_n^{(2)}(x,y) = \frac{1}{n}\sum_{j=1}^{n} |\log x_i - \log y_i|$$

Income flux has also been gauged in studies by Abowd and Card (1989), Gottschalk and Moffitt (1994) and Stevens (2001), among others.

Directional income movement

An observer may be more interested in the directions and magnitudes of income changes than in absolute values. For such an observer, the concept of interest is directional income movement.

Several ad hoc directional indices are in use, such as the fraction of upward or downward movers, the average amount gained by the winners, and the average amount lost by the losers. Moving beyond these ad hoc measures, Fields and Ok (1999b) axiomatized directional income movement and devised as a measure the mean change in log-incomes:

$$m_n^{(3)}(x,y) = \frac{1}{n}\sum_{j=1}^{n}(\log x_i - \log y_i)$$

This measure combines income gains and losses taking account of the income levels of each of the gainers and each of the losers.

Mobility as an equalizer of longer-term incomes relative to the base

One of the primary motivations for economic mobility studies is to gauge the extent to which longer-term incomes are distributed more or less equally than are single year incomes. Slemrod (1992), for instance, has maintained that what he graphically calls 'time-exposure income' gives a better picture of inequality than does 'snapshot income'. Continuing in a similar vein, Krugman (1992) has written: 'If income mobility were very high, the degree of inequality in any given year would be unimportant, because the distribution of lifetime income would be very even ... An increase in income mobility tends to make the distribution of lifetime income more equal'. Similar statements have been made by, among others, Shorrocks (1978), Maasoumi and Zandvakili (1986), Atkinson *et al.* (1992) and Jarvis and Jenkins (1998).

What unites these and other authors is a concern with income mobility as an equalizer of longer-term incomes along with the judgement that the extent of such equalization is of ethical relevance. In Fields (2004), it is shown that although the established mobility measures do a good job of measuring other mobility concepts, they do not adequately gauge this one.[4]

In the absence of a good measure of this concept, a new class of measures representing this concept were worked out. One easily implementable measure in this class is the equalization measure

$$\mathrm{E} \equiv 1 - (\mathrm{I(a)}/\mathrm{I}(\mathrm{Y}_1))$$

where a is the vector of average incomes, Y_1 is the vector of base year incomes, and I(.) is an inequality measure.[5] When incomes over a longer period are distributed as unequally as base year incomes are, $\mathrm{E} = 0$. When incomes over a longer period are distributed more equally than base year incomes, $\mathrm{E} > 0$, signifying that the income mobility that took place caused longer-term incomes to be more equally distributed than were base year incomes. Lastly, when incomes over a longer period are distributed less equally than base year incomes, $\mathrm{E} < 0$; that is, the pattern of changes has been in the disequalizing direction.

Summary

Six mobility concepts have been presented in this section: time-independence, positional movement, share movement, income flux, directional income movement, and mobility as an equalizer of longer-term incomes relative to the base. Because each of these concepts is different from the others, a measure of one concept does not necessarily accord with the measure of another. Whether they give the same qualitative answers in practice is an empirical question, to which we now turn.

Comparing the mobility concepts

Consider how two or more mobility situations compare with one another. Which has more mobility? That different mobility indexes can produce different ordinal rankings is well-known; see for instance Dardanoni (1993), Maasoumi (1998) and Checchi and Dardanoni (2003). What is unclear, though, is whether the different indexes produce different ordinal rankings because they are gauging fundamentally different concepts (as, for example, income inequality is a fundamentally different concept from poverty) or because they produce different ordinal rankings for the same concept (as, for example, may arise for two different Lorenz consistent inequality indexes when Lorenz curves cross).

Let us compare the six mobility concepts, using one index of each. These indices are defined formally in Table 6.1. Statistical software for calculating a number of these indices (and others) is available in Van Kerm (2002).

Table 6.1 Measures of six mobility concepts used in the empirical work

Concept	**Measure used in the two-period case**
Time–independence	$1 - r(y_1, y_2)$, where r is the Pearson correlation coefficient
Positional–movement	$1 - \rho(y_1, y_2)$, where ρ is the rank correlation coefficient
Per capita share movement	$(1/n)\Sigma\lvert s(y_{2i}) - s(y_{1i})\rvert$, where s(.) denotes i's share of total income
Per capita income flux	$(1/n)\Sigma\lvert y_{2i} - y_{1i}\rvert$
Per capita directional movement	$(1/n)\Sigma(\log y_{2i} - \log y_{1i})$
Mobility as an equalizer of longer-term income	$E \equiv 1 - (I(a)/I(Y_1))$, where a is the vector of average incomes, Y_1 is the vector of base year incomes, and I(.) is an inequality measure (either the Gini coefficient or the Theil index)

How do the six mobility concepts compare in the two examples?

First, let us take the hotel example, in which guests move between rooms of different quality from one night to the next. Because guests are switching rooms and some rooms are better than others, there is not perfect *time-dependence.*[6] The movements among rooms of different quality means that *positional movement* takes place, as does *share movement.* There is *flux,* because some guests experience different rooms of different quality. Those who move up in the hotel experience *upward directional movement;* the opposite is the case for those moving down in the hotel. Finally, because the average quality experienced over a number of nights is distributed more equally than the quality on the first night or on any other night, mobility has *equalized longer-term outcomes* relative to any given night's distribution.

Consider next the two-person income example. Either the personalized change was

$$\text{(a)}\ \underset{\alpha\ \beta}{(1,3)} \rightarrow \underset{\alpha\ \beta}{(1,5)}$$

or it was

$$\text{(b)}\ \underset{\alpha\ \beta}{(1,3)} \rightarrow \underset{\alpha\ \beta}{(5,1)}$$

Here, the recipients' names (in Greek letters) have been inserted so that it is easier to talk about who is who.

What has happened in Case (a)? There is perfect *time-dependence* in ranks but not in incomes. There has been no *positional movement.* There has been *share movement,* upward in the case of individual β and downward in the case of individual α. Incomes changed and therefore *income flux* took place; its average absolute value was \$1. As for *directional income movement,* income growth took place for individual β but not for individual α. Finally, the distribution of average income over the longer term is (1, 4), which is more unequal than the base year distribution (1, 3). Mobility therefore *disequalized longer-term incomes relative to base year incomes.*

What if the underlying situation had been that of Case (b)? In this case, some of the preceding indicators are the same and some are different. Once again, there is perfect *time-dependence* in ranks but not in incomes, but this time the dependence is negative, not positive. Now, there has been *positional movement.* There also have been *share movements. Income flux* took place; now, its average magnitude is \$3. *Directional income movements* took place for both individuals in this case, upward for individual α and downward for individual β. Finally, the distribution of average income over the longer term is (3, 2), which is more equally distributed than either base year or final year income, and mobility therefore *equalized longer-term incomes relative to base year incomes*.

This subsection has asked: Was there mobility in the hotel? Was there mobility in the two-person economy? In each case, what was its nature? The answers to these questions have been shown to depend on which mobility concept is used. As shall now be demonstrated, different conclusions on the nature of mobility also arise in empirical applications for the United States and France.

How do the six mobility concepts compare in two country cases?

Examples have been given showing that the six mobility concepts can convey different impressions from one another. This subsection shows that different concepts also produce different patterns in actual countries' experiences.

The first empirical application is from the United States. Measures of the six concepts are used to gauge five-year income mobility from 1970–75 to 1990–95. Data are drawn from the Panel Study of Income Dynamics on earnings (including overtime and bonuses) for men aged 25–60 in the base year who were not students, retired or self-employed and who had positive earnings in both years. Further details are presented in Fields *et al.* (2000) and Fields (2004).

Figure 6.1 plots the time paths of five-year earnings mobility (in real dollars) for one measure of each of the six concepts, as presented in Table 6.1. Measures of time-independence, positional movement, share movement, and income flux are all seen to exhibit the same pattern: rising until 1980–85, falling thereafter. However, these time paths do not hold for the other two concepts. The measure of directional income movement exhibits a saw-tooth pattern. On the other hand, the measure of mobility as an equalizer of longer-term incomes exhibits a peak followed by a valley. Moreover, this last measure crossed over from positive values in the 1970s to zero or negative values in the 1980s and 1990s. In other words, earnings mobility among US men acted to equalize longer-term incomes relative to base year income in the 1970s and stopped doing so since. So, contrary to Krugman's conjecture and the others cited above, mobility may not be making the distribution of longer-term income more equal in the United States any longer.

A second empirical application is from France, drawing on the work of Buchinsky *et al.* (2003). The French data come from employers' declarations to the government of wages and salaries paid to each of their employees (now known as the Déclarations Annuelles de Données Sociales, formerly called the Déclarations Annuelles de Salaires). These data were merged with the information on sex, age and education level from the government's demographic registry (Echantillon Démographique Permanent). Two-year mobility in real francs was calculated beginning in 1967–69 and ending in 1997–99.

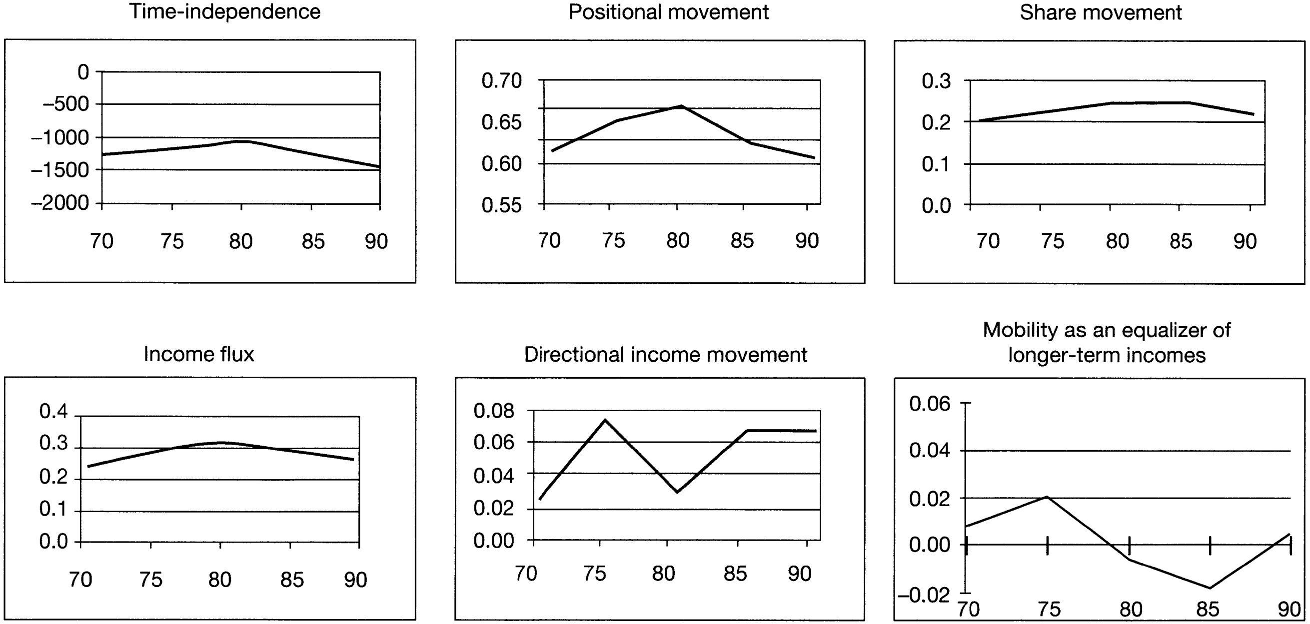

Figure 6.1 United States: evolution of earnings mobility, 1970–95

The first question for France is the same as for the US: How did mobility evolve over time for each of these six mobility concepts? In this case, Figure 6.2 demonstrates that measures of the first five mobility concepts all exhibit the same pattern: higher mobility at the beginning, followed by a sharp drop, and then a levelling-off at a new lower level. However, the sixth mobility concept – mobility as an equalizer of longer-term income – shows a different pattern. This type of mobility reversed course and has now reached its earlier levels. Note, too, that in France, unlike the United States, these values are always positive; that is, two-year average earnings have always been more equally distributed than base year earnings were.

The second and third questions for France concern demographic differences. Who has more mobility: women or men; better educated or less educated workers? The data are presented in Figures 6.3 and 6.4. For both questions, the answers differ depending on which mobility concept is used. By gender, women have more time-independence and positional movement than men, less share movement than men, about the same flux and directional movement in logs, and about the same amount of mobility as an equalizer of longer-term incomes. By education, those with the highest educational attainments have less time-independence and positional movement and, if anything, more share movement, flux, and directional income movement in logs. Finally, mobility equalized longer-term incomes less for the best educated and moderately educated than for the least educated in the early years, but this difference appears to have disappeared more recently.

In summary, these results for the United States and France show that the different mobility concepts produce qualitatively different empirical patterns. For some mobility concepts, mobility has fallen over time; for others, it has not. For some mobility concepts, women have more mobility than men; for others, men have more mobility than women. For some mobility concepts, mobility rises with education; for others, it falls.

These different results imply that researchers must be very cautious before saying that mobility is higher here than there, for this group as compared with that group, or now as compared with before. Mobility studies must specify which mobility concept, or concepts, is under discussion. Rather than talking about 'mobility', analysts would be able to communicate more effectively if we were to speak in terms of 'positional movement', 'income flux', or whatever. Let analysts decide which aspect(s) of mobility is (are) of greatest interest and choose the mobility indices accordingly. As these empirical results demonstrate, it does make a difference which concept(s) one chooses to measure.

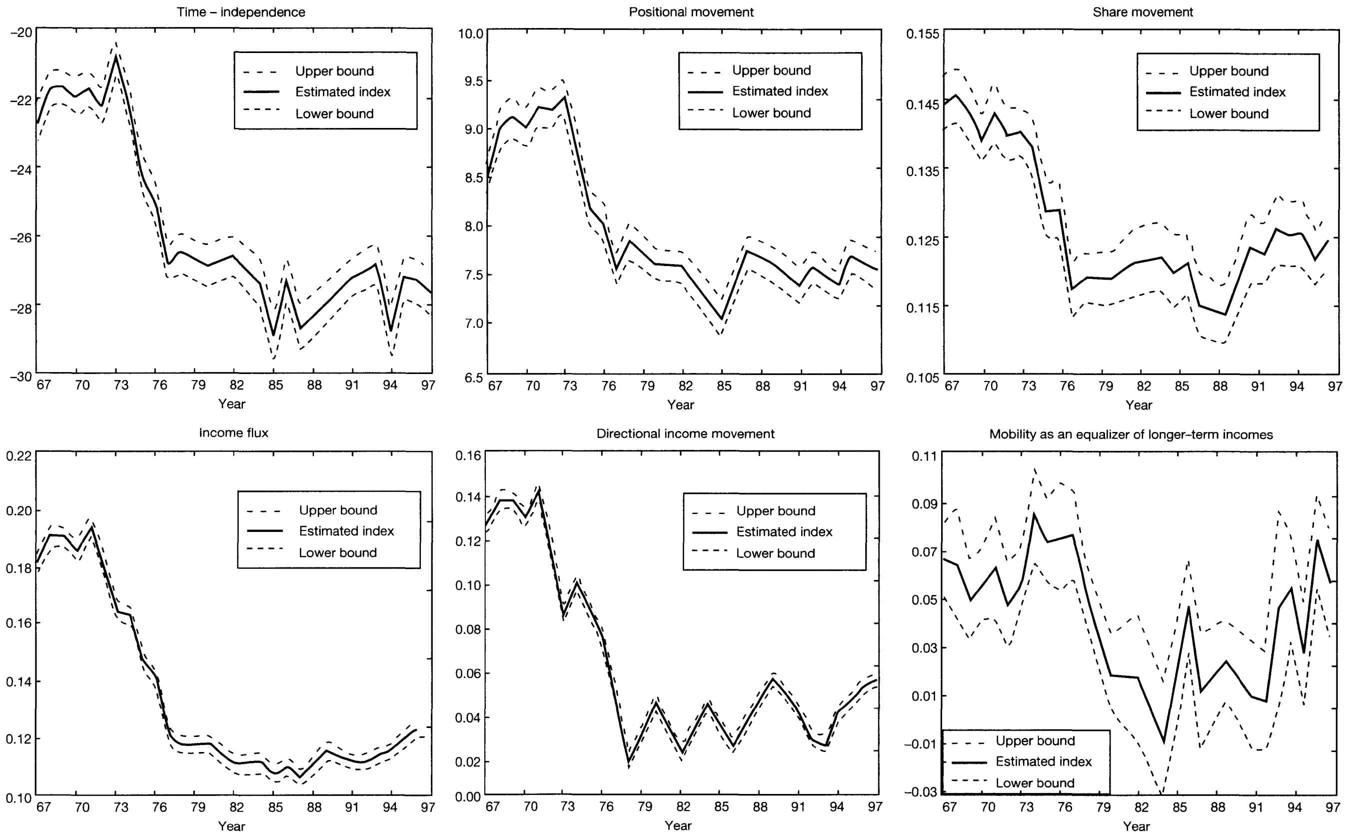

Figure 6.2 France: evolution of wage mobility, 1967–99, overall (with 95 per cent confidence intervals)

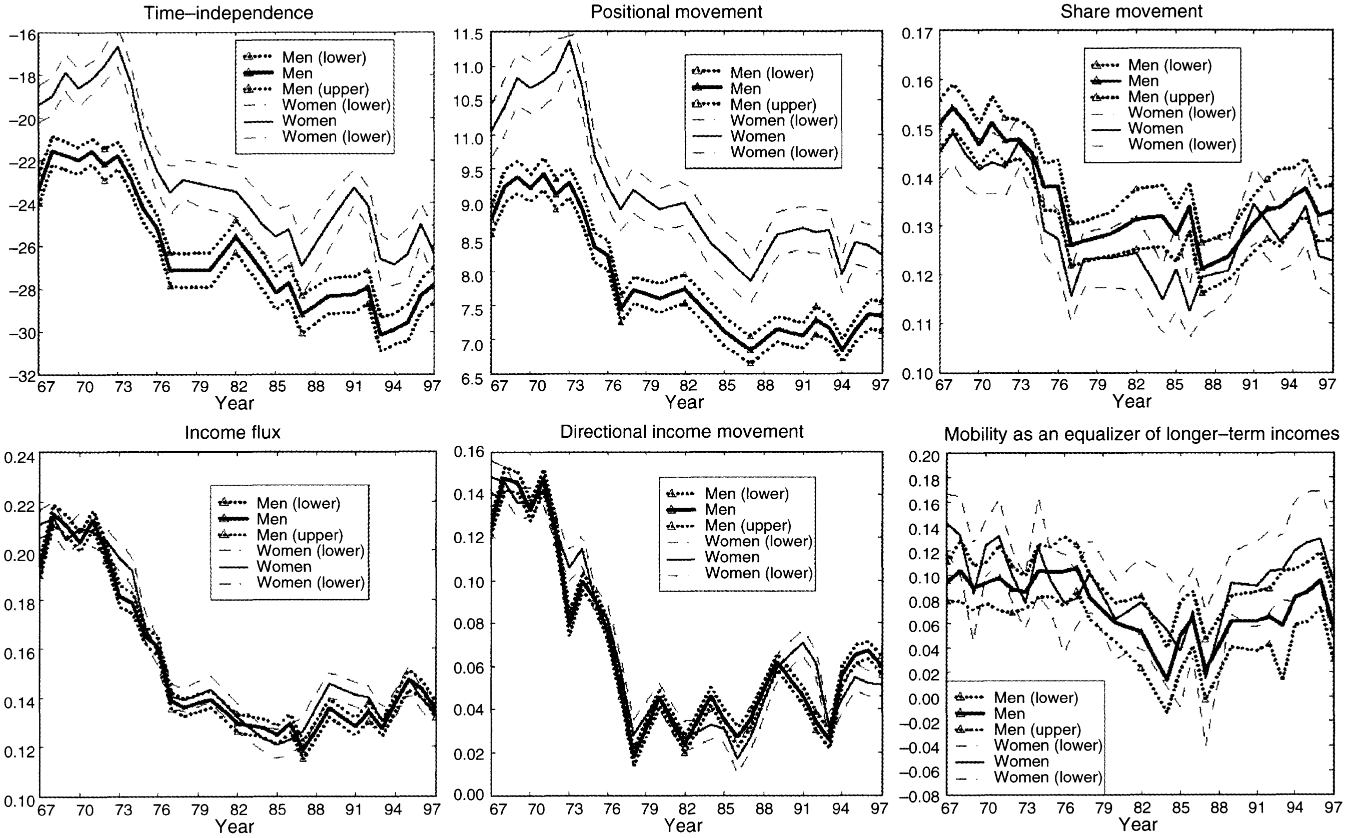

Figure 6.3 France: evolution of wage mobility, 1967–99, by gender (with 95 per cent confidence intervals)

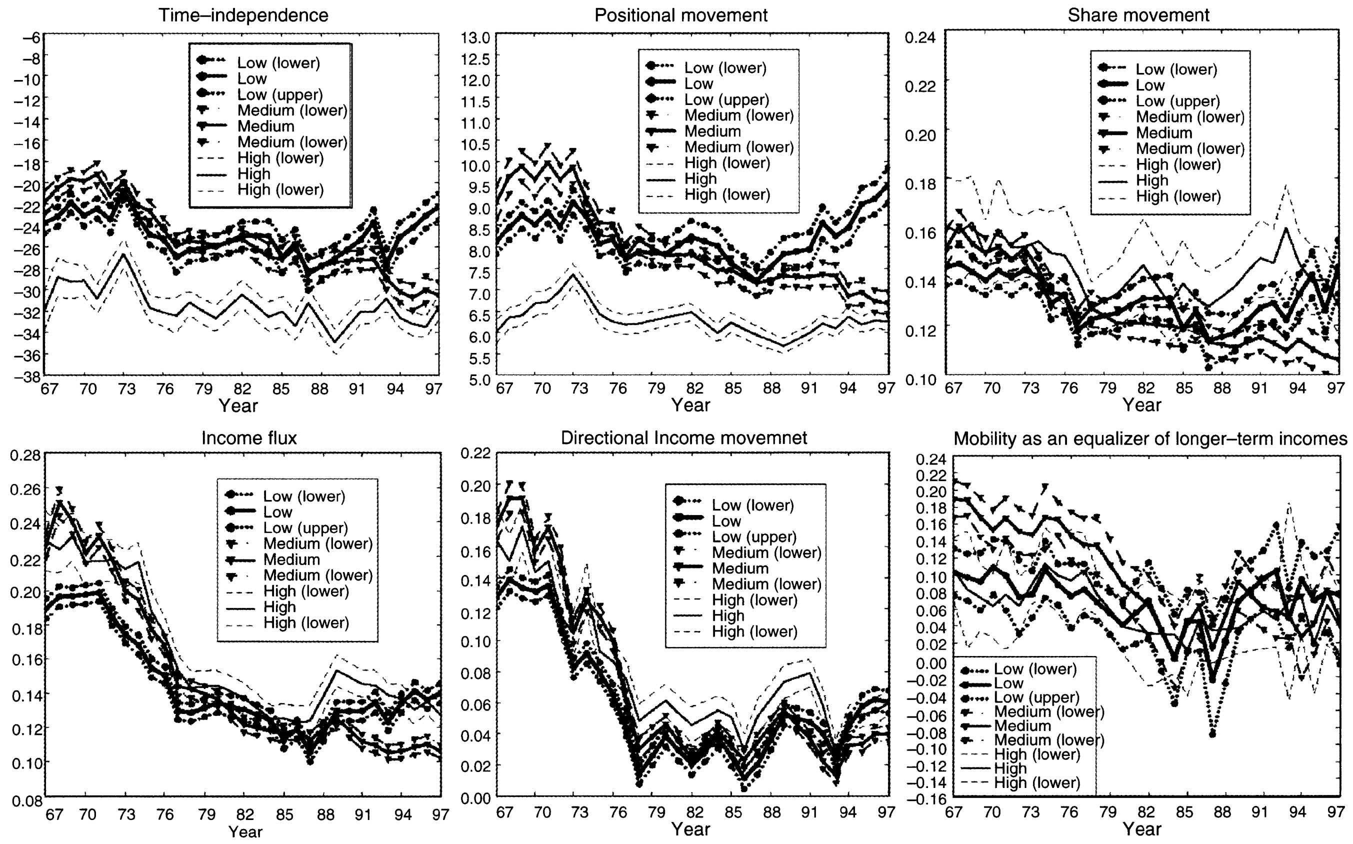

Figure 6.4 France: evolution of wage mobility, 1967–99, by education (low: primary or junior high sch.; medium: technical sch.; high secondary dipl. (baccalaureat and beyond)

Conclusion

This chapter demonstrated that economic mobility truly is multifaceted. The six facets are time-independence, positional movement, share movement, flux, directional income-movement, and mobility as an equalizer of longer-term incomes. These six concepts were explored, and it was shown that they can produce very different qualitative answers to such basic questions as whether economic mobility is increasing or decreasing over time, whether women have more or less economic mobility than men, and whether the better educated have more or less mobility than those with less education.

It follows that mobility comparisons can only be made once the mobility concept under examination has been made precise. An unqualified statement of the form 'Mobility is higher in *A* than in *B*' is as vague and meaningless as saying that 'Income distribution is better in *X* than in *Y*'. In the same way that researchers have learned to talk about which aspect of the income distribution is better in *X* than in *Y* (for example, location, dispersion or economic well-being) according to a particular measure of that aspect, researchers also need to learn to talk about which aspect of mobility is higher in *A* than in *B*.

The various mobility concepts used in the literature differ from one another in ways that are only imperfectly understood. Likewise, the various measures of a given concept also differ from one another in ways that are only imperfectly understood. A task for future research would be to explore these differences and systematize them.

Notes

1. The original citation is Schumpeter (1955: 126). Schumpeter is cited in Sawhill and Condon (1992) and Danziger and Gottschalk (1995). Jarvis and Jenkins (1996) use the same analogy.
2. Surveys of the literature on economic mobility may be found in Atkinson *et al.* (1992), Maasoumi (1998), Fields and Ok (1999a) and Morgan (2005).
3. It might be better to say nothing believable can be learned from such anonymous data. Some researchers have not been content to say nothing when only anonymous data are available, instead making assumptions as to how particular individuals' incomes change in comparable cross-sections. The answers merely reflect the assumptions maintained in deriving them. The results of such exercises are literally unbelievable and should be given no credence.
4. In particular, the measure proposed by Shorrocks (1978) and generalized by Maasoumi and Zandvakili (1986) gauges the inequality of longer-term incomes relative to the weighted average of inequality in each period, not inequality in the base period. On the other hand, while the index proposed by Chakravarty *et al.* (1985) does relate the inequality of longer-term incomes to inequality in the base period, when they put their index to use, they assign welfare significance only to the change in inequality and not to any change in the level of income. See Fields (2004) for further discussion.
5. In the empirical work below, the Gini coefficient is used for the United States, and Theil's L index for France.

6. Rejecting perfect *time-dependence* does not mean that we do or do not have perfect *time-independence*. Whether we do or not depends on whether all guests are randomly assigned to rooms night after night or whether the assignment one night is linked to the previous assignment.

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7

Social Groups and Economic Poverty: A Problem in Measurement

S. Subramanian

Motivation

This chapter combines the themes of poverty and inequality, within a measurement setting, with a view to elucidating some of the complications that can arise, and how these might be addressed, when we allow for a certain elementary obtrusion of considerations of 'society' into routinely mainstream notions of the 'economy'. Specifically, the concern is with reckoning aspects of distributive justice from a group perspective, in addition to the more standardly individualistic perspective, with an emphasis on the sorts of conflicts which these alternative perspectives could engender, and how these conflicts might be reconciled in the process of seeking a real-valued measure of income poverty. The two perspectives of distributive justice just alluded to are handily described by Stewart (2002) in the terms, respectively, of horizontal inequality and vertical inequality. Much of received theorizing has been concerned almost exclusively with vertical inequality, which has tended to confine horizontal inequality, in a relative sense, to the unhappy status of what Stewart (*op. cit.*) calls 'a neglected dimension of development'. The question of why groups deserve a great deal more analytical and empirical attention than they would appear to have received in the discourse on poverty, inequality and development has been dealt with fairly exhaustively in Stewart's work, and therefore represents ground that one does not need to cover again here. Reference, in this context, must also be made to earlier work, notably from the viewpoint of measurement, by Anand and Sen (1995), Jayaraj and Subramanian (1999), Majumdar (1999), Majumdar and Subramanian (2001), and Subramanian and Majumdar (2002).

It is perhaps important to stress that the analytical content of this chapter – whether in the matter of the existence results it advances or the specific poverty measures it discusses – is not motivated by any illusion as to either its revelation or novelty value. The arguments in this chapter are, on the whole, uniformly simple and obvious. It is perhaps precisely because of this obviousness that attention needs to be drawn to the pervasive reality and centrality of groups in any assessment of social welfare; and the motivational concern of this chapter is, simply, to point to the obvious so that it is not overlooked. There is nothing very paradoxical in this: it is just another instance of Edgar Allan Poe's purloined letter. If the notion of horizontal inequality has met with traditionally little engagement in exercises dealing with the assessment of overall deprivation or well-being, this fact probably has much to do with the common failure of being blind to what stares one in the face (excepting, of course, those instances of a deliberate ideological opposition to the notion of groups and their relevance in the scheme of things). The motivational objective of this chapter, therefore, is to highlight an issue for reasons that arise not from its complexity, but from a combination of its importance and its relative historical neglect.

Measuring poverty in a stratified society

An issue of potential interest in the measurement of poverty has to do with the way in which poverty is distributed across different well-defined subgroups within the population.[1] Foster and Shorrocks (1991) have advanced and studied a property of poverty indices which they call *subgroup consistency* and which demands that, other things equal, an increase in any subgroup's poverty should increase overall poverty. In motivating their discussion of this property, the authors (1991: 687) state: 'Subgroup consistency may ... be regarded as a natural analogue of the monotonicity condition of Sen (1976), since monotonicity requires that aggregate poverty fall ... if one *person's* poverty is reduced, *ceteris paribus*, while subgroup consistency demands that aggregate poverty fall if one *subgroup's* poverty is reduced, *ceteris paribus*.' In this connection, it is immediately tempting to seek also an analogy between the conventional *transfer* axiom and a corresponding one which could be defined for subgroups.

The transfer condition requires that, *ceteris paribus*, a progressive rank preserving transfer between two poor individuals should be accompanied by a reduction in poverty. In a similar spirit, one could require – speaking loosely for the moment – that aggregate poverty should decline with a move toward equalization, through income redistribution, of subgroup poverty levels, other things remaining the same. The requirement is formalized, in this note, through the postulation of a property called *subgroup sensitivity*.

The relevance of subgroup sensitivity is captured in the following illustration. Suppose poverty to be measured by the simple headcount ratio. Imagine that the population is partitioned into two subgroups, A and B, where A stands for a historically disadvantaged social group, say, and B stands for the rest. Suppose the headcount ratio of poverty for subgroup A to be 0.7 and that for subgroup B to be 0.3. If subgroup A's share in total population is 50 per cent, then the headcount ratio for the population as a whole would be $0.5 (= 0.5 * 0.7 + 0.5 * 0.3)$. If now there is a pure redistribution of income from subgroup B to subgroup A, whereby A's headcount ratio is reduced to 0.6 while B's headcount ratio is raised to 0.4, then we may be disposed to judge that such a movement toward equalization of poverty across subgroups should lead to an overall reduction in measured poverty. This, precisely, is the sort of judgement that would be endorsed by the axiom of subgroup sensitivity. Such a possibility, however, is not accommodated by the headcount ratio which, in the context of the present example, continues to remain at $0.5 (= 0.5 * 0.6 + 0.5 * 0.4)$. This simple example points to a possible limitation underlying conventional approaches to the measurement of poverty.

The difficulty in question resides in the fact that certain axioms for poverty measurement have been advanced on the implicit assumption that there is no more than one group – that constituted by the population as a whole – that needs to be reckoned in an overall assessment of the extent of poverty in a society. This assumption is particularly salient in the so-called 'symmetry' and 'transfer' axioms. The former property demands, in essence, that in making poverty comparisons across income profiles, the personal identities of individuals should be of no account. This property is also a standard feature of the literature in social choice theory, where it more commonly goes by the name of the 'anonymity' axiom. One can immediately see that if the identity of an individual is linked to the fact of group affiliation, then a poverty index that is sensitive to the group composition of a population could well militate against the requirement of anonymity imposed by the symmetry axiom. The axiom in question is widely regarded as being completely innocuous and self-evidently desirable from an ethical point of view: it is, indeed, so much taken for granted that its social choice version – anonymity – has come in for specifically targeted criticism in a carefully argued assessment by Loury (2000), who refers to the anonymity axiom as a stark example of 'liberal neutrality'. What could constitute a possible objection to symmetry/anonymity which, after all, echoes a requirement that is a feature of many liberal constitutions – the requirement that no person may be discriminated against on the grounds of birth, race, class, caste, religion or sex?

Here is an objection: one may wish to discriminate in favour of members of an historically oppressed and consequently currently disadvantaged group; but in order to discriminate *in favour of* somebody, one will have to discriminate *against* somebody else on the ground of the latter's group affiliation – an avenue of redress for the former individual which is denied by the symmetry axiom. Briefly, symmetry cannot be reconciled with group-based principles of distributive justice such as are embodied in provisions like 'compensatory discrimination' or 'affirmative action'. The preceding discussion suggests that symmetry is an unquestionably desirable property when one is assessing inequality or poverty or welfare in the context of a homogeneous population; however – and possibly because of repeated, mechanical use – often the qualifying attribute of homogeneity seems implicitly to be forgotten when the axiom is invoked. Indeed, one of the few authors who are careful to rationalize the symmetry axiom on the grounds of its appeal in the context of homogeneous populations is Shorrocks (1988). One could, of course, suppose that symmetry is so widely specified as a desirable property only because of a formally unstated assumption as to the homogeneity of a population; but somehow, this is a less than convincing explanation when the axiom is also routinely invoked in the context of exercises that explicitly accommodate groups into the analysis, and therefore are concerned with heterogeneous populations.

Similarly, where the transfer axiom is concerned, one can see that considerations of intergroup equality could, in specific cases, conflict with considerations of interpersonal equality. Regressive transfers between members of a homogeneous group may naturally be taken to reduce welfare and enhance inequality and poverty; but should the same outcome necessarily hold for transfers between individuals belonging to different groups in a heterogeneous population? It is these sorts of difficulties attending the measurement of poverty that are sought to be made transparent in this chapter.

The next section deals with certain relevant preliminary formalities of concepts and definitions. A couple of general possibility theorems on poverty indices follow which highlight the complications that can arise from taking the issue of group-wise poverty distribution seriously. Then comes a brief interpretation and assessment of the results presented in the previous section. Examples from the literature of 'group-sensitive' poverty measures are discussed, followed by a brief discussion of the implications of 'group-sensitivity' for budgetary intervention in poverty redress exercises. The chapter closes with conclusions drawn.

Formalities[2]

Let N be the set of all positive integers. For every $n \in N$, let X_n be the set of non-negative vectors $\{(x_1, \ldots, x_i, \ldots, x_n)\}$, and define X to be the set $\cup_{n \in N} X_n$. A typical element of the set X is an income vector **x**, a typical element x_i of which stands for the income of person i ($i \in N$). For every $\mathbf{x} \in X$, $n(\mathbf{x})$ stands for the dimensionality of **x**. The *poverty line*, z, is a positive level of income such that individuals with incomes less than z are certified to be poor. For every $(z, \mathbf{x}) \in T \times X$ (where T is the set of positive reals), $\mathbf{x}_P(z, \mathbf{x})$ will stand for the vector of poor incomes; $\mathbf{x}_R(z, \mathbf{x})$ will stand for the vector of non-poor incomes; and $\mu^P(z, \mathbf{x})$ will stand for the average income of the poor population. A vector $\mathbf{x} \in X$ will be said to be derived as a *permutation* of a vector $\mathbf{y} \in X$ if $\mathbf{x} = \mathbf{y}\Pi$ for some permutation matrix Π; and $\mathbf{x}^o$ is the *ordered* version of **x** if $\mathbf{x}^o$ is derived from **x** by a permutation for which $x_i^o \leq x_{i+1}^o (i = 1, \ldots, n(\mathbf{x}) - 1)$.[3] For all $\mathbf{x}, \mathbf{y} \in X$, it will be said that **x** *vector-dominates* **y** – written **x**V **y** – if **x** and **y** are equi-dimensional, $x_i^o \geq y_i^o$ for all i and $x_i^o > y_i^o$ for some i; **x** will be said to be derived from **y** through *an increment to a person's income* if **x** and **y** are equi-dimensional, $x_i = y_i$ for all $i \neq k$ and $x_k > y_k$ for some k; and **x** will be said to be derived from **y** through *a permissible progressive transfer* if **x** and **y** are equi-dimensional, $x_i = y_i$ for all $i \neq j, k$ for some j,k satisfying $y_j < y_k$, $x_j = y_j + \delta$ and $x_k = y_k - \delta$ where $0 \leq \delta \leq (y_k - y_j)/2$.

Since a major concern of this chapter is with the notion of reference groups, I turn now to this latter issue. For every $n \in N$, let $\mathbf{G}_n$ be the set of all possible partitions of the set $\{1, \ldots, n\}$, and define **G** to be the set $\cup_{n \in N} \mathbf{G}_n$. A typical element of the set **G** is a partition **g** of the population, and a typical element of **g** is a subgroup, denoted by the running index j. (It is immediate, of course, that for any partition **g** of the population, the number of subgroups must be at least one, and cannot exceed the number of individuals in the population). Notice that any $\mathbf{g} \in \mathbf{G}$ is induced by some appropriate *grouping* of the population, by which is meant some well-defined scheme of categorization (such as by height, age, gender, caste, religion, and so on), in accordance with which the population can be classified in a mutually exclusive and completely exhaustive fashion. (It is possible, of course, that two or more groupings can induce the *same* partitioning of a given population: for example, if in some society the only illiterate individuals all happen to be females, then a grouping according to gender [{male, female}] will yield up the same partition as a grouping according to literacy status [{literate, illiterate}].) For all $\mathbf{x} \in \mathbf{X}$ and $\mathbf{g} \in \mathbf{G}$, the pair (**x**, **g**) will be said to be *compatible* if and only if **g** partitions a population of the same size as the dimensionality of **x**. Given any compatible pair (**x**, **g**) belonging to **X** × **G**, subgroup j's vector of incomes will be represented by $\mathbf{x}^j$, for every j belonging to **g**.

Two polar cases of grouping are of interest. The first is what one may call the *atomistic grouping*, which induces the *finest* partition $\mathbf{g}^a = \{\{1\}, \ldots, \{i\}, \ldots, \{n\}\}$ of $\{1, \ldots, n\}$: this is the case of 'complete heterogeneity', such as might be precipitated by a classification according to 'finger-print type'. The second is what one may call the *universal grouping*, which induces the *coarsest* partition $\mathbf{g}^u = \{\{1, \ldots, n\}\}$ of $\{1, \ldots, n\}$: this is the case of 'complete homogeneity', such as might be precipitated by a classification according, say, to membership to the human race.

We are now in a position to define a *poverty index*, by which shall be meant a mapping $P : T \times \mathbf{X} \times \mathbf{G} \rightarrow R$ (where R is the real line), such that, for all $z \in T$, and all compatible $(\mathbf{x}, \mathbf{g}) \in \mathbf{X} \times \mathbf{G}$, $P(z, \mathbf{x}, \mathbf{g})$ is a unique real number which is intended to signify the extent of poverty that obtains in the regime $(z, \mathbf{x}, \mathbf{g})$. To invest P with more structure, we need to constrain it with a set of properties that we may require the poverty index to satisfy. What follows is a restricted set of six axioms, of which the sixth[4] is a relatively recent addition to the stock of known axioms.

Symmetry (Axiom S) For all $z \in T$, all $x, y \in X$ and all $\mathbf{g} \in \mathbf{G}$ such that $(x, \mathbf{g})$ and $(y, \mathbf{g})$ are compatible pairs, if x is derived from y by a permutation, then $P(z, x, \mathbf{g}) = P(z, y, \mathbf{g})$.

Monotonicity (Axiom M) For all $z \in T$, all $x, y \in X$ and all $\mathbf{g} \in \mathbf{G}$ such that $(x, \mathbf{g})$ and $(y, \mathbf{g})$ are compatible pairs, if x is derived from y by an increment to a poor person's income, then $P(z, x, \mathbf{g}) < P(z, y, \mathbf{g})$.

Respect for Income Dominance (Axiom D) (See Amiel and Cowell 1994.) For all $z \in T$, all $x, y \in X$ and all $\mathbf{g} \in \mathbf{G}$ such that $(x, \mathbf{g})$ and $(y, \mathbf{g})$ are compatible pairs, if $x_P V y_P$, then $P(z, x, \mathbf{g}) < P(z, y, \mathbf{g})$.

> (Note: Amiel and Cowell (1994) point out that Axioms M and D are independent, but are rendered equivalent in the presence of the symmetry axiom.)

Transfer (Axiom T) For all $z \in T$, all $x, y \in X$ and all $\mathbf{g} \in \mathbf{G}$ such that $(x, \mathbf{g})$ and $(y, \mathbf{g})$ are compatible pairs, if $x_R = y_R$ and x_P is derived from y_P by a permissible progressive transfer, then $P(z, x, \mathbf{g}) < P(z, y, \mathbf{g})$.

Subgroup Consistency (Axiom SC) (See Foster and Shorrocks 1991.) For all $z \in T$, all $x, y \in X$ and all $\mathbf{g} \in \mathbf{G}$ such that $(x, \mathbf{g})$ and $(y, \mathbf{g})$ are compatible pairs, if x^j and y^j are of the same dimensionality for all $j \in \mathbf{g}$ and $[P(z, x^j, \mathbf{g}^u) = P(z, y^j, \mathbf{g}^u)$ for all $j \in \mathbf{g} \backslash \{k\}$ and $P(z, x^k, \mathbf{g}^u) < P(z, y^k, \mathbf{g}^u)$ for some $k \in \mathbf{g}]$, then $P(z, x, \mathbf{g}) < P(z, y, \mathbf{g})$.

Subgroup Sensitivity (Axiom SS) For all $z \in T$, all $x, y \in X$ and all $\mathbf{g} \in \mathbf{G}$ such that $(x, \mathbf{g})$ and $(y, \mathbf{g})$ are compatible pairs, if (i) x^j and y^j are of the same dimensionality for all $j \in \mathbf{g}$; (ii) $\mu^P(z, x) = \mu^P(z, y)$; and (iii) $x^j = y^j$ for all $j \in \mathbf{g} \backslash \{s, t\}$ for some s,t satisfying $P(z, x^t, \mathbf{g}^u) < P(z, y^t, \mathbf{g}^u) \leq P(z, y^s, \mathbf{g}^u) < P(z, x^s, \mathbf{g}^u)$; then it is the case that $P(z, x, \mathbf{g}) > P(z, y, \mathbf{g})$.

(That is, other things remaining the same, if by a pure redistribution of poor incomes the relatively disadvantaged subgroup s becomes less poor and the relatively advantaged subgroup t becomes poorer, while maintaining the relative poverty rankings of the two subgroups, then overall poverty should decline.)

What is the class of poverty indices which satisfy the property of subgroup sensitivity in conjunction with some combination of other desirable properties discussed earlier? This question is now addressed.

Two general possibility results for poverty indices

This is a brief section in which it is demonstrated that certain standard axioms employed in the measurement of poverty could prove to be less innocuous than they appear to be, when social groups are explicitly factored into the measurement of aggregate poverty. In particular, the following two propositions are true:

Proposition 1 *There exists no poverty index* $P : T \times \mathbf{X} \times \mathbf{G} \to R$ *satisfying Axioms S, M and SS.*

Proof What follows is a proof by contradiction. Consider a situation in which $z = 50$. Let $\mathbf{g}$ be such that the population is partitioned into exactly two subgroups which are indexed 1 and 2 respectively. Consider a pair of income vectors $\mathbf{x}, \mathbf{y}$ such that $n(x_P) = n(y_P) = n(x_R) = n(y_R) = 2$. Further, assume that $x^1_R = x^2_R = y^1_R = y^2_R = (60, 60)$; and that $x^1_P = (10, 20)$; $x^2_P = (30, 40)$; $y^1_P = (10, 30)$; and $y^2_P = (20, 40)$. It is immediately clear that

$$\mu^P(z, \mathbf{x}) = \mu^P(z, \mathbf{y})(= 25) \tag{7.1}$$

Further, since P satisfies Axioms S and M, it must satisfy Axiom D as well; and by Axiom D, given that $n(x^1) = n(x^2) = n(y^1) = n(y^2)$ and $x^2_P V y^2_P V y^1_P V x^1_P$, we have

$$P(z, x^2, \mathbf{g}^u) < P(z, y^2, \mathbf{g}^u) < P(z, y^1, \mathbf{g}^u) < P(z, x^1, \mathbf{g}^u) \tag{7.2}$$

In view of (7.1) and (7.2), Axiom SS will dictate that

$$P(z, \mathbf{x}, \mathbf{g}) > P(z, \mathbf{y}, \mathbf{g}) \tag{7.3}$$

Next, notice that $\mathbf{x} = (x^1_P, x^2_P, x^1_R, x^2_R)$ is just a permutation of $\mathbf{y} = (y^1_P, y^2_P, y^1_R, y^2_R)$: if the person with income 20 in x^1_P swaps places with the person with income 30 in x^2_P, then x^1_P becomes y^1_P and x^2_P becomes y^2_P. By Axiom S, one must have

$$P(z, \mathbf{x}, \mathbf{g}) = P(z, \mathbf{y}, \mathbf{g}) \tag{7.4}$$

(7.3) and (7.4) are mutually incompatible, and this completes the proof of the proposition. □

Proposition 2 *There exists no poverty index* $P : T \times \mathbf{X} \times \mathbf{G} \to R$ *satisfying Axioms D, T and SS.*

Proof Again, we have a proof by contradiction. As in the proof of Proposition 1, imagine a situation in which $z = 50$, and $\mathbf{g}$ is such as to partition the population into two subgroups, 1 and 2. Let $\mathbf{x}$ and $\mathbf{y}$ be two income vectors satisfying $n(x_P) = n(y_P) = n(x_R) = n(y_R) = 2$, and let it be the case that $x^1_R = x^2_R = y^1_R = y^2_R = (60, 60)$; $x^1_P = (10, 20); x^2_P = (20, 30)$; $y^1_P = (10, 25)$; and $y^2_P = (15, 30)$. Notice first that

$$\mu^P(z, \mathbf{x}, \mathbf{g}) = \mu^P(z, \mathbf{y}, \mathbf{g})(= 20) \tag{7.5}$$

Further, since $n(x^1) = n(x^2) = n(y^1) = n(y^2)$, and $x_p^2 V y_p^2 V y_p^1 V x_p^1$, Axiom D will require that

$$P(z, x^2, g) < P(z, y^2, g) < P(z, y^1, g) < P(z, x^1, g) \tag{7.6}$$

In view of (7.5) and (7.6), Axiom SS will dictate that

$$P(z, x, g) > P(z, y, g) \tag{7.7}$$

Next, it is easy to see that $x_P = (x_p^1, x_p^2)$ has been derived from $y_P = (y_p^1, y_p^2)$ by a permissible progressive transfer (of 5 from the person with income 25 in the vector y_p^1 to the person with income 15 in the vector y_p^2). Given, additionally, that $n(x) = n(y)$, Axiom T will demand that

$$P(z, x, g) < P(z, y, g) \tag{7.8}$$

From (7.7) and (7.8), we obtain a contradiction. This completes the proof of the proposition. □

Assessment

The two non-existence results presented in the preceding section confirm the wisdom of a moral (suitably translated to the poverty-measurement context) that has been upheld by Sen in his discussion of impossibility results in social choice theory. This moral (Sen 1970: 178) points to 'the sever[ity] of the problem of postulating absolute principles ... that are supposed to hold in every situation'. Confronted with an impossibility theorem, it is not always a simple matter to be able convincingly to identify the 'villain of the piece'; namely, the guilty axiom that is driving the result under review. Specifically, given the present context, and by way of illustration, neither the transfer nor the subgroup sensitivity axiom is as persuasive as either may appear in a context-free environment. For example, in particular cases the antecedents of Axiom SS can be satisfied by a very regressive transfer (from an acutely impoverished person belonging to a relatively advantaged subgroup to a much better off individual belonging to a relatively disadvantaged subgroup), and in such cases we may find it hard to endorse Axiom SS. By the same token, *any* permissible progressive transfer between two poor individuals would be encouraged by the transfer axiom, and in particular cases, wherein such transfers exacerbate inter-group poverty differentials, we may find it hard to accept Axiom T.

Given this general difficulty of discerning an unqualified virtue in any given axiom under all plausible circumstances, a possible way out may be to restrict the applicability of the axiom to a domain that is relatively non-controversial. In the specific instance of the *symmetry* axiom, for example, there may be a case for requiring only that *within* any subgroup, swapping incomes across members of the subgroup should leave the value of the poverty index for the subgroup unchanged. In a *between-group* context, one might further wish to impose the restriction that measured poverty should be invariant with respect to the precise labels that are attached to subgroups. Similarly, in the case of the *transfer* axiom, one may wish to restrict its operation, in its conventional form, only to interpersonal redistributions of income *within* each subgroup. Is there also a way of ensuring some requirement of equity, from a *between-group* perspective, in the distribution of poverty across subgroups? How this can be done is perhaps best exemplified by means of an illustration, which is now discussed.

'Group-sensitive' poverty indices: an example

Let $\mathbf{P}^*$ be the set of all symmetric, monotonic, transfer-satisfying, and decomposable poverty indices. (A poverty index is said to be *decomposable* – see Foster *et al.* (FGT hereafter) 1984 – if overall poverty can be expressed as a population-share weighted sum of subgroup poverty levels). Let $(\mathbf{x}, \mathbf{g})$ be a compatible pair belonging to $\mathbf{X} \times \mathbf{G}$, and let $\mathbf{g}$ partition the population into K distinct subgroups. The poverty line, as usual, is given by z. Consider $P^* \in \mathbf{P}^*$, and let P_j^* serve as a shorthand for $P^*(z, \mathbf{x}^j, \mathbf{g}^u)$, which is the poverty level, as measured by the index P^*, of the jth subgroup $(j = 1, \ldots, K)$. Assume, further, that the subgroups have been indexed in non-increasing order of poverty, so that $P_j^* \geq P_{j+1}^*$, $j = 1, \ldots, K-1$. Let θ_j be the population share of the jth subgroup, and Θ_j the proportion of the population that belongs to groups whose poverty levels are less than or equal to the poverty level of the jth subgroup. Then, two examples of aggregate poverty measures that directly incorporate considerations of 'inter-group equity' are the indices $_1P$ and $_2P$ below:

$$_1P(z, \mathbf{x}, \mathbf{g}) = [1/(K-1)]\Sigma_{j=1}^{K}[(K-1-j)\theta_j + \Theta_j]P_j^* \text{ and} \tag{7.9}$$

$$_2P(z, \mathbf{x}, \mathbf{g}) = [\Sigma_{j \in \mathbf{g}}\theta_j(P_j^*)^2]^{1/2} \tag{7.10}$$

The index $_1P$ is of a type discussed in Jayaraj and Subramanian (1999), Majumdar and Subramanian (2001), and Subramanian and Majumdar (2002), while the index $_2P$ is of a type discussed in Anand and Sen (1995). In what follows, we shall confine attention to the class of indices embodied in (7.10); and, in the interests of specificity, it would be useful to particularize the index P^* to a familiar poverty measure. To this end, consider the so-called P_α class of indices proposed by FGT (1984) (FGT P_α), and given, for all $z \in T$ and all compatible $(\mathbf{x}, \mathbf{g}^u) \in \mathbf{X} \times \mathbf{G}$, by

$$P_\alpha(z, \mathbf{x}, \mathbf{g}^u) = (1/n(\mathbf{x}))\Sigma_{i \in Q(\mathbf{x})}[(z - x_i)/z]^\alpha, \alpha \geq 0 \tag{7.11}$$

where $Q(\mathbf{x})$ is the set of poor individuals in $\mathbf{x}$. Each member of the class of indices P_α (see FGT 1984) is known to be symmetric and decomposable; and the index P_2, in addition, satisfies the monotonicity and transfer axioms. Let us designate by F the index P_2. A specialization of the class of 'between-group equity-conscious' poverty measures encompassed in (7.10) is yielded by the following index,[5] F^a, which is given, for all $z \in T$, and all compatible $(\mathbf{x}, \mathbf{g}) \in \mathbf{X} \times \mathbf{G}$, by

$$F^a(z, \mathbf{x}, \mathbf{g}) = [\Sigma_{j \in \mathbf{g}} \theta_j F_j^2]^{1/2} \tag{7.12}$$

In what sense does F^a attend to the concern for inter-group equity? One way of seeing this is, first, to note that $C^2 \equiv [(1/F^2)\Sigma_{j \in \mathbf{g}} \theta_j F_j^2 - 1]$ is the squared coefficient of variation in the distribution of the group-specific poverty levels F_j. Then, it is clear that $\Sigma_{j \in \mathbf{g}} \theta_j F_j^2 = F^2(1 + C^2)$ whence, in view of (7.12), and after making the appropriate substitution, we have

$$F^a(z, \mathbf{x}, \mathbf{g}) = F(1 + C^2)^{1/2} \tag{7.13}$$

Notice from (7.13) that the index F^a is just the average (across subgroups) level of poverty, as measured by the index F, enhanced by a factor incorporating the squared coefficient of variation in the inter-group distribution of poverty as measured by the F_j. F^a is mean poverty 'adjusted' for inter-group inequality. (It may be noted that for the class of poverty indices subsumed in (7.9), the 'adjustment' for inequality in the inter-group distribution of poverty levels is via a 'Gini-type', rather than 'coefficient of variation-type', inequality measure.)

It is not difficult to check that the index F^a satisfies both the 'within-group' and the 'between-group' versions of the symmetry property discussed earlier. 'Within-group', because it is a known property of the FGT P_α class of indices that each member satisfies symmetry, and each of the F_j is just the FGT index realized for $\alpha = 2$. 'Between-group', because one can see from inspection of (7.12) that switching around the labels of the subgroups will make no difference to the value of F^a. Further, F^a clearly also satisfies the 'within-group' version of the transfer property, since each of the F_j is known to be transfer-respecting (indeed, the P_α indices all satisfy transfer for every $\alpha > 1$); while in a 'between-group' context, as the expression for F^a in (7.11) makes clear, it *is* sensitive to inter-group inequality in the distribution of poverty: when the latter (as measured by C^2) rises, other things equal, F^a also registers an increase in value.

An advantage with a poverty index such as F^a resides in a specific sort of 'flexibility' it possesses, in that it effects a trade off between the conventional transfer axiom and Axiom SS, by allowing the former to 'trump' the latter when interpersonal transfers across groups are rather more than less 'progressive', and allowing the latter to 'trump' the former when interpersonal transfers across groups are rather less than more 'progressive'. A simple example, similar to one in Jayaraj and Subramanian (1999), might help to elucidate this point.

Let the poverty line z be given by 10. Let $\mathbf{x}$ and $\mathbf{y}$ be two 4-dimensional income vectors, and let $\mathbf{g}^o$ be such as to partition the population into two subgroups 1 and 2 respectively, with each subgroup having two persons in it. Similarly, let $\mathbf{u}$ and $\mathbf{v}$ be any other two 4-dimensional vectors, and let $\mathbf{g}^o$ again partition the population into the subgroups 1 and 2, with each subgroup having two members. The vectors $\mathbf{x}$, $\mathbf{y}$, $\mathbf{u}$ and $\mathbf{v}$ can be written as $\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2), \mathbf{y} = (\mathbf{y}^1, \mathbf{y}^2), \mathbf{u} = (\mathbf{u}^1, \mathbf{u}^2)$, and $\mathbf{v} = (\mathbf{v}^1, \mathbf{v}^2)$. Let it be the case that $\mathbf{x}^1 = (1, 9)$, $\mathbf{x}^2 = (1.5, 3.5)$, $\mathbf{y}^1 = (0, 9)$ and $\mathbf{y}^2 = (1.5, 4.5)$; and $\mathbf{u}^1 = (1, 9)$, $\mathbf{u}^2 = (1.5, 2), \mathbf{v}^1 = (0, 9)$ and $\mathbf{v}^2 = (1.5, 3)$. Suppose we measure poverty by the 'adjusted' index F^a of (7.12). Some routine computation will confirm that $F(z, \mathbf{x}^1, \mathbf{g}^u) = 0.205$; $F(z, \mathbf{x}^2, \mathbf{g}^u) = 0.28625$; $F(z, \mathbf{y}^1, \mathbf{g}^u) = 0.2525$; $F(z, \mathbf{y}^2, \mathbf{g}^u) = 0.25625$; $F^a(z, \mathbf{x}, \mathbf{g}^o) = 0.2490$; $F^a(z, \mathbf{y}, \mathbf{g}^o) = 0.2541$; $F(z, \mathbf{u}^1, \mathbf{g}^u) = 0.205$; $F(z, \mathbf{u}^2, \mathbf{g}^u) = 0.340625$; $F(z, \mathbf{v}^1, \mathbf{g}^u) = 0.2525$; $F(z, \mathbf{v}^2, \mathbf{g}^u) = 0.303125$; $F^a(z, \mathbf{u}, \mathbf{g}^o) = 0.2811$; and $F^a(z, \mathbf{v}, \mathbf{g}^o) = 0.2790$. Notice now that $\mathbf{y}$ is derived from $\mathbf{x}$ through a regressive transfer, exactly as $\mathbf{v}$ is derived from $\mathbf{u}$ through a regressive transfer; in both cases, the transfer is from a poor person belonging to a relatively advantaged group to a richer poor person belonging to a relatively disadvantaged group. The transfer axiom will dictate that $F^a(z, \mathbf{y}, \mathbf{g}^o) > F^a(z, \mathbf{x}, \mathbf{g}^o)$, while the subgroup sensitivity axiom will dictate that $F^a(z, \mathbf{y}, \mathbf{g}^o) < F^a(z, \mathbf{x}, \mathbf{g}^o)$; in exactly similar fashion, the transfer axiom will dictate that $F^a(z, \mathbf{v}, \mathbf{g}^o) > F^a(z, \mathbf{u}, \mathbf{g}^o)$, while the subgroup sensitivity axiom will dictate that $F^a(z, \mathbf{v}, \mathbf{g}^o) < F^a(z, \mathbf{u}, \mathbf{g}^o)$. What actually obtains is a situation in which $F^a(z, \mathbf{y}, \mathbf{g}^o)(= 0.2541) > F^a(z, \mathbf{x}, \mathbf{g}^o)(= 0.2490)$, and $F^a(z, \mathbf{v}, \mathbf{g}^o)(= 0.2790) < F^a(z, \mathbf{u}, \mathbf{g}^o)(= 0.2811)$. That is to say, in the transition from $\mathbf{x}$ to $\mathbf{y}$, Axiom T is upheld and Axiom SS violated, while in the transition from $\mathbf{u}$ to $\mathbf{v}$, Axiom T is violated and Axiom SS upheld. In both cases, an interpersonally regressive income transfer has been accompanied by a diminution in the inter-group poverty differential; only, in the first case the transfer has been more regressive than in the second (the income difference between those involved in the transfer is greater in the first case than in the second), and the poverty index F^a has effected a trade off in favour of the transfer axiom in the first case, and a trade off in favour of the subgroup sensitivity axiom in the second case. This does not accord ill with intuition for, as was pointed out on p. 150:

> in particular cases the antecedents of Axiom SS can be satisfied by a very regressive transfer (from an acutely impoverished person belonging to a relatively advantaged subgroup to a much better off individual belonging to a relatively disadvantaged subgroup), and in such cases we may find it hard to endorse Axiom SS, [while] by the same token, *any* permissible progressive transfer between two poor individuals would be encouraged by the transfer axiom, and in particular cases, wherein such transfers exacerbate inter-group poverty differentials, we may find it hard to accept Axiom T.

Additionally, F^a has the convenient property of precipitating the index F as a special case, which happens when the grouping employed is the universal grouping which induces the coarsest partition $\mathbf{g}^u$ of the population. At the other extreme, when the grouping employed is the atomistic one, which induces the finest partition $\mathbf{g}^a$ of the population, it can be verified that F^a just becomes $(P_4)^{1/2}$, where P_4 is the P_α index for $\alpha = 4$. (The details are available in Jayaraj and Subramanian 1999.)

Finally, a concrete empirical illustration of the information value of an index such as F^a may be useful. Making use of data on the cross-country distribution of per capita gross national product (GNP) available in the United Nations Development Programme's *Human Development Report* (*HDR*), one can construct a picture of global poverty. Such an exercise has been carried out in Subramanian (2003), and I draw on the results of that exercise. The *HDR 1999* provides information on per capita GNP, in 'purchasing power parity dollars', for each of 174 countries for the year 1997. The global average per capita GNP works out to a little in excess of PPP$6,000, and we shall take the 'international poverty line' to be PPP$3,000 per capita per annum – which is less than one-half of the global average per capita GNP. We shall designate this poverty line by z^*; and let x^* denote the country-wise distribution of income as presented in the *HDR 1999*. Since information on intra-country distribution is unavailable, we shall simply assume that each person in each country receives the country's average per capita GNP. A grouping of countries resorted to in the *HDR 1999* is a classification comprising the following seven groups: sub-Saharan Africa, Asia and the Pacific, the Arab states, Latin America and the Caribbean, Eastern Europe and the Commonwealth of Independent States, Southern Europe, and Industrialized countries. Let us denote by $\mathbf{g}^*$ the partition of the world's population induced by this particular grouping. Poverty for each country-group j will now be measured by an index which one may call the *triage headcount ratio* h_j: for a poor country-group j (that is, a country-group whose per capita GNP is less than the poverty line), h_j is simply the proportion of the country-group's population that must be allocated an income of zero so that the average income of the rest of the population in the country-group is enabled to rise to the poverty line level of income z^*; and for a non-poor country-group j, we shall take h_j to be zero. That is, if x_j^* is the per capita GNP of country-group j, then the triage headcount ratio for country j is defined as: $h_j \equiv \max[(z^* - x_j^*)/z^*, 0]$. By this reckoning, the proportion of the world's population – call it h – that must just cease to exist in order that every remaining person may receive an income of z^* works out to 18.6 per cent – the details are provided in Table 7.1. More specifically, this is the value of the triage headcount ratio corresponding to the universal grouping $\mathbf{g}^u$: $h(z^*, x^*, \mathbf{g}^u) = 0.186$. What if we employed the grouping $\mathbf{g}^*$ resorted to in the *HDR 1999*? The *adjusted* triage headcount ratio $h^a(z^*, x^*, \mathbf{g}^*) \equiv [\Sigma_{j \in \mathbf{g}^*} \theta_j h_j^2]^{1/2}$ turns to be 0.247, as can be confirmed

Table 7.1 Grouped data on global poverty, 1997

Country group	Population (in millions)	Population share	Triage headcount ratio
Sub-Saharan Africa	555.20	0.0967	0.6048
Asia and the Pacific	3140.30	0.5467	0.2126
Arab states	252.30	0.0439	0.1012
Latin America and the Caribbean (including Mexico)	490.50	0.0854	0.0167
Eastern Europe and the CIS	398.90	0.0695	0.0823
Southern Europe	64.20	0.0112	0.0000
Industrialized countries	842.30	0.1466	0.0000
World	5743.70	1.0000	0.1863

Note: The poverty line is taken to be PPP$3,000 per capita per annum (roughly one half of the average per capita GNP). The 'triage headcount ratio' is the proportion of the population that must be allowed to perish so that each member of the surviving population is enabled to achieve just a poverty-line level of income.

Source: Based on UNDP (1999).

from the figures presented in Table 7.1. The rise in the triage headcount ratio from 18.6 per cent to 24.7 per cent is a substantial one, and an indicator of the considerable inequality in the cross-country distribution of deprivation. If this outcome is an unattractive one, the picture, presumably, would be even worse if the grouping we employed classified the world not into seven groups, but into 174 groups – each group being represented by an individual country. Incorporating inter-group differentials into an overall assessment of deprivation certainly shows up in the deeply stratified world in which we live.

Two implications of 'group-sensitivity' in a poverty measure

The particular grouping of a population we resort to must be informed by an appreciation of the sociological salience of the classificatory scheme we adopt. More than one classificatory scheme may be relevant, depending on the precise context in, and purpose for which, deprivation is being sought to be measured. It is therefore a matter of some importance, in making poverty comparisons, to be explicit not only about the distributions under comparison and the poverty line(s) employed, but also about the grouping invoked. A certain ranking, valid for poverty measured with a particular grouping in mind, could in principle be inverted by a ranking that is valid for poverty measured with some other grouping in mind. An example of such rank reversal has in fact already been considered on p. 153. Harking back to the income vectors **u** and **v** previously reviewed, it can be verified that, when $z = 10$, $F^a(z, \mathbf{u}, g^u)(= 0.2728) < F^a(z, \mathbf{v}, g^u)(= 0.2778)$, but

$F^a(z, \mathbf{u}, \mathbf{g}^o)(= 0.2811) > F^a(z, \mathbf{v}, \mathbf{g}^o)(= 0.2790)$. A first implication of working with 'group-sensitive' poverty measures, therefore, is that the particular grouping we employ can make a substantial difference to the evaluative outcome of poverty comparisons.

Second, the grouping that is employed also has implications for 'targeting' in poverty redress schemes. Again, a simple numerical illustration might be helpful in explicating the idea. Let the poverty line be $z' = 10$, and let $\mathbf{a}$ be a 4-dimensional income vector (namely $n(\mathbf{a}) = 4$). Let $\mathbf{g}'$ be a partition of the population, based, let us say, on a grouping by caste, which divides it into two groups, 1 and 2 respectively. We shall write $\mathbf{a} = (\mathbf{a}^1, \mathbf{a}^2)$, with $\mathbf{a}^1 = (a_{11}, a_{12})$ and $\mathbf{a}^2 = (a_{21}, a_{22})$, where a_{ji} stands for the income of the ith person in the jth group ($j = 1, 2$ and $i = 1, 2$). Suppose, for specificity, that $\mathbf{a}^1 = (4, 6)$ and $\mathbf{a}^2 = (3, 9)$. If θ_j is the population share of subgroup j ($j = 1, 2$), then it is clear that in the present instance $\theta_1 = \theta_2 = 1/2$. Let $\mathbf{g}^u$ be an alternative partitioning of the population, induced by the universal grouping, which recognizes only one group, that constituted by the grand coalition of individuals. Suppose a budgetary allocation of $T = 5$ is available for poverty alleviation. Which is the best way of allocating the budget among the individuals in the population?

To complicate matters, we shall imagine that there are two policymakers, A and B, of whom A – who has no time for sociological affectations – believes that $\mathbf{g}^u$ is the only valid partitioning of the population, while B – who herself is a member of an underprivileged caste – believes that $\mathbf{g}'$ is a meaningful partition of the population. Policymaker A is comfortable with using the poverty index $F(z, \mathbf{x})$ (which, to recall, is the same as the index $F^a(z, \mathbf{x}, \mathbf{g}^u)$), while policymaker B is comfortable with using the index $F^a(z, \mathbf{x}, \mathbf{g})$. Let t_{ji}, in B's notation, be the amount of the budgetary allocation T which goes to the ith individual in the jth group ($j = 1, 2$ and $i = 1, 2$), and denote by $\mathbf{t}$ the vector $(t_{11}, t_{12}, t_{21}, t_{22})$; further, let $\mathbf{t}^1$ and $\mathbf{t}^2$ stand for the vectors (t_{11}, t_{12}) and (t_{21}, t_{22}) respectively. (Of course, the individuals that B calls 11, 12, 21 and 22 will probably be called just 1, 2, 3 and 4 by A, but since the latter believes in a thoroughgoing version of the symmetry axiom, his philosophy of 'what's in a name?' should be compatible with an acceptance of B's eccentric mode of labelling the individuals.)

A's objective is to solve the following programming problem:

Problem A

$$\underset{\{t_{11}, t_{12}, t_{21}, t_{22}\}}{\text{Minimize}}\ F^a(z', \mathbf{a} + \mathbf{t}, \mathbf{g}^u) = [1/n(\mathbf{a})(z')^2][\{z' - (a_{11} + t_{11})\}^2 + \{z' - (a_{12} + t_{12})\}^2 + \{z' - (a_{21} + t_{21})\}^2 + \{z' - (a_{22} + t_{22})\}^2]$$

subject to

$t_{11} + t_{12} + t_{21} + t_{22} \leq T$ and

$0 \leq t_{11} \leq z' - a_{11}, 0 \leq t_{12} \leq z' - a_{12}, 0 \leq t_{21} \leq z' - a_{21}$ and $0 \leq t_{22} \leq z' - a_{22}$

The optimal solution to this problem is the so-called 'lexicographic maximin' solution (see Bourguignon and Fields 1990, and Gangopadhyay and Subramanian 1992). The solution consists in raising the poorest person's income to the income of the second poorest person if the budget will permit, or to the highest feasible level not exceeding the second poorest person's income; if the budgetary outlay is thereby exhausted, we stop the exercise here, and if not, the incomes of the two poorest individuals are raised to the income of the third poorest person if the budget will permit, or to the highest feasible level not exceeding the third poorest person's income; and so on, until we reach that marginal individual with whom the budget is exhausted. Given the specific numerical values we have assigned to the poverty line z', the income vector **a**, and the budgetary outlay T, it can be verified that the optimal solution to Problem A is provided by

$$t^*_{11} = 2, t^*_{12} = 0, t^*_{21} = 3 \text{ and } t^*_{22} = 0 \tag{7.14}$$

The resulting, post transfer income vector is given by

$$\mathbf{a}^* = (a_{11} + t^*_{11}, a_{12} + t^*_{12}, a_{21} + t^*_{21}, a_{22} + t^*_{22}) = (6, 6, 6, 9) \tag{7.15}$$

Next, policymaker B's problem can be written as follows:

Problem B

$$\underset{\{t_{11}, t_{12}, t_{21}, t_{22}\}}{\text{Minimize}}\ F^a(z', \mathbf{a} + \mathbf{t}, \mathbf{g}') = [\theta_1\{F^a(z', \mathbf{a}^1 + \mathbf{t}^1, \mathbf{g}^u)\}^2 + \theta_2\{F^a(z', \mathbf{a}^2 + \mathbf{t}^2, \mathbf{g}^u)\}^2]^{1/2}$$

subject to

$t_{11} + t_{12} + t_{21} + t_{22} \leq T$ and

$0 \leq t_{11} \leq z' - a_{11}, 0 \leq t_{12} \leq z' - a_{12}, 0 \leq t_{21} \leq z' - a_{21}$ and $0 \leq t_{22} \leq z' - a_{22}$

Will the transfer schedule $\mathbf{t}^*$ presented in (7.14) also be an optimal solution to Problem B? It can be verified, given the numerical assumptions we have made regarding θ_1, θ_2, z' and T, that $F^a(z', \mathbf{a} + \mathbf{t}^*, \mathbf{g}') = 0.1281$. If there is no other allocation $\mathbf{t}^{**}$ such that $F^a(z', \mathbf{a}+\mathbf{t}^{**}, \mathbf{g}') < 0.1281$, then $\mathbf{t}^*$ is an optimal solution to Problem B. However, consider the transfer schedule $\mathbf{t}^{**}$ given by

$$t^{**}_{11} = 2.4558, t^{**}_{12} = 0.4558, t^{**}_{21} = 2.0884 \text{ and } t^{**}_{22} = 0 \tag{7.16}$$

It is easy to check that $F^a(z', \mathbf{a}+\mathbf{t}^{**}, \mathbf{g}') = 0.1256 < F^a(z', \mathbf{a}+\mathbf{t}^*, \mathbf{g}') = 0.1281$. Since poverty is lower with the transfer schedule $\mathbf{t}^{**}$ than with the schedule $\mathbf{t}^*$, it is clear that $\mathbf{t}^*$ is not an optimal solution to Problem B. (In fact, $\mathbf{t}^{**}$ solves Problem B. This claim will not be proved here, but it may be noted that an intuitive sufficient condition for an optimum is a feasible transfer schedule that (a) respects the lexicographic maximin principle of allocation *within* each subgroup; and (b) simultaneously ensures equalization of poverty levels *between* subgroups. In the present instance, note that

$\mathbf{a}^1 + \mathbf{t}^{1**} = (6.4558, 6.4558)$ and $\mathbf{a}^1 + \mathbf{t}^{2**} = (5.0884, 9)$: the lexicographic maximin outcome obtains within each subgroup, and further, subgroup poverty levels are equalized since, as can be confirmed, $F^a(z', \mathbf{a}^1 + \mathbf{t}^{1**}, \mathbf{g}^u) = F^a(z', \mathbf{a}^2 + \mathbf{t}^{2**}, \mathbf{g}^u) = 0.1256$.)

Briefly, policymakers A and B have a quarrel – and a substantial one at that – on their hands. While quarrels over the choice of the poverty line z have been numerous, quarrels over the choice of **g** have been relatively muted, with the implicit consensus favouring policymaker A's approach to the problem. Yet, both choices have implications not only for poverty comparisons but also for the proper targeting of scarce resources in poverty alleviation programmes. There would thus appear to be a case for an explicit statement of the precise choice of **g** that is made, and for a justification of that choice. The issue assumes a particular salience in the context of societies characterized by stratification arising from the historically cumulated maldistribution of burdens and benefits across identifiable subgroups of the population. For analysts concerned with the measurement of poverty and anti-poverty policy based on such measurement, the issue resolves itself into not just a problem in logic but into a larger problem in social ethics.

Conclusions

In much of mainstream economic theorizing, the only 'marker' of identity is income. This is quite clearly evident in, for example, standard approaches to the measurement of poverty. The point is made explicit in Sen's (1976) seminal paper dealing with the derivation of an ordinal measure of poverty, in which he draws specific attention to an assumption which is at the welfare basis of many poverty measures, and which he calls the *Monotonic Welfare* axiom. According to this axiom, given any income vector **x** and any pair of individuals j and k with incomes x_j and x_k respectively, if $x_j > x_k$, then $W_j(\mathbf{x}) > W_k(\mathbf{x})$, where W_j (respectively, W_k) is the welfare level of individual j (respectively, individual k). It should be emphasized that Sen is himself sceptical of the universal validity of this axiom, and employs it largely in the spirit of assembling material for a characterization theorem. In a richer framework of welfare, the latter would presumably be a function of arguments other than just income. Specifically, room would have to be made for the notion, as Akerlof and Kranton (2000: 718) put it, that 'identity is based on social categories', and the fact that income classes do not exhaust social categories. If a person's identity, and the welfare she experiences, depends not only on her income but also, for example, on the colour of her skin, then it is entirely conceivable that, given an income vector **x** and an n-tuple *s* describing each individual's skin colour, one can have a pair of individuals j and k such that $x_j > x_k$, j is black and k is white, and $V_j(\mathbf{x}, s) < V_k(\mathbf{x}, s)$, where $V_i(i = 1, \ldots, n)$ stands for person i's welfare level, and each person's welfare level is assumed to be increasing in her/his income, other things equal. In terms of the standard symmetry axiom, aggregate welfare and poverty levels should remain unchanged if j and k were to swap their incomes; however, in terms of the welfare index V, one can easily see that black j would be rendered worse off and white k would be rendered better off if j and k were to swap incomes. Similarly, the standard transfer axiom would endorse a permissible progressive income transfer from j to k; however, such a transfer would only serve to widen the welfare gap (when welfare is measured by V) between the two individuals. It is clear then that allowing for a plurality of groups in society does have non-trivial implications for the measurement of society-wide deprivation, a point that is emphasized by Thurow (1981: 179, 180, 182):

> Is the correct economic strategy to resist group welfare measures and group redistribution programmes wherever possible? Or do groups have a role to play in economic justice? ... [I]t is not possible for society to determine whether it is or is not an equal opportunity society without collecting and analyzing economic data on groups ... Individuals have to be judged based on group data ... A concern for groups is unavoidable.

This note has been concerned to explore an aspect of the analytics of poverty measurement as a specific application of the exercise of complicating mainstream accounts of the economy by allowing for the pervasive reality of the stratification of society into groups. In the process, it points to two issues that could be salient in a consideration of how to accommodate subgroup poverty in the aggregation exercise of measuring income deprivation. First, it suggests the desirability of the poverty index being a variable, rather than trivial or constant, function of the precise grouping that is employed in partitioning a population into subgroups. By entering the grouping explicitly as an argument in the poverty function, the domain of the function is informationally expanded in a way that enriches a group sensitive assessment of poverty. Second, it suggests that if it is considered desirable to incorporate directly into the measurement of poverty considerations relating to the intergroup distribution of poverty, then certain conventional axioms of poverty measurement may have to be modified, via restrictions on their domains of applicability, in order to avoid problems of internal consistency in the aggregation exercise.

Notes

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1. For analyses of poverty measurement when different groups are perceived to have different needs, as reflected in variations in subgroup poverty lines, see Atkinson (1987) and Keen (1992). While the concern in these papers, as in the present chapter, is with reckoning subgroup poverty in the measurement of aggregate poverty, the underlying motivations are rather different.
2. This section is heavily dependent on Jayaraj and Subramanian (1999: especially 197–200).
3. See Foster and Shorrocks (1991).
4. The axiom of 'subgroup sensitivity' has been advanced in Jayaraj and Subramanian (1999).
5. The underlying logic of the poverty index F^a is motivationally similar to that of the 'gender-adjusted human development index' of Anand and Sen (1995), which has been discussed in UNDP (1995).

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8

Intersociety Literacy Comparisons

Satya R. Chakravarty and Amita Majumder

Introduction

Literacy is an individual's first step in knowledge building. Therefore, literacy figures are essential in any quantification of human development. For instance, in the construction of the human development index, UNDP (1990–2005) used literacy as one of the key indicators of human development.

The most well-known measure of literacy (MOL) is the literacy rate, the proportion of adult population that is literate. However, as pointed out by Basu and Foster (1998, BF hereafter), this measure ignores the positive impact of the presence of a literate person in a household on the illiterate persons of the household. More precisely, this measure does not take into account the fact that the illiterate persons of a household can benefit from the knowledge of a literate person in the household. The essential idea underlying this notion of benefit is that a literate person confers positive externality identically on all illiterate persons of the household to which he belongs. In other words, within a household literacy can be regarded as something like a pure public good, which is characterized by non-rivalry and non-exclusiveness. By non-rivalry, we mean here that one illiterate person's benefit from the knowledge of a literate person in the household does not reduce the amount of benefit that another illiterate person in the same household can derive. On the other hand, non-exclusiveness means that the benefit an illiterate member of a household receives from having a literate person in the household does not exclude another illiterate person of the household from enjoying the same benefit. This kind of intra-household externality can arise in many ways. For instance, for filling in an official form an illiterate person can take the help of a literate person in the household.[1]

Now, an illiterate person will belong to either (i) a household that has one or more literate persons or (ii) a household that has no literate person. BF referred to the first type of illiterate as a proximate illiterate (since he has proximity to literacy because of the presence of a literate person in the household) and the second type of illiterate as an isolated illiterate. In order to distinguish between these two types of illiterates, BF assumed that each proximate illiterate person counts for α literate persons, where α is a number lying between zero and one and an isolated illiterate is regarded as a 'zero literate' person. Thus, in 'literacy equivalent' terms, every proximate illiterate person has a status that lies somewhere in between that of complete illiteracy and that of complete literacy. They also suggested a measure, the 'effective literacy rate' that takes this into account. This measure is the usual literacy rate plus α times the fraction of proximate illiterates in the population.

BF provided a set of axioms that exactly characterizes the effective literacy rate. These axioms are externality, anonymity, monotonicity, normalization and decomposability. Externality requires, under *ceteris paribus* assumption, literacy of a population to decrease or remain unaltered according as a split of a household in the population is externality reducing or externality neutral. A household split is called externality reducing (externality neutral) if it creates (does not create) isolated illiterates. Anonymity demands that any characteristic other than literacy status of individuals or household is irrelevant to the measurement of literacy. Monotonicity means that the level of literacy of a population rises if, given other things, an illiterate person becomes literate. According to normalization, the literacy measure should take on the values zero and one in extreme cases of complete illiteracy and complete literacy, respectively. Finally, decomposability says that for any partitioning of the population into subgroups with respect to characteristics like race, region, religion and so on, the overall literacy of the population is the weighted average of subgroup literacy levels, where the weights are the population shares of the subgroups.

While anonymity, monotonicity and externality are quite appealing, the remaining two axioms appear to be debatable to some extent. Any bounded measure of literacy can be made to satisfy the normalization axiom under suitable transformations. Sen (1992: 106) questioned the appropriateness of a poverty separability condition that parallels the decomposability axiom. He believed that one subgroup's poverty may be affected by what happens to other subgroups.[2] Clearly, if the literate persons of a subgroup can positively affect the literacy status of another subgroup, then the appropriateness of the literacy decomposability axiom also becomes questionable. To understand this more explicitly, note that one implication of subgroup decomposability is that for any society the overall literacy level can be expressed as the weighted average of literacy levels of individual households, where the weights are the adult population proportions of the respective households. Now, in India, particularly in rural areas, a person belonging to an illiterate family usually takes literary help from a literate neighbour in many respects; for example, for filling in a bank withdrawal form, reading and writing letters and so on. Therefore, in such a situation subgroup decomposability does not appear to be a suitable property. While we do not wish to claim that normalization and decomposability are always undesirable postulates, in view of the above discussion we can probably maintain the view that if a literacy index fails to meet these two properties but satisfies the remaining three, it cannot be discarded outright and may be suitable in some appropriate situations where normalization and decomposability do not hold.

Therefore, in this chapter we first suggest a class of literacy measures whose members may or may not satisfy decomposability but satisfies the remaining four axioms. Evidently, if some member of this class meets decomposability as well, then it must be the BF measure. This class can then be extended to a larger class of measures whose members will fulfil anonymity, monotonicity and externality but not necessarily the other two BF postulates.

To analyze the BF axioms further, we establish their independence, where independence means that none of these axioms implies or is implied by one or more of the other four. This shows that none of the BF axioms is redundant for deriving their measure. Next, as an extension of the BF exercise, we also propose two population principles for comparing literacy across societies. The first principle demands that if each household in a society is replicated any number of times, then the literacy levels of the original and replicated societies are the same. According to the second principle, literacy remains unchanged if replication occurs within all households not affecting their number. However, it turns out that decomposability along with anonymity implies the first principle. Thus, decomposability subsumes a weaker property, the first population principle, within an anonymous framework. Analogously, the second principle drops out as an implication of decomposability, anonymity and neutral part of externality. However, the reverse implications are not true. Note that since the BF index is decomposable, anonymous and invariant to externality neutral splits, it is capable of making intersociety literacy comparison by either of the two population principles. The two classes of measures that we suggest in this chapter also satisfy the two population principles.

We also illustrate different measures numerically using the national sample survey household level data for the rural sectors of seven states in India for the year 1993–94 and derive some policy implications of intra-household positive externality from literacy. It emerges that the literacy ranking of the states by a measure is sensitive to the values of α as well as to the functional forms of measures for the same value of α.

The following section starts with a discussion and demonstration of the BF axioms, and then we discuss the two population principles and their analytical relationship with decomposability before introducing the new classes of literacy measures. We then provide the numerical illustration, present a discussion on the applicability of the index to a more general set up and finally offer our conclusions.

Properties for a measure of literacy, their implications and several new measures

Consider a society consisting of k households that contain n adults. The literacy profile of household h ($h = 1, \dots k$) is given by the vector x^h, where x_j^h, the j-th coordinate of x^h, takes on the value 1 or 0 according as the jth member of household h is literate or illiterate. When we say that a person is literate, we are assuming that he/she has fulfilled some unambiguous criterion for literacy. For instance, according to Census of India (1991) a person is regarded as literate if he/she 'can read or write with understanding in any language. However, a person who can merely read but cannot write is not literate'. The term 'society' is used to refer to the vector of household literacy profiles $x = (x^1, \dots, x^k)$. Evidently, x represents the literacy levels of the society as well as a household structure. The literacy profile x^0 associated with x is obtained by concatenating the household vectors in x. For instance, consider a society of three households with two, three and four adult members respectively. Assuming that there is no literate in the first household and that only the first two members of the second household and the first member of the third household are literates, we have $x = ((0,0),(1,1,0),(1,0,0,0))$ and $x^0 = (0,0,1,1,0,1,0,0,0)$.

Let D^n be the set of all societies with adult population size n. Note that two arbitrary societies in D^n may or may not have the same number of households. The set of all possible societies with arbitrary adult population size and number of households is $D = U_{n \in M} D^n$, where M is the set of positive integers. For any function f: $D \to R^1$, the restriction of f on D^n is denoted by f^n, where $n \in M$ is arbitrary and R^1 is the real line. For any $n \in M$, $x \in D^n$ we write $n_l(x)$ for the number of literate persons in the society x. For any k-household society $(x^1 \dots x^k) \in D^n, n \in M$, the number of adult persons in x^h, $h = 1, \dots, k$, is denoted by a_h. Evidently, $\sum_{h=1}^{k} a_h = n$.

Now, as stated in the introduction, an illiterate person is a member of a household that contains either at least one literate person or no literate persons. Clearly, in the former case the illiterate person can derive literacy benefit from having a literate person in the household. We can, therefore, call this person proximate illiterate and regard him/her as an $\alpha (0 < \alpha < 1)$ literate person. In contrast, in the latter case the illiterate person can be called isolated illiterate because he/she does not get any literacy benefit from a member of the household. Evidently, α determines the extent to which a literate person's knowledge is able to help the illiterates literally. As BF argued, the value of α is to be determined from empirical estimation (see also Basu *et al.* 2000). For any $x \in D^n$, where $n \in M$ is arbitrary, the number of proximate literates in x is denoted by $n_p(x)$.

The effective literacy profile of household h, $\hat{x}^h$, can be defined as

$$\hat{x}_j^h = \begin{cases} 1 & \text{if } x_j^h = 1, \\ \alpha & \text{if } x_j^h = 0 \text{ and } x_q^h = 1 \text{ for some } q \neq j, \\ 0 & \text{if } x_q^h = 0 \text{ for every } q \end{cases}$$

The society effective literacy profile is given by $x^* = (\hat{x}^1, \hat{x}^2, \dots, \hat{x}^k)^0$. The number of effective literates in household h in the vector x is $\sum_{j \in h} \hat{x}_j^h = e_h(x)$. For the purpose at hand, we need some more preliminaries. For all $n \in M$, $x \in D^n$, we say that $y \in D^n$ is obtained from x by a 'simple increment' if $y_j^h = 1$ and $x_j^h = 0$, while $y_i^r = x_i^r$ for all $(r, i) \neq (h, j)$ and we write yCx to indicate this. That is, the societies x and y are identical except that one illiterate person (j) in x^h is becoming literate in y^h. We say that the $(k + 1)$-household society $y \in D^n$ is obtained from the k-household society $x \in D^n$ through a household split if some household in x, say r, is broken down into two households, which are households r and $(r + 1)$ in y, while all the households numbered from 1 to $(r - 1)$ are the same in x and y, and household h in x is same as household $(h+1)$ in y, where $h = r+1, \dots, k$. Equivalently, x is obtained from y as follows: the households numbered 1 to $(r - 1)$ in x are the corresponding households in y, household r in x is a concatenation of households r and $(r + 1)$ in y, and the h-th household in x is the $(h + 1)$th household in y, where $h = r + 1, \dots, k$. Thus

$$\begin{aligned} x^h &= y^h, 1 \leq h < r \\ &= y^h \, T y^{h+1}, h = r \\ &= y^{h+1}, r < h \leq k \end{aligned}$$

where $y^h \, T y^{h+1}$ denotes the concatenation of the vectors y^h and y^{h+1}. For the given example of x, let $y = ((0, 0), (1, 1), (0), (1, 0, 0, 0))$. Then we have $x^1 = y^1$, $x^3 = y^4$ and $x^2 = y^2 T y^3$.

The household split will be called 'externality neutral' if either (i) both y^r and y^{r+1} contain a literate person, or (ii) neither of y^r or y^{r+1} contains a literate person. We denote this relationship between x and y by yNx. The split is called 'externality reducing' if exactly one of y^r and y^{r+1} contains a literate person and this is denoted by yEx. In the example considered above, the relationship yEx holds. A society $x \in D^n$ is called 'completely literate' if $x_j^h = 1$ for all j and for all h; x is 'completely illiterate' if $x_j^h = 0$ for all j and h.

A measure of literacy H is a real valued function defined on D; that is, $H: D \to R^1$. Thus, for any $x \in D^n$, where $n \in M$ is arbitrary, $H^n(x)$ denotes the extent of literacy of the society x.

BF laid down the following desiderata for an MOL:

Anonymity (ANY) For all $n \in M$, $x \in D^n$, if $y \in D^n$ is obtained from x by either a reordering of households or a reordering of members of a household, then $H^n(y) = H^n(x)$.

Monotonicity (MON) If $x, y \in D^n$, where $n \in M$ is arbitrary, are related as yCx, then $H^n(y) > H^n(x)$.

Externality (EXT) For all $n \in M$, $x \in D^n$, suppose that $y \in D^n$ is obtained from x through a household split. Then (a) $H^n(y) = H^n(x)$ if yNx holds; and (b) $H^n(y) < H^n(x)$ if yEx holds.

Normalization (NOM) For all $n \in M$, $x \in D^n$, $H^n(x) = 1$ if x is completely literate and $H^n(x) = 0$ if x is completely illiterate.

Subgroup decomposability (SUD) For $x^{(n_i)} \in D^{n_i}$, $i = 1, 2, \ldots m$, where $x^{(n_i)}$ is an n_i-member adult person society,

$$H^n(x) = \sum_{i=1}^{m} \frac{n_i}{n} H^{n_i}(x^{(n_i)}), \text{ where } x = (x^{(n_1)}, x^{(n_2)}, \ldots x^{(n_m)}) \text{ and } n = \sum_{i=1}^{m} n_i$$

Since we have already mentioned these properties in the introduction, and they have also been discussed earlier by BF, we skip any further discussion on them here.

In addition to the above axioms considered by BF, we also suggest the following as a postulate for an MOL.

Principle of population (POP) For all $n \in M$, $x \in D^n$, $H^n(x) = H^{mn}(y)$, where y is the society obtained by replicating each household in x m times.

According to POP, for an m-fold replication of each household of the society, the degrees of literacy of the replicated and the original populations are the same, where $m \geq 2$ is arbitrary. Thus, POP leads us to view literacy in proportional terms. Evidently, POP is helpful for cross population comparisons of literacy.

An alternative to POP can be the following:

Alternative population principle (APP) For all $n \in M$, $x \in D^n$, $H^n(x) = H^{mn}(z)$, where z is the society obtained from x by replicating the individuals within each household m times.

Given $x \in D^n$, y in POP and z in APP have the same population size mn. But the essential difference between y and z is that y has a higher number of households than z. In fact, the number of households in y is mk, where k is the common number of households in x and z.

The most commonly used MOL is the literacy rate, the proportion of adult population that is literate. Formally, the literacy rate $A: D \to R^1$ is defined as

$$\begin{aligned} A^n(x) &= \sum_{i=1}^{n} \frac{x_i^0}{n} \\ &= \frac{n_\ell(x)}{n} \end{aligned} \tag{8.1}$$

where $n \in M$ and $x \in D^n$ are arbitrary. As observed by BF, A^n satisfies all their axioms except part (b) of EXT. It also satisfies POP and APP.

BF suggested a more sophisticated MOL, the effective literacy rate $B: D \to R^1$, where for all $n \in M, x \in D^n$

$$\begin{aligned} B^n(x) &= \sum_{i=1}^{n} \frac{x_i^*}{n} \\ &= A^n(x) + \alpha P^n(x) \end{aligned} \tag{8.2}$$

where $P^n(x) = n_p(x)/n$ is the proportion of proximate illiterates in x. B^n can be rewritten as

$$B^n(x) = \alpha(1 - C^n(x)) + (1 - \alpha)A^n(x) \tag{8.3}$$

where $C^n(x)$ is the fraction of isolated illiterates in x. The number $1 - C^n(x)$ gives the proportion of population in x with one or more literate persons in the household and is equal to $A^n(x) + P^n(x)$. This has also been used as an indicator of literacy (see Rogers and Herzog 1966, Sharma and Retherford 1993). Thus, B^n is a convex mix of the two indicators of literacy $1 - C^n$ and A^n, with weights α and $1 - \alpha$, respectively. If $\alpha = 0$, $B^n(x) = A^n(x)$. On the other hand if $\alpha = 1$, B^n becomes $1 - C^n$. Subramanian (2000) suggested a measure S^n, which is given by the product of A^n and $1 - C^n$. In the particular case $\alpha = A^n$, B^n and S^n are related by $B^n(x) - S^n(x) = A^n(x)(1 - A^n(x))$. BF demonstrated that B^n is the only MOL that verifies their five axioms. It also meets POP and APP.

We now show that the five BF axioms are independent. The demonstration involves construction of an indicator that will satisfy any four of these axioms but not the remaining one. Thus, independence means that if we drop any one of these five axioms, then the resulting MOL will not be the BF index.

Proposition 1 *Axioms ANY, MON, EXT, NOM and SUD are independent.*

Proof

(i) The measure $(1 - (C^n(x))$ may not satisfy MON but satisfies others;

(ii) For a k-household society with n adult persons, the measure $\sum_{h=1}^{k} a_h \frac{\dot{e}_h}{n}$ will not remain unchanged under an externality neutral split, thus violating part (a) of EXT. It, however, satisfies the other postulates.

As observed by BF, $A^n(x)$ violates part (b) of EXT but meets others;

(iii) The MOL $\delta B^n(x)$, where $\delta > 0$, $\delta \neq 1$, does not fulfil the part of NOM that corresponds to complete literacy, but fulfils others. The MOL $\frac{1+B^n(x)}{2}$ meets all properties except the part of NOM which corresponds to complete illiteracy;

(iv) The indicator $\alpha(1 - C^n(x))^\theta + (1 - \alpha)(A^n(x))^\theta$, where $\theta > 0, \theta \neq 1$, is a violator of SUD but not of others;

(v) The measure $(1 - \alpha)\sum_{i=1}^{n} \frac{(x_i^*)^{\eta_i}}{n} + \alpha(1 - C^n(x))$ where $\eta_i > 0$, $\eta_i \neq \eta_j$ if $i \neq j$, fulfils all the five except ANY. □

The next proposition shows that SUD combined with one or more of the remaining BF axioms implies POP and APP.

Proposition 2 *(a) If a literacy measure $H: D \to R^1$ satisfies ANY and SUD, then it satisfies POP; (b) if a literacy measure $H: D \to R^1$ fulfils ANY, SUD and part (a) of EXT, then it fulfils APP.*

Proof (a) Suppose society y is obtained by replicating t times all households in $x \in D^n$ that consists of k households, where household h with literacy profile x^h includes a_h individuals giving a total of n individuals for x. Society y, which has a population size of tn, is composed of tk households. Let $H^{tn}(y)$ be the MOL for y. By SUD, we have $H^{tn}(y) = \sum_{h=1}^{tk} \frac{b_h}{tn} H^{b_h}(y^h)$, where y^h is the literacy profile of household h in y, b_h is the number of persons in y^h and $tn = \sum_{h=1}^{tk} b_h$. Applying anonymity all the MOLs associated to similar replicated households give the same value. Therefore,

$$H^{tn}(y) = \sum_{h=1}^{k} \frac{ta_h}{tn} H^{a_h}(x^h) = \sum_{h=1}^{k} \frac{a_h}{n} H^{a_h}(x^h) = H^n(x),$$

where the last equality is obtained by applying SUD. $H^{tn}(y) = H^n(x)$ is precisely the requirement of POP;

(b) Suppose society z is obtained by replicating t times all the individuals in every household in society $x \in D^n$. Society x is composed of k households where household h with literacy profile x^h includes a_h individuals, giving a total of n individuals for entire x.

Similarly, society z, by definition, consists of k households, where household h includes ta_h individuals giving a total of tn individuals for entire z. Let $H^{tn}(z)$ be the MOL for z. We can now split each household of ta_h individuals in z into t identical households of a_h individuals to obtain society y. Since the operation is externality neutral, in view of part (a) of EXT, we have

$H^{tn}(z) = H^{tn}(y)$. Applying the result in part (a) of the proposition, by SUD and ANY, we get $H^{tn}(y) = H^{n}(x)$, which gives $H^{tn}(z) = H^{n}(x)$, the requirement of APP. □

It may be important to note that the converse of this proposition is not true. That is, POP and APP do not imply the postulates from which they are derived in Proposition 2. To see this, note that the MOL in (iv) in the proof of Proposition 1 verifies POP and APP, but not SUD. Similarly, the MOL in (v) in the same proof meets POP and APP but not ANY. Finally, we use the first MOL in (ii) in that proof to demonstrate the remaining part of the claim. It is also easy to construct examples that will satisfy POP (APP) but not ANY and SUD (ANY, SUD and EXT(a)) simultaneously.

Since the BF index B^n is a convex combination of the MOLs A^n and $(1-C^n)$, a natural generalization of B^n can be the same convex combination of a transformation of A^n and $(1 - C^n)$. More precisely, as a generalization of B we may suggest the use of the MOL $G: D \rightarrow R^1$ where for all $n \in M, x \in D^n$,

$$G^n(x) = \alpha f(1 - C^n(x)) + (1 - \alpha) f(A^n(x)) \tag{8.4}$$

where $f: Q \rightarrow R^1$, with Q being the set of rational numbers in $[0, 1]$.

The following proposition identifies the class of all real valued functions defined on Q for which G verifies NOM and part (b) of EXT.

Proposition 3 *The general MOL* G *defined in (8.4) satisfies normalization and part (b) of externality if and only if* $f(0) = 0$, $f(1) = 1$ *and* f *is increasing.*

Proof Suppose that $A^n(x) = 0$. This in turn implies that $C^n(x) = 1$. Then

$$G^n(x) = \alpha f(0) + (1 - \alpha) f(0) = f(0) \tag{8.5}$$

But in this extreme case by normalization $G^n(x) = 0$. This along with (8.5) gives $f(0) = 0$. Next, suppose that $A^n(x) = 1$ which gives $C^n(x) = 0$. Then

$$G^n(x) = \alpha f(1) + (1 - \alpha) f(1) = f(1) \tag{8.6}$$

But normalization says that $G^n(x) = 1$ in this case. Using this information in (8.6) we get $f(1) = 1$.

Now, suppose that y has been obtained from x by a household split and the split is externality reducing. Then, the split decreases the number of proximate illiterates. Part (b) of EXT then demands

$$\begin{aligned} G^n(y) &= \alpha f(1 - C^n(y)) + (1 - \alpha) f(A^n(y)) < \alpha f(1 - C^n(x)) + (1 - \alpha) f(A^n(x)) \\ &= G^n(x) \end{aligned} \tag{8.7}$$

Since the split does not reduce the number of literates, we have $A^n(y) = A^n(x)$. Therefore, $G^n(y) < G^n(x)$ means that $f(1 - C^n(y)) < f(1 - C^n(x))$; that is, $f(A^n(y) + P^n(y)) < f(A^n(x) + P^n(x))$. Since $P^n(y) < P^n(x)$ and $A^n(y) = A^n(x)$, we need increasingness of f for the inequality in (8.7) to hold. This establishes the necessity part of the proposition. The sufficiency is easy to verify.[3] □

Let F be the class of all real valued increasing functions defined on Q that take on the values 0 and 1 at 0 and 1, respectively. More precisely, $f: Q \to R^1$ is a member of F if f is increasing, and $f(0) = 0$ and $f(1) = 1$. It is clear that to every $f \in F$ there corresponds a different index of the form (8.4). These indices will differ only in the manner how we specify f. For any $f \in F$, the underlying MOL G^n, in addition to being affirmatively responsive to part (b) of EXT and NOM, is anonymous, population replication invariant (since A^n and C^n are so), monotonic (since f is increasing) and invariant to externality neutral splits. However, G^n may not fulfil SUD.

Examples of functions that are members of F are

(i) $f_1(t) = t^\theta, \theta > 0$
(ii) $f_2(t) = (e^t - 1)/(e - 1)$
(iii) $f_3(t) = 2t/(1 + t)$

Now, the MOL of the form (8.4) associated with f_1 in (i) becomes $G^n_\theta(x) = \alpha(1 - C^n(x))^\theta + (1 - \alpha)(A^n(x))^\theta, \theta > 0$. Clearly for $\theta = 1$, which gives $f_1(t) = t$, G^n_θ becomes the BF MOL B^n in (8.2). Note that for a given society x, which is neither completely literate, nor completely illiterate, $G^n_\theta(x)$ is decreasing in θ. As $\theta \to 0$, $G^n_\theta(x) \to 1$.

We will now consider the explicit forms of MOLs corresponding to the functions f_2 and f_3 specified above. Clearly, we can have infinitely many functions to choose from F. The choice gets widened if in (8.4) we give up the assumptions $f(0) = 0$ and $f(1) = 1$ but retain increasingness of f. In such a case G^n will fulfil ANY, MON, EXT, POP and APP, but not NOM and also not necessarily SUD. Let $\overline{F}$ be the class of all real valued increasing functions defined on Q. Then $F \subset \overline{F}$. Examples of functions that belong to $\overline{F} - F$ can simply be constructed by adding a non-zero constant term to the examples given above. That is, if we define $g_i(t) = f_i(t) + \delta_i$, where $\delta_i \neq 0$ is a constant, $i = 1, 2, 3$; then g_i's are members of $\overline{F} - F$.

Literacy in rural India: an illustration

The purpose of this section is to illustrate numerically the measures suggested on pp. 170–1 of the chapter using statewise household level literacy data thrown up by the National Sample Survey Organisation (NSSO) for rural India for the period 1993–94. The states considered are Andhra Pradesh (AP), Arunachal Pradesh (AR), Haryana (HA), Karnataka (KA), Manipur (MA), Rajasthan (RA), and Sikkim (SI). The literacy measures chosen for the purpose are the generalized BF index G_θ, and the measures H_2 and H_3, whose restrictions on D^n are given by

$$H_2^n(x) = \alpha \frac{e^{1-C^n(x)} - 1}{e - 1} + (1 - \alpha)\frac{e^{A^n(x)} - 1}{e - 1} \tag{8.8}$$

and

$$H_3^n(x) = \frac{2\alpha(1 - C^n(x))}{2 - C^n(x)} + \frac{2(1 - \alpha)A^n(x)}{1 + A^n(x)} \tag{8.9}$$

where $n \in M$ and $x \in D^n$ are arbitrary. It may be noted that H_2 and H_3 are special cases of G in (8.4), corresponding respectively to the functional forms $f_2, f_3 \in F$ considered on p. 171.

Numerical estimates of literacy for rural India for the period considered are presented in Table 8.1. The first column of the table gives the names of the states for which the calculations are made. In columns 2 and 3 we show, for each state, the literacy and proximate illiteracy rates A and P. (Since interstate comparisons involve different population sizes, we drop superscript n from all indices.) Assuming that $\alpha = 0.25$, statewise generalized BF index for three different values of θ ($\theta = 0.5$, 1.0 and 1.5) are presented in columns 4–6. It may be recalled that for $\theta = 1$, G_θ becomes the BF index B. In columns 7 and 8 we show values of H_2 and H_3 for the same value of α. Columns 9–13 present the values of G_θ ($\theta = 0.5$, 1.0 and 1.5), H_2 and H_3 corresponding to $\alpha = 0.75$. Thus, while in the former case a proximate illiterate is equivalent to one quarter of a literate, in the latter case we have three-quarters equivalence.

Several interesting features emerge from Table 8.1. For all ${}^7C_2 = 21$ pairwise comparable situations, we have uniform ranking by all the three measures for $\alpha = 0.25$. The same ranking of the states is obtained for H_3 and $G_{0.5}$ for $\alpha = 0.75$ also. For this latter value of α, uniform ranking by all measures is generated in 20 cases, a disagreement arises for the pair (AP, RA). More precisely, while H_3 and $G_{0.5}$ make AP more literate than RA for both $\alpha = 0.25$ and 0.75, G_θ ($\theta = 1.0$ and 1.5) and H_2 agree (disagree) with this ordering for $\alpha = 0.25$ (0.75). From this observation we can conclude two features on literacy ranking. First, alternative measures may generate different orderings of societies for the same value of α (as in the case of $\alpha = 0.75$). Second, a change in the value of α may give rise to a change in ranking by the same measures (as G_θ for $\theta = 1.0$, 1.5 and H_2 demonstrate this for the pair AP and RA, when α increases from 0.25 to 0.75). This second feature was noted by BF as well. Note that for the pair (AP, RA), we have $A(\text{AP}) > A(\text{RA})$, but $P(\text{AP}) < P(\text{RA})$. The dominance of the component αP in the calculation of the MOLs (G_θ for $\theta = 1.0$, 1.5 and H_2) for high values of α may be the underlying factor for the reverse ordering generated. However, for the remaining MOLs ($G_{0.5}$ and H_3) αP does not become a dominant factor. This clearly shows that the ranking of the societies is sensitive to the value of α.

Table 8.1 Literacy in rural India, 1993–94

	Proportion of		Values of index										
			$\alpha = 0.25$					$\alpha = 0.75$					
State	Literates	Proximate illiterates	G_θ					G_θ					
	(A)	(P)	$\theta = 0.5$	$\theta = 1.0$	$\theta = 1.5$	H_2	H_3	$\theta = 0.5$	$\theta = 1.0$	$\theta = 1.5$	H_2	H_3
1	2	3	4	5	6	7	8	9	10	11	12	13
Andhra Pradesh (AP)	0.3988	0.4595	0.7052	0.5137	0.3877	0.4116	0.6586	0.8527	0.7434	0.6593	0.6645	0.8354
Arunachal Pradesh (AR)	0.3133	0.5357	0.6501	0.4472	0.3271	0.3552	0.5874	0.8310	0.7151	0.6306	0.6373	0.8080
Haryana (HA)	0.4874	0.4528	0.7660	0.6006	0.4831	0.5012	0.7338	0.9018	0.8270	0.7688	0.7725	0.8907
Karnataka (KA)	0.4799	0.4320	0.7583	0.5879	0.4671	0.4855	0.7249	0.8894	0.8039	0.7362	0.7396	0.8776
Manipur (MA)	0.6496	0.3212	0.8508	0.7299	0.6318	0.6379	0.8370	0.9405	0.8906	0.8484	0.8490	0.9358
Rajasthan (RA)	0.3431	0.5409	0.6744	0.4783	0.3685	0.3853	0.6178	0.8516	0.7488	0.6736	0.6796	0.8316
Sikkim (SI)	0.6321	0.3030	0.8381	0.7079	0.6030	0.6100	0.8226	0.9240	0.8594	0.8039	0.8038	0.9185

A comparison between AR and RA shows that RA has higher values of A and P than AR and since the three indices combine increasing transformations of A and P in a positive way, RA becomes more literate than AR by all the measures. The same phenomenon holds for the pair (HA, KA). For no other pair of states presented in the table has this characteristic been found. In such cases, we have to consider specific indices for literacy ranking of states.

We conclude this section with some additional observations on the figures presented in the table. By all the indices considered, MA turns out to be the most literate state, whereas AR has the minimum literacy level. The situations for the remaining states are in between these two extremes. It might be of interest to note that although MA has the highest proportion of literates, its proportion of proximate illiterates is rather low. The converse is true for AR. For many states, the proportion of proximate illiterates is significantly greater than the proportion of isolated illiterates. Thus, a substantial proportion of population in these states has immediate access to literacy because of intra-household externality. For each state, the excess of an index over the literacy rate A shows the quantitative impact of the intra-household externality on assessment of literacy. These observations correspond closely to an important policy implication. Consider a cost-constrained literacy campaign programme in a region. Under the programme, one person from each illiterate household can be made literate so that other members in the household can take advantage of intra-household positive externality from literacy. Since higher number of households can now be covered by the programme, the society becomes effective literates to a larger extent, which in turn demonstrates a greater success of the programme.

Discussion

The results developed in the chapter are based on the assumption that within a household, literacy can be regarded as a pure public good characterized by non-rivalry and non-exclusiveness. That is, all illiterate persons in a household derive literacy benefit from the presence of one or more literate persons in the household under the conditions that no illiterate person in the household can be excluded from receiving this benefit and one person's benefit does not reduce the level of benefit for another person. As a result, it has been assumed that each illiterate person in a household with one or more literates can be regarded as an $\alpha (0 < \alpha < 1)$ literate person. Thus, irrespective of the number of literate persons in the household, an illiterate person becomes an α literate person. Further, intrinsic to this beneficial connection between literates and illiterates is the assumption that there is no constraint on the time that a literate person can spend helping illiterates.

Now, it is quite likely that a higher fraction of literate persons may ensure a greater access to literacy skills of illiterates. The time constraint of a literate person is also likely to influence this access. Another issue is gender sensitivity to literacy analysis. The positive externality generated by literacy is likely to be higher if the source of literacy is a female instead of a male. (BF suggested a modification of their measure *B* along this line.) Furthermore, in a dynamic context, the lifetime externality of a younger person will be higher than that of an older person. Thus, characteristics other than the sheer existence of a literate person in a household may be quite important in determining the value of α.

The next point concerns the domain of positive impact that a literate person can have on illiterates. In addition to conferring literal benefits to their own household members who are illiterate, a literate person may be able to have a positive affect on the literacy status of their own society, caste, and so on. This will depend on the individual's social connection in the community/caste. Extensions of our analysis along these lines will be worthwhile.

Since alternative literacy measures may rank two societies in different directions, another line of investigation can be the development of a quasi-ordering such that the ranking of the societies by a set of measures satisfying certain postulates will coincide with that generated by the ordering. This is left as a future research programme.

Conclusion

Basu and Foster (1998) characterized a sophisticated literacy measure using five axioms. In this chapter, we argued that if a measure satisfies three of their five axioms (namely, anonymity, monotonicity and externality), then it also becomes suitable in certain other applications. We, therefore, introduced two classes of measures whose members satisfy at least these three axioms. Two population principles for intersociety literacy comparisons have also been suggested and their relationships with the Basu–Foster axioms have been established. Finally, we illustrated our results numerically using Indian data and discussed some policy implications.

Notes

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1. It may be the case that a few literate household members may choose not to share skill. However, for the sake of simplicity, here we have followed the approach of Basu and Foster (1998).
2. For detailed discussion on the poverty separability axiom, see Anand (1983); Chakravarty (1983, 1990); Foster *et al.* (1984); Cowell (1988) and Foster and Shorrocks (1991).
3. Note that if the household split is externality neutral, then increasingness of f does not follow as a necessary condition for EXT to hold. However, if f is increasing then the underlying $G^n(x)$ satisfies both parts (a) and (b) of EXT.

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9

The Human Development Index Adjusted for Efficient Resource Utilization

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Introduction

The human development index (HDI) developed by the United Nations Development Programme (UNDP 2003) is computed as the average of three equally weighted outcome measures or indices of human development: life expectancy (LI), educational attainment (EI) and income (WI). However, this computational process is independent of the resource endowment being devoted by each country to the achievement of the three outcome levels (Raab *et al.* 2000). Hence, it is conceivable that two different countries consume vastly different amount of resources in achieving the same, say, LI, whereas this difference in the efficiency of resource utilization is not reflected in the HDI. The purpose of this chapter is to address this efficiency issue. Here, the term *efficiency* corresponds to the concept of Pareto–Koopmans efficiency in economics (Varian 1999). Thus, it measures the ability of each country to transform the minimum possible units of its own resources into the maximum possible levels of the three outcomes. As a result, a country or decision-making unit (DMU) 'is fully efficient if and only if it is not possible to improve any input or output without worsening some other input or output' (Cooper *et al.* 2000: 45). This definition is operationalized through the development of a benchmarking model, where each country's three HDI outcome measures, LI, EI and WI, are evaluated relative to an efficient or 'best-practice' production frontier, formed by the benchmarking (that is, most efficient) countries. The determination of this frontier is achieved through the use of the data envelopment analysis (or DEA) methodology (Cooper *et al.* 2000; Thanassoulis 2001; Zhu 2003).

The methodological underpinnings of the HDI are straightforward and appear as a technical note to the various *Human Development Reports* (UNDP 2003). For each country, the LI is measured by the life expectancy at birth. EI is based upon the weighted average of the adult literacy rate (2/3 weight) and the combined gross enrolment in primary, secondary and tertiary education (1/3 weight). WI uses the adjusted, per capita GDP (PPP, US$). All three are deprivation indexes. As such, LI and the two components of EI are computed as the ratio of the difference between each country's observed value and a minimum goalpost value to the difference between a maximum and the minimum goalposts. A similar procedure is followed for the computation of WI, but using the log of GDP and of the two goalposts. The use of logs is intended to account for the diminishing returns exhibited by the income component towards the enhancement of human development.

Since its inception in 1990, the HDI has spawned a wide gamut of studies that may be classified into three categories. The first deals with attempts to enhance the understanding and justification of the methodological construct. Included here are studies directed towards:

(i) detailing the evolution of the construction methodology of the HDI as a measure of human well-being, its impact on policymaking and possible directions for future research (Jahan 2003);
(ii) analyzing the characteristics of the HDI as an index, across a variety of dimensions (Ivanova *et al.* 1999; Alkire 2002);
(iii) bridging the gap between the 1990 and the 1994 methods of computing the goalposts (Mazumdar 2003);
(iv) extending the diminishing-returns methodology to the computation of the EI (Noorbakhsh 1998);
(v) testing with a moderate amount of success the assumption of the HDI construct that per capita GDP exhibits diminishing returns to development (Cahill 2002);
(vi) studying in more depth the relationship between human development and economic growth (Ranis *et al.* 2000), as a way of justifying the use of HDI over that of per capita GDP as 'a measure of average achievement in basic human capabilities' (Jahan 2003: 3); and
(vii) stating reasons why the HDI construct may not have kept up with current global concerns (Sagar and Najam 1998).

The second category of studies explores the role of HDI in explaining specific issues related to human development in specific countries. Recent examples of this rather voluminous literature include assessing the extent of regional disparities in Iran (Noorbakhsh 2002) or the state of human development in China (Dejian 2003).

The third and final category attempts to extend the HDI's range of applicability through the incorporation of other dimensions likely to impact upon a country's human development. Examples of this literature are:

(i) introducing environmental factors designed to identify the extent to which countries are willing to accept environmental degradation to obtain current income at the expense of future economic expansion (Lasso de la Vega and Urrutia 2001; Neumayer 2001);

(ii) measuring cross-country divergence in the standard of living (Mazumdar 2003);
(iii) assessing the advantages and disadvantages of using the HDI as a monitor of human rights worldwide (Fukuda-Parr 2001);
(iv) presenting evidence of HDI dominance over per capita GDP as a measure of human welfare, on the grounds that the former is better suited to capture 'how long the economy can keep the average person alive to experience [given levels] of welfare' (Berg 2002: 193), whereas the latter 'fails to measure the lifetime welfare of the individuals' (Berg *op. cit.*: 182);
(v) assessing the HDI's suitability as a measure of a nation's competitiveness (Ivanova *et al.* 1997, 1998);
(vi) evaluating the HDI's role in measuring a child's quality of life (Raab *et al.* 2000); and
(vii) using HDI as a yardstick in the computation of alternative achievement and improvement indexes as measures of quality of life (Zaim *et al.* 2001).

One of the gaps that becomes apparent in this brief review of the literature is the dearth of studies on the level of effort, in terms of resources allocation, devoted by various countries in their pursuit of the three objectives embedded in the HDI and thus in the achievement of specific HDI targets. The current study attempts to bridge this gap. More specifically, the objectives of this study are (i) to assess the efficiency of each country's resource allocation policies in generating the given outcome levels of the three outcomes, LI, EI and WI; (ii) to produce an HDI for each country, adjusted for the efficiency of the resource allocation process; and (iii) to test for any statistical difference between the two.

Research framework

This section sets the stage for the efficiency analysis of the next section. It describes the inputs hypothesized to be affecting each output, summarizes the DEA model used in the estimation of efficiency and outlines the possible sources of efficiency.

The model's inputs and outputs

The outputs to be considered in this chapter are the three components of the HDI, namely LI, EI and WI. The model also includes several inputs hypothesized to impact upon each output. Table 9.1 lists these inputs. Several considerations have guided the input-selection criteria. First, it should be observed that to prevent the data consistency problems common to studies of this type (Ivanova *et al.* 1997), the inputs have been selected from among those present in the website for UNDP (2003), with the two exceptions noted in Table 9.1.

Table 9.1 Inputs and outputs of the model

Outputs	Inputs	
LI – Life expectancy index	PHYS	Number of physicians per 100,000 population
	HEC	Health expenditures per capita
	M/F LEB	Male/female life expectancy at birth
EI – Educational attainment	PEDEX	Public education expenditures
	A/Y LR	Adult/youth literacy rate
	F/M ALR	Female/male literacy rate
	F/M PST	Female/male combined primary, secondary, tertiary enrolment
WI – Income index	NFDII	Net foreign direct investment flows (% of GDP)
	ECPC	Electricity consumption per capita
	GDPE	GDP per unit of energy use
	F/M EEI	Female/male expected earned income
	GDCF	Gross domestic capital formation
	IU%P	Internet users (% of population)

Sources: UNDP (2003) for all variables except for GDCF and IU%P; UN (various years) for GDCF; Globstat (n.d.) for IU%P.

Second, in the selection of these particular sets of inputs, special care has been taken to account for the dynamic inter-relationships among the inputs and outputs. For example, in the LI case, current health expenditures are obviously not the only health expenditures to impact upon LI. The pattern of past years' health policies are going to affect this year's life expectancy and thus such pattern should be included in the formulation. This is the well-known problem in economics of selecting the appropriate lag structure to each dynamic setting. Given the impossibility of the task, for each output, a series of stock variables, such as PHYS, are used as proxies for the cumulative effects of past expenditure flows.

Third, included here are various male/female or female/male ratios. The rationale for these ratios is that the closer they are to 1, the higher the additional expenditure flows to achieve gender equality and, hence, the higher the corresponding output index. As a result, these ratios are being used as proxies for stock variables, measuring how much investment has already been undertaken to achieve gender equality. A similar argument may be made in the case of adult/youth literacy rate (A/Y LR). The closer the ratio is to one, the higher the success of the alphabetization campaigns aimed at closing the age gap in education prevalent in many countries. Fourth, the FDI variable has been normalized through its division by GDP, thus substantially palliating the problem of unusual year-to-year fluctuations. It is to be recognized that the efficiency measures will be better if more basic inputs can be used. Another caution that one need to exercise in interpreting the results is that some of the institutional differences may distort the results. Public expenditure may not fully capture the inputs for countries that are primarily private sector oriented. However, data limitations force us to settle for proxies.

The DEA framework

For each HDI component, the efficiency of each country or DMU is measured by its ability to transform the appropriate inputs into the corresponding output. The starting point of the analysis is the construction of an efficient production frontier, for each of the three outputs, formed by the 'best practice' benchmarking countries. For this purpose, the DEA formulations of the chapter include a set of C DMUs or countries. The outputs are denoted by y_{co}, where the index o represents a given output ($o = 1, 2, 3$ for outputs LI, EI and WI, respectively). For each output o, there are I_o inputs, denoted by x_{ci}, where the index $i = 1, \dots, I_o$ represents the appropriate inputs, as listed in Table 9.1 and $c = 1, \dots, C$ represents the countries. Only 80 countries had the entire dataset and, hence, for the purposes of this chapter $C = 80$. The rationale for selecting a single output DEA formulation, each representing a particular HDI dimension, instead of a multiple output framework, lies in the fact that, as Table 9.1 indicates, some inputs are unique to a particular output and thus the policy implications differ for each HDI dimension.

To achieve this chapter's objectives, several characteristics of the input/output relationship need first to be described. These are based upon standard notions of production economics (Coelli *et al.* 1998; Cooper *et al.* 2000; Thanassoulis 2001). The first deals with how efficiency should be measured. For this purpose, observe that a DEA formulation may adopt an output or an input orientation. With the former, the efficiency of an economic unit is measured in terms of the output levels produced with a given level of inputs and of its ability to increase those output levels up to those of the benchmark. This is in contrast to an input orientation, where the efficiency of an economic unit is assessed in terms of the levels of the various inputs utilized to produce given levels of output and of its ability to reduce those input levels down to those of the benchmark. This chapter uses the input orientation, as being closer to the stated purpose of this chapter of developing a resource-adjusted HDI estimates. Further, an input orientation appears more desirable since countries have a greater ability to control their inputs than their outputs. The second characteristic deals with returns to scale. If the underlying system is characterized by a constant returns to scale (CRS) technology, with inputs and outputs increasing or decreasing at the same rate, both orientations ought to yield the same efficiency level. Otherwise, when the rates of change differ for the inputs and the output, variable returns to scale (VRS) are manifestations of scale, which should be purged before the appropriate measure of inefficiency can be obtained. Of particular importance for this chapter are the cases of non-increasing (NRS), decreasing (DRS) and increasing (IRS) returns to scale.

Another important characteristic of these formulations is the presence or absence of congestion. 'Evidence of congestion is present when reductions in one or more inputs can be associated with increases in one or more outputs – or, proceeding in reverse, when increases in one or more inputs can be associated with decreases in one or more outputs – without worsening any other input or output' (Cooper *et al.* 2000: 2). Coelli *et al.* (1998: section 7.5) gives some examples of input congestion in cases of government or union-based controls on the use of certain inputs. Output congestion is not relevant for this chapter, since the outputs, LI, EI and WI, are being evaluated separately. The problem with congestion is that it is not costless to dispose of unwanted inputs. Hence, resources that would otherwise be used towards the production of the desired outputs must be devoted for such disposal. In the language of production economics, the terms 'weak disposability' (WD) and 'strong disposability' (SD) are used to denote the presence or absence, respectively, of congestion.

The DEA formulations exhibiting various combinations of characteristics are listed in Table 9.2. Each model is identified by two criteria: (i) WD or SD, in terms of congestion; and (ii) CRS, NRS or VRS for scale. The references listed earlier (Coelli *et al.* 1998; Cooper *et al.* 2000; Thanassoulis 2001) provide theoretical justification for their use. For the purpose of this chapter, the information of interest consists of the optimum values of the efficiency index,

Table 9.2 DEA formulations and efficiency decompositions

	$Min\ k_n$
For all formulations	$\sum_{c=1}^{C} \lambda_c y_c \geq y_n$
	$\lambda_c \geq 0, c = 1, \ldots, C$
For SD, add	$\sum_{c=1}^{C} \lambda_c x_{ci} \leq k_n x_{in} \quad i = 1, \ldots, I$
For WD, add	$\sum_{c=1}^{C} \lambda_c x_{ci} = k_n x_{in} \quad i = 1, \ldots, I$
For VRS, add	$\sum_{c=1}^{C} \lambda_c = 1$
For NRS, add	$\sum_{c=1}^{C} \lambda_c \geq 1$
Efficiency decompositions	k_n (CRS, SD) = (Congestion)(Scale)(PTE) Scale = k_n (CRS, SD)/k_n (VRS, SD) Congestion = k_n (VRS, SD)/k_n (VRS, WD) PTE = k_n (VRS, WD)
DR if	Scale <1 and k_n (NRS, SD) >k_n (CRS, SD)
IR if	Scale <1 and k_n (NRS, SD) = k_n (CRS, SD)
CRS if	Scale = 1

k_n, as well as an assessment of the effect of congestion and scale on efficiency. The subscript n identifies the country/nation that is going to be evaluated in terms of its ability to generate more output or use fewer inputs than a composite of all countries. All models have two constraints in common. One is the non-negativity constraint for the weights; that is, for $\lambda_c, c = 1, \ldots, C$. The other indicates that the level of output of the composite country, computed as the weighted average of all the countries' output, has to be at least as large as that of the country being evaluated. The other four rows provide the additional constraint(s) to be added, depending upon the congestion and scale characteristics desired. Observe that the above decomposition is performed for each output separately. Joint effects of the inputs on the outputs are left for future research.

Decomposing the efficiency indexes

The decomposition used in this study follows the methodology in Färe *et al.* (1994). A summary appears in Färe and Grosskopf (1998a) and an application in Nasierowski and Arcelus (2003). The starting point is to decompose the optimal efficiency of the model in Charnes *et al.* (1981), with constant-returns-to scale, strong disposability (SD, CRS) into three factors. These factors are also listed in Table 9.2. The first two control for scale and for congestion. The last is the pure technical efficiency (PTE), a residual unexplained by the other two factors and thus perhaps a better measure of resource utilization than the VRS or CRS formulations. It should be observed that the NRS case does not play a role in the decomposition process. Its usefulness lies in the role it plays when determining whether the scale factor, if it exists, is due to increasing (IRS) or decreasing (DRS) returns to scale. Table 9.2 also sets the conditions for this dichotomy.

Analysis of results

This section describes the two-part numerical analysis undertaken in support of the efficiency related model of the manuscript. The first part evaluates the implications of the efficiency decomposition listed in Table 9.2. The second uses this information to derive the various efficiency-related HDI estimates and the corresponding country ranks and discusses the statistical evidence for and against the usefulness of the various estimates.

The efficiency decomposition

Table 9.3 presents the numerical results of the efficiency decomposition of Table 9.2. The results were obtained with the *OnFront* package (Färe and Grosskopf 1998b). For each output, be it LI, EI or WI, and each country, Table 9.3 presents the efficiency measure (EFF) under CRS and SD, and its decomposition, in terms of scale (SC), congestion (CON) and pure technical

Table 9.3 Efficiency decompositions

	LI					EI					WI				
Country	**EFF**	**SC**	**CON**	**PTE**	**RS**	**EFF**	**SC**	**CON**	**PTE**	**RS**	**EFF**	**SC**	**CON**	**PTE**	**RS**
Norway	0.95	0.97	0.97	1	IR	0.99	1	0.99	1	CR	1	1	1	1	CR
Sweden	0.96	0.99	0.99	0.98	IR	1	1	1	1	CR	0.93	0.93	1	1	DR
Canada	0.97	0.99	1	0.98	IR	0.99	1	1	0.99	CR	0.85	0.85	1	1	DR
Belgium	0.96	0.98	0.99	0.99	IR	1	1	1	1	CR	0.94	0.94	1	1	DR
Australia	0.98	1	1	0.98	CR	1	1	1	1	CR	0.81	0.83	0.98	1	DR
United States	0.93	0.96	0.96	1	IR	0.99	1	1	0.99	CR	0.97	0.97	1	1	DR
Iceland	0.95	0.98	0.99	0.97	IR	0.97	1	1	0.97	CR	1	1	1	1	CR
Netherlands	0.95	0.98	1	0.97	IR	0.99	0.99	1	1	DR	0.86	0.86	1	1	DR
Japan	1	1	1	1	CR	0.94	0.99	1	0.95	IR	1	1	1	1	CR
Finland	0.97	0.98	1	0.99	IR	1	1	1	1	CR	0.88	0.92	0.96	1	DR
Switzerland	0.97	0.98	0.99	1	IR	0.94	1	1	0.95	CR	0.9	0.9	1	1	DR
France	0.97	0.97	0.99	1	IR	0.98	1	1	0.98	CR	0.93	0.93	1	1	DR
United Kingdom	0.96	0.99	1	0.97	IR	1	1	1	1	CR	0.88	0.88	1	1	DR
Denmark	0.89	0.95	0.98	0.96	IR	0.99	1	1	0.99	CR	0.77	0.77	1	1	DR
Austria	0.96	0.98	1	0.99	IR	0.97	1	1	0.97	CR	0.85	0.85	1	1	DR
Germany	0.95	0.97	0.99	0.99	IR	0.98	1	1	0.98	CR	0.88	0.88	1	0.99	DR
Ireland	0.94	0.98	1	0.97	IR	0.97	1	1	0.97	CR	0.98	0.98	1	1	DR
New Zealand	0.98	1	1	0.98	CR	1	1	1	1	CR	0.83	0.95	0.87	1	DR
Italy	0.97	0.99	0.98	1	IR	0.96	0.98	1	0.98	IR	1	1	1	1	CR
Spain	1	1	1	1	CR	1	1	1	1	CR	0.9	0.9	1	1	DR
Israel	0.95	1	1	0.96	CR	0.96	0.99	1	0.97	IR	0.92	0.96	0.99	0.97	DR
Greece	0.99	0.99	1	1	DR	0.96	0.98	1	0.98	IR	0.94	0.94	1	1	DR
Singapore	1	1	1	1	CR	0.95	0.98	1	0.98	IR	0.86	0.86	1	1	DR
Korea, Rep.	0.99	0.99	1	1	IR	0.98	1	0.98	1	CR	0.92	0.92	1	1	DR
Portugal	0.96	0.99	1	0.98	IR	1	1	1	1	CR	0.79	0.79	1	1	DR
Slovenia	0.97	0.99	1	0.99	IR	0.95	0.98	1	0.97	IR	0.83	0.83	1	1	DR

Argentina	0.94	0.98	1	0.96	IR	0.95	0.96	1	0.99	IR	1	1	1	1	CR
Hungary	0.94	0.98	1	0.96	IR	0.94	0.97	1	0.97	IR	0.81	0.81	1	1	DR
Poland	0.99	0.99	1	1	IR	0.95	0.99	1	0.95	IR	0.76	0.76	1	1	DR
Chile	1	1	1	1	CR	0.93	0.98	0.98	0.97	IR	0.93	0.95	0.98	1	DR
Uruguay	0.96	0.99	1	0.98	IR	0.96	0.96	1	1	IR	1	1	1	1	CR
Costa Rica	1	1	1	1	CR	0.87	0.94	1	0.93	IR	0.95	0.95	1	1	DR
Lithuania	1	1	1	1	CR	0.94	0.97	1	0.97	IR	0.85	0.85	1	1	DR
Trinidad and Tobago	0.98	0.98	1	1	DR	0.88	0.94	1	0.94	IR	1	1	1	1	CR
Latvia	1	1	1	1	CR	0.94	0.98	1	0.95	IR	0.72	0.81	0.88	1	DR
Mexico	0.94	0.98	1	0.96	IR	0.88	0.94	1	0.95	IR	0.96	0.96	1	1	DR
Belarus	0.99	0.99	1	1	IR	0.93	0.97	1	0.96	IR	1	1	1	1	CR
Panama	0.96	0.99	1	0.96	IR	0.89	0.92	1	0.97	IR	0.67	0.91	0.97	0.75	DR
Malaysia	0.98	0.99	1	0.98	IR	0.89	0.9	1	0.99	IR	0.86	0.87	1	0.99	DR
Bulgaria	0.97	1	0.98	1	CR	0.95	0.95	1	1	IR	0.8	0.94	0.85	1	DR
Romania	0.94	0.97	1	0.97	IR	0.91	0.95	1	0.95	IR	0.91	0.91	1	1	DR
Colombia	0.93	0.95	1	0.99	IR	0.89	0.94	0.99	0.95	IR	0.84	0.84	1	1	DR
Venezuela	0.97	1	1	0.97	CR	0.87	0.9	1	0.97	IR	1	1	1	1	CR
Thailand	1	1	1	1	CR	0.87	0.92	1	0.95	IR	0.79	0.88	0.9	1	DR
Brazil	0.87	0.87	1	1	IR	0.89	0.92	0.98	0.98	IR	0.87	1	0.98	0.89	CR
Lebanon	0.9	0.97	1	0.92	IR	0.98	0.98	1	1	IR	0.94	0.99	0.95	1	DR
Philippines	0.91	0.96	1	0.96	IR	0.94	0.96	0.99	0.99	IR	0.81	0.92	0.88	1	DR
Ukraine	0.99	0.99	1	1	IR	0.93	0.99	1	0.94	IR	1	1	1	1	CR
Peru	0.89	0.91	1	0.97	IR	0.97	0.99	1	0.98	IR	1	1	1	1	CR
Turkey	0.91	0.94	1	0.96	IR	0.98	0.99	0.98	1	IR	1	1	1	1	CR
Jamaica	0.99	0.99	1	1	DR	0.84	0.89	0.94	1	IR	0.72	0.91	0.93	0.84	DR
Sri Lanka	1	1	1	1	CR	0.9	0.93	1	0.97	IR	0.97	0.97	1	1	DR
Paraguay	0.91	0.95	1	0.96	IR	0.86	0.92	1	0.94	IR	0.94	0.94	1	1	DR
Ecuador	0.93	0.98	1	0.95	IR	0.93	0.97	0.98	0.98	IR	1	1	1	1	CR
Dominican Rep.	0.86	0.92	1	0.94	IR	0.95	0.95	1	1	IR	1	1	1	1	CR
Uzbekistan	0.92	0.98	0.94	1	IR	0.92	1	0.97	0.95	CR	1	1	1	1	CR

Continued

Table 9.3 Continued

Country	LI					EI					WI				
	EFF	SC	CON	PTE	RS	EFF	SC	CON	PTE	RS	EFF	SC	CON	PTE	RS
China	0.94	0.99	1	0.95	IR	1	1	1	1	CR	0.69	0.74	0.94	1	DR
Tunisia	0.88	0.94	1	0.94	IR	0.97	0.97	1	1	IR	0.86	0.92	0.93	1	DR
Jordan	0.89	0.97	1	0.92	IR	0.89	0.89	1	1	IR	0.94	1	0.94	1	CR
El Salvador	0.92	0.94	1	0.98	IR	0.86	0.87	0.99	1	IR	1	1	1	1	CR
South Africa	0.55	0.56	0.98	1	IR	0.93	0.94	1	1	IR	1	1	1	1	CR
Syrian Arab Rep.	0.9	0.98	1	0.92	IR	0.98	0.98	1	1	IR	1	1	1	1	CR
Viet Nam	0.97	0.97	1	0.99	IR	0.88	0.96	0.99	0.93	IR	0.75	0.81	0.93	1	DR
Indonesia	1	1	1	1	CR	1	1	1	1	CR	0.94	0.94	1	1	DR
Tajikistan	1	1	1	1	CR	0.95	0.97	0.98	1	DR	1	1	1	1	CR
Bolivia	0.75	0.8	1	0.94	IR	0.92	0.98	1	0.95	IR	0.87	0.95	0.91	1	DR
Honduras	0.85	0.86	1	0.98	IR	0.79	0.79	1	1	IR	0.91	0.99	0.92	1	IR
Nicaragua	0.9	0.92	1	0.97	IR	0.7	0.71	0.98	1	IR	0.95	0.95	1	1	DR
Guatemala	0.83	0.84	1	0.99	IR	0.84	0.88	0.95	1	IR	1	1	1	1	CR
Zimbabwe	0.43	0.47	0.92	0.98	IR	0.89	0.96	1	0.93	IR	0.92	1	0.93	1	CR
Ghana	0.94	0.94	1	1	IR	0.8	0.82	0.99	0.97	IR	0.83	0.83	1	1	DR
Kenya	0.65	0.68	0.98	0.97	IR	0.84	0.88	0.99	0.97	IR	1	1	1	1	CR
Congo	0.6	0.6	1	1	IR	0.89	0.93	0.96	1	IR	1	1	1	1	CR
Pakistan	0.73	0.79	1	0.91	IR	0.78	0.8	0.98	1	IR	1	1	1	1	CR
Nepal	1	1	1	1	CR	1	1	1	1	CR	1	1	1	1	CR
Bangladesh	0.83	0.83	1	1	IR	0.69	0.69	1	1	IR	1	1	1	1	CR
Nigeria	0.59	0.63	0.99	0.94	IR	1	1	1	1	CR	1	1	1	1	CR
Zambia	0.45	0.48	0.92	1	IR	0.85	0.9	0.99	0.94	IR	0.8	0.82	0.98	1	IR
Senegal	0.82	0.82	1	1	IR	0.59	0.62	0.96	1	IR	0.87	0.96	0.91	1	DR
Benin	0.87	0.87	1	1	IR	0.77	0.78	0.99	1	IR	1	1	1	1	CR

efficiency (PTE). The last column for each output indicates whether the country in question exhibits the type of returns to scale (RS) that can be classified as increasing (IRS), constant (CRS) or decreasing (DRS), in accordance with the criteria listed at the end of Table 9.2. The results indicate that congestion is not much of a problem for any country. Even for those with CON below 1, the actual value is over 0.9 and even higher for LI and EI. A few exceptions exist in the WI case (New Zealand, Latvia, Bulgaria and Philippines), but even then the CON values are all in the high 0.80s. Most of the inefficiency, when in existence, appears to be scale related. This is true, even in highly inefficient countries, for one or more outputs – as is the case, for example, with South Africa, Zimbabwe, Nigeria and Zambia for LI, or Senegal for EI. Once congestion and scale are controlled for, the high values in the PTE columns indicate scant evidence of inefficiency left to be explained by exogenous factors. Further, with a few exceptions, the evidence indicates that any further resource investment and/or reallocation in most inefficient countries should be directed towards health and education, to judge by the overwhelming majority of IR in the LI and EI columns and the mostly DR in WI. These results are also consistent with key tenets of human capital theory (Schultz 1993).

The HDI estimates

With the efficiency coefficients listed in Table 9.3, three different HDI estimates are computed. The first adds up the values of LI, EI and WI, weighted by the corresponding EFF (model CRS, SD) estimates of Table 9.3 and divides the resulting sum by three. The equal weight given to each output follows the original HDI computational procedure. This process yields the values of the HCRS column of Table 9.4. A similar procedure is used with the EFF estimates for the (VRS, SD) model, to yield the values of the HVRS column. Similarly, the use of the PTE weights results in the estimates of the HPTE column. Table 9.4 includes the necessary information together with the original HDI and the gender-related HDI (GHDI) values and the country ranks resulting from each set of estimates.

Table 9.4 provides a wide assortment of index values and of ranks, but no hint as to whether there are any statistically significant differences among them. The issue here is whether the various indexes exhibit any information content over and above that provided by the original HDI. The statistical analysis is summarized in Table 9.5. The data in Table 9.4 are used in the computation of the Pearson correlation coefficients between the values of any two indexes (the pair comparison t-test was also used, with similar conclusions and hence are not reported) and the nonparametric Spearman correlation coefficients for the corresponding ranks. These and other statistical tests appear in most textbooks on the subject (Lind *et al.* 2003). The null hypothesis tests for the existence of pairwise correlation. Low p-values indicate presence of such correlation.

Table 9.4 HDI values and associated country ranks

	HDI		HCRS		HVRS		HPTE		GHDI	
Country	**Value**	**Rank**	**Value**	**Rank**	**Value**	**Rank**	**Value**	**Rank**	**Value**	**Rank**
Norway	0.9420	1	0.9219	1	0.9278	4	0.9400	1	0.9410	1
Sweden	0.9410	2	0.9064	6	0.9309	2	0.9339	5	0.9360	5
Canada	0.9400	3	0.8807	12	0.9307	3	0.9307	6	0.9380	2
Belgium	0.9390	4	0.9093	4	0.9341	1	0.9370	2	0.9330	7
Australia	0.9390	4	0.8751	15	0.9278	5	0.9340	4	0.9380	2
United States	0.9390	4	0.9067	5	0.9251	7	0.9367	3	0.9370	4
Iceland	0.9360	7	0.9121	2	0.9181	8	0.9181	10	0.9340	6
Netherlands	0.9350	8	0.8751	14	0.9278	6	0.9278	7	0.9300	8
Japan	0.9330	9	0.9114	3	0.9145	12	0.9145	12	0.9270	10
Finland	0.9300	10	0.8844	10	0.9148	11	0.9271	8	0.9280	9
Switzerland	0.9280	11	0.8675	16	0.9080	15	0.9110	15	0.9230	14
France	0.9280	11	0.8898	9	0.9172	10	0.9202	9	0.9260	11
UK	0.9280	11	0.8785	13	0.9179	9	0.9179	11	0.9250	12
Denmark	0.9260	14	0.8168	24	0.9031	17	0.9087	18	0.9240	13
Austria	0.9260	14	0.8587	19	0.9111	14	0.9141	13	0.9210	15
Germany	0.9250	16	0.8654	17	0.9079	16	0.9109	16	0.9200	16
Ireland	0.9250	16	0.8902	8	0.9023	18	0.9051	20	0.9170	17
New Zealand	0.9170	18	0.8609	18	0.8727	24	0.9108	17	0.9140	18
Italy	0.9130	19	0.8919	7	0.9011	19	0.9071	19	0.9070	19
Spain	0.9130	19	0.8840	11	0.9133	13	0.9133	14	0.9060	20
Israel	0.8960	21	0.8491	21	0.8670	26	0.8700	25	0.8910	21
Greece	0.8850	22	0.8544	20	0.8805	20	0.8805	22	0.8790	23
Singapore	0.8850	22	0.8297	23	0.8780	21	0.8809	21	0.8800	22
Korea, Rep.	0.8820	24	0.8480	22	0.8737	23	0.8800	23	0.8750	26
Portugal	0.8800	25	0.8086	26	0.8744	22	0.8744	24	0.8760	25
Slovenia	0.8790	26	0.8072	27	0.8678	25	0.8678	26	0.8770	24
Argentina	0.8440	27	0.8118	25	0.8295	27	0.8295	27	0.8360	27
Hungary	0.8350	28	0.7487	35	0.8107	31	0.8138	31	0.8330	28
Poland	0.8330	29	0.7550	33	0.8177	29	0.8177	30	0.8310	29
Chile	0.8310	30	0.7946	29	0.8163	30	0.8243	29	0.8240	31
Uruguay	0.8310	30	0.8068	28	0.8218	28	0.8245	28	0.8280	30
Costa Rica	0.8200	32	0.7704	30	0.7999	32	0.7999	32	0.8140	32
Lithuania	0.8080	33	0.7526	34	0.7974	33	0.7974	33	0.8060	33
Trinidad and Tobago	0.8050	34	0.7643	32	0.7865	34	0.7865	34	0.7980	34
Latvia	0.8000	35	0.7151	40	0.7561	38	0.7845	35	0.7980	34
Mexico	0.7960	36	0.7339	36	0.7660	37	0.7688	39	0.7890	36
Belarus	0.7880	37	0.7661	31	0.7777	35	0.7777	37	0.7860	37
Panama	0.7870	38	0.6694	54	0.7059	54	0.7105	55	0.7840	38
Malaysia	0.7820	39	0.7104	44	0.7696	36	0.7696	38	0.7760	40
Bulgaria	0.7790	40	0.7121	42	0.7409	42	0.7800	36	0.7780	39
Romania	0.7750	41	0.7112	43	0.7512	40	0.7512	42	0.7730	41

Continued

Table 9.4 Continued

	HDI		HCRS		HVRS		HPTE		GHDI	
Country	**Value**	**Rank**	**Value**	**Rank**	**Value**	**Rank**	**Value**	**Rank**	**Value**	**Rank**
Colombia	0.7720	42	0.6841	48	0.7533	39	0.7533	41	0.7670	42
Venezuela	0.7700	43	0.7260	38	0.7509	41	0.7537	40	0.7640	43
Thailand	0.7620	44	0.6753	51	0.7230	47	0.7460	43	0.7600	44
Brazil	0.7570	45	0.6609	57	0.7135	49	0.7214	50	0.7510	45
Lebanon	0.7550	46	0.7085	45	0.7215	48	0.7320	47	0.7390	48
Philippines	0.7540	47	0.6743	52	0.7130	50	0.7404	44	0.7510	45
Ukraine	0.7480	48	0.7261	37	0.7316	44	0.7316	48	0.7440	47
Peru	0.7470	49	0.7145	41	0.7369	43	0.7369	45	0.7290	52
Turkey	0.7420	50	0.7157	39	0.7282	46	0.7333	46	0.7340	51
Jamaica	0.7420	50	0.6424	63	0.6855	59	0.7113	53	0.7390	48
Sri Lanka	0.7410	52	0.7061	46	0.7316	44	0.7316	48	0.7370	50
Paraguay	0.7400	53	0.6628	56	0.7101	52	0.7101	56	0.7270	53
Ecuador	0.7320	54	0.6955	47	0.7092	53	0.7150	51	0.7180	56
Dominican Rep.	0.7270	55	0.6807	49	0.7127	51	0.7127	52	0.7180	56
Uzbekistan	0.7270	55	0.6796	50	0.6845	60	0.7082	57	0.7250	54
China	0.7260	57	0.6451	62	0.6985	56	0.7107	54	0.7240	55
Tunisia	0.7220	58	0.6506	60	0.6889	58	0.7050	58	0.7090	58
Jordan	0.7170	59	0.6480	61	0.6842	61	0.6964	61	0.7010	59
El Salvador	0.7060	60	0.6555	58	0.7025	55	0.7050	58	0.6960	60
South Africa	0.6950	61	0.6086	64	0.6937	57	0.6967	60	0.6890	61
Syrian Arab Rep.	0.6910	62	0.6629	55	0.6728	63	0.6728	63	0.6690	64
Viet Nam	0.6880	63	0.6042	65	0.6502	65	0.6647	65	0.6870	62
Indonesia	0.6840	64	0.6719	53	0.6833	62	0.6833	62	0.6780	63
Tajikistan	0.6670	65	0.6520	59	0.6608	64	0.6667	64	0.6640	65
Bolivia	0.6530	66	0.5540	67	0.6084	69	0.6243	69	0.6450	66
Honduras	0.6380	67	0.5378	68	0.6180	67	0.6321	66	0.6280	68
Nicaragua	0.6350	68	0.5355	69	0.6218	66	0.6261	68	0.6290	67
Guatemala	0.6310	69	0.5595	66	0.6175	68	0.6278	67	0.6170	69
Zimbabwe	0.5510	70	0.4520	72	0.5126	71	0.5324	71	0.5450	70
Ghana	0.5480	71	0.4697	71	0.5417	70	0.5438	70	0.5440	71
Kenya	0.5130	72	0.4248	74	0.4980	73	0.5018	73	0.5110	72
Congo	0.5120	73	0.4272	73	0.5033	72	0.5133	72	0.5060	73
Pakistan	0.4990	74	0.4137	75	0.4765	76	0.4793	75	0.4680	75
Nepal	0.4900	75	0.4900	70	0.4900	74	0.4900	74	0.4700	74
Bangladesh	0.4780	76	0.4030	77	0.4767	75	0.4767	76	0.4680	75
Nigeria	0.4620	77	0.4032	76	0.4531	77	0.4545	77	0.4490	77
Zambia	0.4330	78	0.3238	80	0.4069	80	0.4164	80	0.4240	78
Senegal	0.4310	79	0.3317	79	0.4116	79	0.4300	78	0.4210	79
Benin	0.4200	80	0.3685	78	0.4187	78	0.4200	79	0.4040	80

Table 9.5 Statistical tests

	Pearson correlation				
	HDI	**GHDI**	**HCRS**	**HVRS**	**HPTE**
HDI	—	0.999	0.987	0.995	0.980
GHDI	0	—	0.984	0.994	0.988
HCRS	0	0	—	0.991	0.997
HVRS	0	0	0	—	0.986
HPTE	0	0	0	0	—
	Spearman correlation				
	HDI	**GHDI**	**HCRS**	**HVRS**	**HPTE**
HDI	—	0.999	0.972	0.990	0.980
GHDI	0	—	0.969	0.988	0.977
HCRS	0	0	—	0.997	0.991
HVRS	0	0	0	—	0.971
HPTE	0	0	0	0	—

Notes: correlation coefficients in upper triangle; p-values in lower triangle.

Several implications of the results in Table 9.5 deserve special consideration. First, GHDI and HDI yield almost identical values and ranks. Hence, GHDI does not exhibit much discriminating power, independent of HDI, in explaining gender-related issues. Second, each component of the decomposition in Table 9.2 may be evaluated in terms of its information content over and above that provided by HDI. This can be readily seen by comparing, both for the values and for the ranks. As a result,

(i) HDI/GHDI versus HCRS can be used for the effect of accounting or not for efficiency;
(ii) HCRS versus HVRS, for the effect of scale;
(iii) HVRS versus HPTE, for the effect of congestion; and
(vi) HDI/GHDI versus HPTE for the effect of controlling for both congestion and scale.

The last comparison is of particular importance, since the comparison is made between the first and last indexes; that is, without any efficiency considerations and after both effects have been accounted for. The results suggest the robustness of the original HDI estimates. All correlations are above 0.9 and highly significant. This indicates that HDI does manage to capture most of the inefficiency of countries in the utilization of their resources. Finally, these interpretations should be tempered by the observation that this stability is certainly due to the behaviour of the countries ranked in approximately the bottom two thirds of the table. The top, say, 20 ranked countries (approximately) do exhibit sufficient variations across the various HDI estimates, to suggest substantial differences in ranking. Thus, the resource-adjusted HDI adds an additional explanatory dimension without distorting the information content of the original HDI. The reasons for this dichotomy are left as avenue for further research.

Conclusion

HDI as an alternative measure of progress of nations has opened up new prospects for analysing socioeconomic development in a cross-country comparative context. However, it is still in need of refinement since development is a complex, dynamic and multidimensional concept. In fact, as noted earlier in this chapter, there is a growing body of literature devoted to this objective. However, this literature appears to have focused mainly on the distributional or equity aspect of development, without any recognition of changes in the resource base. But equity without efficiency is not sustainable over time. It is thus important to analyze whether a given level of human development of a nation is achieved using available resources optimally. The DEA methodology addresses this problem by recognizing and analyzing the output levels and resource commitments in the estimation of efficiency adjusted HDIs. Further, such analysis has been undertaken relative to the performance of other countries, rather than on the basis of some predetermined objective. In this way, the modern benchmarking methodology may be brought to the fore for a large cross-section of countries. One of the policy implications of this study, then, is that countries can find their human development achievements relative to resource utilization, and take a more pro-active approach to improve efficiency in such events where inefficient use of resources is discernible. As a result, to increase the HDI, an efficient country may need more resources, whereas an inefficient one may start by considering the need for structural change. Further, from the RS results of Table 9.3, the resource allocation should be directed towards health and education, the two dimensions where the overwhelming majority of inefficient countries exhibit increasing returns on their investment.

This study also calls for an extension of the debate on HDI by bringing the efficiency dimension into discussions. In essence, it has attempted to integrate welfare economics and production economics to study the globally significant issue of development. Within these two branches of economics, there exist many facets of human development issues that remain unexplored. More research along this integrative line may open up possibilities for important theoretical and practical developments. For example, it may lead to the calculation of HDI that may be more in tune with new concerns, such as the environment or, as in this chapter, gender equality. The advantage of this development is that comparing across a variety of these HDIs leads to the identification of the countries that may rank higher in the achievement of a particular objective than in another. In this way, the selection of inputs and outputs can provide a better match to society's values. Such an approach is also more in tune to Sen's (1990, 1992) concept of development as an expansion of the capabilities of a country and of its citizens.

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10

A Framework for Incorporating Environmental Indicators in the Measurement of Human Well-Being

Osman Zaim

Introduction

The last decade has witnessed major improvements in the measurement of sustainable human development. Considerable time and research effort have been devoted to both extending the dimensions of the measurement and the methodology used to compute sustainable human development indices. Now, the measurement of human well-being is not only limited to economic indicators but also takes into account social, institutional and ecological background, thus utilizing over 130 indicators approved by the United Nations in April 1995 (UN 2001). Improvements in the data collection of indicators, while triggering the construction of indexes from a series of constituent indicators such as human development index (HDI) with component indicators on longevity, educational attainment and income, have also led to aggregation of indexes of different dimensions. As a typical example of the latter, one can cite Prescott-Allen's (2001) human well-being index (HWI), which is an equal weighted average of the human well-being index and ecosystem well-being index (EWI), integrating two indices with social-economical and environmental dimensions.

On the academic front, research during recent years has considerably improved our understanding of sustainable human development, but at the expense of generating a certain amount of controversy. The concerns range from not being able to construct a totally objective index of sustainable human development (because both the indicator selection and weights assigned to these reflect normative judgements of those who developed the index), to whether these indices satisfy the certain axiomatic properties required of any index (Sen 1976; Zheng 1993). Specifically, a consensus has emerged that existing indexes, such as the HDI, fail to measure performance comparisons across time, because these are designed to measure performance comparisons at a point of time rather than being a measure of over-time comparisons.[1]

The purpose of this chapter is to expand on earlier works by Zaim *et al.* (2001) and Grosskopf *et al.* (2002) to provide a framework for incorporating environmental indicators to the measurement of human well-being. Although some prior studies such as Nath *et al.* (1998), Lasso de la Vega and Urrutia (2001) and Prescott-Allen (2001) all had the same objective of reconciling human well-being with environmental indicators, this study with its economic-theoretical approach to index number theory is a deviation. This chapter proposes a useful alternative to the 'aggregate deprivation index' used to measure the well-being of individuals in different countries or geographic locations. Furthermore, an improvement index which alleviates the well-known difficulties associated with over-time comparisons of the aggregate deprivation index is also proposed. The achievement index in this study relies heavily on the theory of quantity indexes whose axiomatic properties are well established. The roots of the improvement index are well grounded in the productivity growth literature. All proposed measures depend on the computation of distance functions, which are a complete characterization of technology. The indexes introduced in this study are an improvement of the empirical literature on social indicators in several respects.

First, unlike previous studies which typically produce a synthetic indicator that aggregates its constituents using artificially assigned weights, the proposed approach implicitly recognizes the underlying production process which transforms inputs (per capita capital) into private goods (which can be proxied by per capita income), social goods (which can be proxied by longevity and knowledge) and undesired goods (such as emissions of environmentally hazardous elements) by putting sufficient emphasis on production with negative externalities. Thus, while providing an economic content to social indicators, the aggregator characteristics of distance functions (which aggregate components with optimally chosen weights determined by the data) are fully exploited. Second, the proposed improvement index, since it is measured with respect to a production technology which is allowed to change over time, can capture the improvement in performance better than alternative indexes which have less tolerance for best achievers.

The next section presents the methodology for constructing a human development index that takes into account differences in the environmental conditions. This is followed by an empirical application and the conclusions drawn.

Methodology

This section follows closely Zaim *et al.* (2001) and Grosskopf *et al.* (2002) in order to summarize the methodology for constructing a human development index void of any environmental considerations, so as to prepare the background for an extension that would allow for the incorporation of environmental indicators. Let us consider a sample of K countries, each of which produces a vector of private goods denoted by $y = (y_1, \ldots, y_M) \in R_+^M$ and a vector of social goods $s = (s_1, \ldots, s_J) \in R_+^J$, using inputs $x = (x_1, \ldots, x_N) \in R_+^N$ with a meta technology represented by output (or production possibilities) set as

$$P(x) = \{(y, s) : x \text{ can produce } (y, s)\} \tag{10.1}$$

which satisfy certain axioms laid out by Shephard (1970).

P.1 $P(0) = \{0, 0\}$.
P.2 $P(x)$ is compact for each $x \in R_+^N$.
P.3 $P(x) \supseteq P(x'), x \geq x'$.
P.4 $(y, s) \in P(x)$ and $y' \leq y$ and $s' \leq s$ imply $(y', s') \in P(x)$.

Properties P.1 state that zero inputs yield zero outputs and that any nonnegative input yields at least zero output. Properties P.2 require that only finite output should be produced given finite inputs. Finally, properties P.3 and P.4 impose free disposability of inputs and outputs, respectively.

Since the ultimate goal is to construct a quantity index for private and social goods that would allow multilateral comparisons across countries, Shephard's output distance functions are a useful tool for both representing technology and also serving as a measure of performance. Hence, given an arbitrary vector of inputs x^0, once the success of countries i and j in expanding their private and social goods with respect to a set of production possibilities (output) common to all countries is measured by means of distance functions,

$$\begin{aligned} D_o^i(x^0, y^i) &= \inf\{\theta^i : x^0, (y^i, s^i)/\theta^i \in P(x)\} \text{ and} \\ D_o^j(x^0, y^j) &= \inf\{\theta^j : x^0, (y^j, s^j)/\theta^j \in P(x)\} \end{aligned} \tag{10.2}$$

this allows for the construction of Malmquist quantity index of (aggregated) social and private goods as

$$Q(x^0, y^i, y^j, s^i, s^j) = \frac{D_o^i(x^0, y^i, s^i)}{D_o^j(x^0, y^j, s^j)} \tag{10.3}$$

This quantity index now compares the provision of social and private goods in country i with respect to a reference country j given an arbitrary vector of inputs x^0 common to both. Since the meta technology that serves as a basis for the computation of distance functions is unobserved, it has to be constructed from the observed inputs and outputs of the countries in our sample. For this purpose, an activity analysis or data envelopment analysis (DEA) approach, which satisfies the properties underlying the technology (see Färe *et al.* 1994), is employed. The piecewise linear output set is

$$P(x) = \{(y,s) : \begin{array}{ll} \sum\limits_{k=1}^{K} z_k y_{km} \geq y_m, & m = 1,\dots,M \\ \sum\limits_{k=1}^{K} z_k s_{kj} \geq s_j, & j = 1,\dots,J \\ \sum\limits_{k=1}^{K} z_k x_{kn} \leq x_n, & n = 1,\dots,N \\ \quad z_k \geq 0 & k = 1,\dots,K\} \end{array} \tag{10.4}$$

where z_k are the intensity variables which serve to form the technology from convex combinations of the data.

This quantity index, which is essentially a Malmquist quantity index (see Färe and Primont 1995), satisfies a number of desirable properties due to Fisher (1922). These are

(1) Homogeneity: $Q(x^0, \lambda y^i, y^j, \lambda s^i, s^j) = \lambda Q(x^0, y^i, y^j, s^i, s^j)$
(2) Time-reversal: $Q_s(x^0, y^i, y^j, s^i, s^j) Q_s(x^0, y^j, y^i, s^j, s^i) = 1$
(3) Transitivity: $Q_s(x^0, y^i, y^j, s^i, s^j) Q_s(x^0, y^j, y^t, s^j, s^t) = Q_s(x^0, y^i, y^t, s^i, s^t)$
(4) Dimensionality: $Q_s(x^0, \lambda y^i, \lambda y^j, \lambda s^i, \lambda s^j) = Q_s(x^0, y^i, y^j, s^i, s^j)$

As for the improvement index, we measure the success of a particular country in expanding its social goods from year t to year $t+1$ with respect to a common (world) benchmark technology constructed for the period t. Our improvement index

$$IMP^{t,t+1} = \frac{D_o^{k,t}(x^{k,t}, y^{k,t+1}, s^{k,t+1})}{D_o^{k,t}(x^{k,t}, y^{k,t}, s^{k,t})} \tag{10.5}$$

is the ratio of two distance functions where

$$D_o^{k,t}(x^{k,t}, y^{k,t+1}, s^{k,t+1}) \\ = \inf\{\theta^{k,t+1} : (x^{k,t}, (y^{k,t+1}, s^{k,t+1})/\theta^{k,t+1}) \in P^t(x^t)\} \tag{10.6}$$

and

$$D_o^{k,t}(x^{k,t}, y^{k,t}, s^{k',t}) = \inf\{\theta^{k,t} : (x^{k,t}, (y^{k,t}, s^{k,t})/\theta^{k,t}) \in P^t(x^t)\}$$

The first-distance function shows the success of an observation, say k, in expanding its private and social goods in year $t+1$ (with respect to a common frontier which represents the technology at t) while using the same level as in year t (that is, $x^{k,t}$). Similarly, the second-distance function measures the success of the same observation in expanding its private and social goods in period t with respect to a common frontier representing the technology at t. Note that, since the distances are measured against the same benchmark (while holding resources and private goods at their year t levels), the ratio indicates the improvement in the provision of private and social goods for observation k.

To incorporate the joint production of bad outputs $b = (b_1, \ldots, b_I) \in R^I_+$ (that is, emissions of environmentally hazardous elements, which would ultimately affect human well-being negatively), this requires modifications both in our definition of performance measure and also in axiomatic properties of the meta technology. Note that while the Shephard output distance function allows one to construct a human well-being index by aggregating the social and private goods without any normative judgements, it still fails to account for the joint production of goods and bads. The generalization of this index to include bad outputs would not be meaningful (by redefining the output distance function as $D_O(x, y, s, b) = \inf\{\theta : x, (y, s, b)/\theta \in P(x)\}$) since it would mean proportionate expansion of bads together with private and social goods as much as feasible without crediting the reduction of bads. Nevertheless, the directional distance function proposed by Chung *et al.* (1997), which suggests asymmetric treatment of good and bad outputs, provides a solution by crediting the expansion of good outputs and the contraction of bad outputs. Letting $g = (g_y, g_s, -g_b)$ be a direction vector, the directional distance function is expressed as

$$\vec{D}_O(x, y, s, b; g_y, g_s, -g_b) = \sup\left[\beta : (y + \beta g_y, s + \beta g_s, b - \beta g_b) \in P(x)\right] \tag{10.7}$$

A nice feature of this directional distance function is that, as shown by Chung *et al.* (1997), it embodies Shephard's output distance function as a special case. Letting $g = (y, s, b)$ and following Chung *et al.* (1997):

$$\begin{aligned} \vec{D}_O(x, y, s, b; y, s, b) &= \sup\{\beta : D_O(x, (y, s, b) + \beta(y, s, b) \leq 1\} \\ &= \sup\{\beta : (1 + \beta) D_O(x, y, s, b) \leq 1\} \\ &= \sup\left\{\beta : \beta \leq \frac{1}{D_O(x, y, s, b)} - 1\right\} \\ &= \frac{1}{D_O(x, y, s, b)} - 1 \end{aligned} \tag{10.8}$$

or equivalently

$$D_O(x, y, s, b) = 1/(1 + \vec{D}_O(x, y, s, b; y, s, b)$$

Then, letting $g = (y, s, -b)$, the bads-incorporating human well-being index can be expressed as

$$Q(x^0, y^i, y^j, s^i, s^j, b^i, b^j) = \frac{1 + \vec{D}^j_O(x^0, y^j, s^j, b^j; y^j, s^j, -b^j)}{1 + \vec{D}^i_O(x^0, y^i, s^i, b^i; y^i, s^i, -b^i)} \tag{10.9}$$

This quantity index now shows the success of country *i* in equiproportionate expansion and contraction of good (social and private) and bad outputs respectively, relative to a reference country *j*. Similarly the bads-incorporating improvement index can be written as

$$IMP^{t,t+1} = \frac{1+\vec{D}_o^{k,t}\left(x^{k,t}, y^{k,t}, s^{k,t}, b^{k,t}; y^{k,t}, s^{k,t}, -b^{k,t}\right)}{1+\vec{D}_o^{k,t}\left(x^{k,t}, y^{k,t+1}, s^{k,t+1}, b^{k,t+1}; y^{k,t+1}, s^{k,t+1}, -b^{k,t+1}\right)} \tag{10.10}$$

Now we turn our attention to the axiomatic properties of the output set when some outputs are undesired or associated with negative externalities. In the production theory, it is common to assume that outputs are strongly disposable, which implies that the disposal of any output can be achieved without incurring any costs in terms of reduced production of other outputs. This is, in fact, the case for desired outputs, private and social goods in P.4. However, the symmetric treatment of outputs in terms of their disposability characteristics loses its justification if one or some of the outputs, along with the desired outputs, are undesired goods such as carbon dioxide production (as a by-product). Especially in regulated environments, where the productive unit is forced to clean up its undesired output or to reduce its levels of undesired output production, undesired and desired outputs have to be treated asymmetrically in terms of their disposability characteristics. Even in the absence of regulations, increased environmental consciousness in society still requires the treatment of undesired goods as weakly disposable; that is, their disposal is achieved by reducing the desired outputs proportionately. The following property

P.5 $(y, s, b) \in P(x)$ and $0 \leq \theta \leq 1$ imply $(\theta y, \theta s, \theta b) \in P(x)$

imposes weak disposability of good and bad outputs.[2] It states that for a given input vector, only a proportional contraction of good and bad outputs is feasible. Finally, one should also recognize the joint product nature of bad outputs. The following property, P.6, is referred to as null-jointness:[3]

P.6 $(y, b) \in P(x)$ and $b = 0$ then $y = 0$

This implies that for a given output vector, if bad output is zero, then so too must be good output. In other words, if one wishes to produce good output, some bad output will also be produced.

The piecewise linear output set associated with properties P.1–P.6 is (see Färe *et al.* 1994):

$$
\begin{aligned}
P(x) = \{(y,s,b) : \quad & \sum_{k=1}^{K} z_k y_{km} \geq y_m, && m = 1,\ldots,M \\
& \sum_{k=1}^{K} z_k s_{kj} \geq s_j, && j = 1,\ldots,J \\
& \sum_{k=1}^{K} z_k b_{ki} = b_i, && i = 1,\ldots,I \\
& \sum_{k=1}^{K} z_k x_{kn} \leq x_n, && n = 1,\ldots,N \\
& z_k \geq 0 && k = 1,\ldots,K\}
\end{aligned}
\tag{10.11}
$$

where the z_k are intensity variables that serve to form the technology from convex combinations of the data. The technology represented in (10.11) also satisfies constant returns to scale; that is

$$P(\lambda x) = \lambda P(x), \lambda > 0 \tag{10.12}$$

The first two inequalities in (10.11) imply that private and social goods are freely disposable. Since the intensity variables $z_k, k = 1,\ldots,K,$ are non-negative and the bad output constraint is a strict equality, one can show that (10.12) satisfies weak disposability.

Null-jointness requires that

(i) $\sum_{k=1}^{K} b_{ki} > 0, i = 1,\ldots,I$

(ii) $\sum_{i=1}^{I} b_{ki} > 0, i = 1,\ldots,K$

The first inequality requires that each bad output is produced by some firm k, while the second inequality states that each firm k produces some bad output.

Having formed the output set for each of the countries in our dataset $k' = 1,\ldots K$, the solution to the following linear programming problem computes the directional distance:

$$
\begin{aligned}
& \vec{D}_O(x^0, y^{k'}, s^{k'}, b^{k'}; g) = \max \beta \\
& \text{such that} \\
& \sum_{k=1}^{K} z_k y_m^k \geq y_m^{k'} + \beta y_m^{k'} && m = 1,\ldots,M \\
& \sum_{k=1}^{K} z_k s_j^k \geq s_j^{k'} + \beta s_j^{k'} && j = 1,\ldots,J \\
& \sum_{k=1}^{K} z_k b_i^k = b_i^{k'} - \beta b_i^{k'} && i = 1,\ldots,I \\
& \sum_{k=1}^{K} z_k x_n^k \leq x_n^o && n = 1,\ldots,N
\end{aligned}
\tag{10.13}
$$

which is in the denominator of (10.9). The directional distance of the reference country in the numerator is computed by replacing the right-hand sides of the first three constraints above with the associated quantities of the country chosen as the reference country, that is, country *j*.

As for the computation of the improvement index, for each *k*′ the following linear programming problem:

$$
\begin{array}{ll}
\vec{D}^{k't}(x^{k'}, s^{k',t+1}, b^{k'',t+1}, y^{k't+1}) = \max \theta^{k',t+1} & \\
\text{such that} & \\
\sum\limits_{k=1}^{K} z_k s^t_{kj} - \theta^{k',t+1} s^{k',t+1}_{k'j} \geq s^{t+1}_{k'j} & j = 1, \ldots, J \\
\sum\limits_{k=1}^{K} z_k y^t_{km} - \theta^{k',t+1} y^{t+1}_{k'm} \geq y^{t+1}_{k'm} & m = 1, \ldots, M \\
\sum\limits_{k=1}^{K} z_k b^t_{ki} + \theta^{k',t+1} b^{t+1}_{k'i} = b^{t+1}_{k'i} & i = 1, \ldots, I \\
\sum\limits_{k=1}^{K} z_k x^t_{kn} \leq x^t_{k'n} & n = 1, \ldots, N \\
z_k \geq 0 & k = 1, \ldots\ldots, K
\end{array}
\qquad (10.14)
$$

solves for the directional distance function in the denominator of $IMP^{t,t+1}$. The numerator can be computed in a similar fashion as

$$
\begin{array}{ll}
\vec{D}^{k't}(x^{k'}, s^{k',t}, b^{k',t}, y^{k',t}) = \max \theta^{k'} & \\
\text{such that} & \\
\sum\limits_{k=1}^{K} z_k s^t_{kj} - \theta^{k',t} s^{k'}_{k'j} \geq s^t_{k'j} & j = 1, \ldots, J \\
\sum\limits_{k=1}^{K} z_k y^t_{km} - \theta^{k',t} y^t_{k'm} \geq y^t_{k'm} & m = 1, \ldots, M \\
\sum\limits_{k=1}^{K} z_k b^t_{ki} + \theta^{k',t} b^t_{k'i} = b^t_{k'i} & i = 1, \ldots, I \\
\sum\limits_{k=1}^{K} z_k x^t_{kn} \leq x^t_{k'n} & n = 1, \ldots, N \\
z_k \geq 0 & k = 1, \ldots\ldots, K
\end{array}
\qquad (10.15)
$$

A numerical example

In constructing the numerical exercise for the human well-being and improvement indexes proposed in this study, data for 22 high income OECD countries are selected for the years 1977, 1980, 1982, 1987, 1990. The reason for restricting the sample to high income countries is twofold. First, since a meta technology that is common to all countries exists, it is desirable that the weak disposability of undesired outputs assumption holds for each country. Otherwise, there is always a danger of over-crediting the comparatively lower emissions of lower income countries who are treating undesired outputs as strongly disposable. In such instances, it may be better to relax the weak disposability assumption in favour of strongly disposable outputs, but still crediting expansion of desired outputs and contraction of undesired. Second, this allows the demonstration of a couple of stylized facts on the informational validity of HDI. Various studies, specifically Ivanova *et al.* (1999), in their assessment of the measurement properties of HDI, find very high correlations between the HDI index and per capita GDP of countries. Furthermore, they show that this high correlation is independent of the weights assigned to constituent indices. Therefore, one would expect this to be amplified within a group of high income countries, resulting in an HDI, which is indistinguishable from per capita income, since all other constituent indices except income are very close to each other. If this is, in fact, the case, it will provide further justification to search for alternative constituent indices or aggregation of indexes of different dimensions.

We proxy the vector of social goods with infant survival rate, life expectancy at birth (total years), primary school enrolment rate (per cent gross) and secondary school enrolment rate (per cent gross). The environmental indicator is chosen as per capita carbon dioxide emission. Our proxy for private goods is real gross domestic product per labour. The resource constraint is represented with an aggregate input, capital stock per labour. The source for the variables representing social goods, and the environmental indicator is the World Bank Social Indicators Database. Other variables, real gross domestic product, capital stock and employment are retrieved from the Penn World Tables which limits extending our data to 1990 only, since revised capital stock estimates are not yet available for later years.

In this particular application, a hypothetical 'average country' (in all variables for every year) is chosen as a reference country. Thus, it is assumed that $j = 0$, which then refers to the associated quantities for the 'average country'.

To provide a means of comparison among indexes with different component indices, a step-by-step approach is followed for the year 1977. First, by holding all variables other than income constant at 'average country' levels, an income index is generated for the year 1977. This index (with associated ranks) is shown in column 1 of Table 10.2. Note that income is almost equally dispersed around the average country with the highest income (USA) and the lowest income (Greece) being approximately 38 per cent above and below average, respectively. Then, the same exercise is repeated for the vector of social goods only, holding income, undesired output and resources at the average country levels. A close examination of column 2 reveals that, with respect to the provision of social goods, high income countries are very similar to each other, the difference between the best and the worst provision being only 5 per cent. Column 3 is reserved for the traditional HDI, a composite index of social goods and income, but with the weights being determined optimally by the data. The comparison of columns 1 and 3 confirms our prior expectation that, with similar provision of social goods, HDI becomes almost identical to per capita labour income and provides justification for Ivanova, Arcelus and Srinivasan's assessment of the measurement properties

Table 10.1 Alternative indexes of human well-being in OECD countries

	$Q(x^0, y^i, y^j, s^0, s^0, b^0, b^0)$		$Q(x^0, y^0, y^0, s^i, s^j, b^0, b^0)$		$Q(x^0, y^i, y^j, s^i, s^j, b^0, b^0)$		$Q(x^0, y^i, y^j, s^i, s^j, b^i, b^j)$	
Australia	1.1303	(7)	1.023	(8)	1.1303	(7)	1.0137	(19)
Austria	0.9702	(13)	0.9934	(22)	0.9702	(13)	1.1195	(11)
Belgium	1.1315	(6)	0.9967	(18)	1.1315	(6)	1.03	(15)
Canada	1.2309	(3)	1.0111	(13)	1.2309	(3)	1.025	(16)
Denmark	0.935	(14)	1.005	(15)	0.9432	(16)	0.95	(21)
Finland	0.831	(19)	1.0379	(2)	0.9574	(14)	1.0235	(17)
France	1.1069	(8)	1.0418	(1)	1.1069	(8)	1.1111	(12)
Greece	0.6278	(22)	0.9951	(20)	0.9069	(21)	1.3146	(1)
Iceland	0.8707	(17)	1.0284	(6)	0.9372	(18)	1.0605	(13)
Ireland	0.7303	(20)	1.0268	(7)	0.936	(19)	1.2294	(7)
Israel	0.8362	(18)	0.9944	(21)	0.9063	(22)	1.2548	(4)
Italy	1.0159	(11)	0.9953	(19)	1.0159	(12)	1.2422	(5)
Japan	0.6286	(21)	1.0379	(2)	0.9459	(15)	1.1253	(10)
Luxembourg	1.1729	(5)	0.9998	(16)	1.1729	(5)	0.891	(22)
Netherlands	1.2532	(2)	1.0341	(4)	1.2532	(2)	1.144	(9)
New Zealand	1.0627	(9)	1.0136	(10)	1.0627	(9)	1.2352	(6)
Norway	1.0142	(12)	1.0308	(5)	1.0229	(11)	1.1684	(8)
Spain	0.933	(15)	1.023	(8)	0.9432	(16)	1.3146	(1)
Sweden	1.0407	(10)	1.0117	(11)	1.0407	(10)	1.0139	(18)
Switzerland	1.1911	(4)	1.0116	(12)	1.1911	(4)	1.3142	(3)
UK	0.8994	(16)	0.9968	(17)	0.9268	(20)	0.9846	(20)
USA	1.3877	(1)	1.0081	(14)	1.3877	(1)	1.055	(14)
Average	1		1		1		1	

Table 10.2 Quantity indexes

	1977	1980	1982	1987	1990	IMP 1977–90
Australia	77.11 (19)	78.45 (16)	74.28 (18)	75.40 (18)	77.75 (22)	1.01
Austria	85.16 (11)	92.29 (7)	89.83 (11)	84.20 (11)	87.12 (10)	1.12
Belgium	78.35 (15)	80.62 (14)	81.22 (15)	81.60 (13)	88.21 (8)	1.19
Canada	77.97 (16)	76.09 (19)	74.23 (19)	79.20 (14)	85.51 (12)	1.13
Denmark	72.27 (21)	79.19 (15)	78.30 (16)	72.74 (20)	79.63 (20)	1.14
Finland	77.86 (17)	77.98 (17)	82.97 (14)	73.96 (19)	82.73 (16)	1.10
France	84.52 (12)	87.33 (9)	92.72 (8)	94.71 (2)	100.00 (1)	1.25
Greece	100.00 (1)	99.53 (3)	99.06 (4)	86.98 (6)	88.45 (7)	0.92
Iceland	80.67 (13)	84.16 (13)	91.12 (9)	85.16 (10)	84.96 (13)	Infeasible
Ireland	93.52 (7)	87.32 (10)	87.49 (12)	77.33 (15)	82.00 (18)	1.01
Israel	95.45 (4)	98.95 (4)	94.92 (6)	85.81 (9)	86.59 (11)	0.95
Italy	94.49 (5)	100.00 (1)	99.18 (3)	94.44 (3)	95.12 (3)	1.12
Japan	85.60 (10)	86.33 (11)	86.42 (13)	82.29 (12)	82.30 (17)	0.99
Luxembourg	67.78 (22)	66.79 (22)	67.39 (21)	71.00 (21)	84.26 (14)	1.24
Netherlands	87.02 (9)	87.41 (8)	94.50 (7)	86.17 (8)	88.07 (9)	1.11
New Zealand	93.96 (6)	95.93 (6)	100.00 (1)	86.27 (7)	89.64 (6)	1.01
Norway	88.88 (8)	67.60 (21)	65.84 (22)	67.44 (22)	81.04 (19)	0.89
Spain	100.00 (1)	100.00 (1)	100.00 (1)	100.00 (1)	100.00 (1)	1.28
Sweden	77.13 (18)	85.89 (12)	90.64 (10)	90.16 (5)	92.38 (5)	1.28
Switzerland	99.97 (3)	98.70 (5)	95.61 (5)	91.04 (4)	94.43 (4)	0.99
UK	74.90 (20)	75.73 (20)	77.16 (17)	75.74 (16)	78.24 (21)	Infeasible
USA	80.25 (14)	76.39 (18)	74.00 (20)	75.67 (17)	83.64 (15)	1.14
Geometric mean	84.62	84.97	85.55	82.19	86.70	1.09

of the HDI. Note that while the HDI index value for half of the countries is the same as GDP per labour, considerable differences still exist between the two indexes for Finland, Greece, Ireland, Iceland and Japan. Armed with sufficient justification for an alternative index, in the last column in Table 10.1, the quantity index in (10.9) – which shows the success of country *i* in equiproportionate expansion and contraction of good (social and private) and bad outputs relative to a reference country *j* – is listed. Note that while this index rewards relatively low polluters such as Greece, Spain, Italy, New Zealand and Ireland, it severely punishes high polluters such as Luxembourg, USA, Sweden, Denmark and Canada.

Since this index is transitive, it allows for bilateral comparisons among all country pairs. To facilitate an easier exposition, the bads-incorporating quantity index (10.9) is normalized for each year by the value of the best performer, so as to assign a value of 100 for the best achiever. These are given in Table 10.2, where although the ranking of individual countries differs from year to year, Switzerland, Spain, and Italy have always maintained their position within the best five performers. As for the worst performers, our bads-incorporating quantity index consistently places UK, Luxembourg,

Table 10.3 Improvement in human well-being over sub-periods

	1977–80	1980–82	1982–87	1987–90	1977–90
Australia	1.00	0.97	1.05	0.99	1.01
Austria	1.09	0.98	1.03	1.02	1.12
Belgium	1.01	1.02	1.08	1.07	1.19
Canada	1.00	1.01	1.10	1.02	1.13
Denmark	1.08	1.02	1.00	1.03	1.14
Finland	0.99	1.11	0.95	1.05	1.10
France	1.01	1.07	1.11	1.04	1.25
Greece	0.99	1.00	0.96	0.97	0.92
Iceland		1.09		1.25	Infeasible
Ireland	0.98	1.03	0.99	1.01	1.01
Israel	1.01	0.96	1.01	0.97	0.95
Italy	1.03	1.02	1.04	1.02	1.12
Japan	1.01	1.02	1.02	0.94	0.99
Luxembourg	1.03	1.04	1.07	1.08	1.24
Netherlands	1.00	1.10	0.99	1.02	1.11
New Zealand	0.99	1.08	0.94	1.01	1.01
Norway	0.72	1.00	1.03	1.20	0.89
Spain	1.04	1.04	1.15	1.04	1.28
Sweden	1.09	1.07	1.08	1.01	1.28
Switzerland	0.96	0.97	1.02	1.03	0.99
UK	1.01	1.01			Infeasible
USA	1.00	1.01	1.04	1.08	1.14
Geometric mean	0.98	1.03	1.02	1.04	1.09

Norway, Australia and Denmark among the last seven. Although the quantity index in (10.9) is not designed to measure performance over time, examination of the ranks pertaining to each year reveals the spectacular performance of France and Sweden, which raised their ranks from 12th to 1st position and from 18th to 5th, respectively. Greece, Israel, Norway, Ireland and Japan, on the other hand, seem to have lost their comparative advantage in the provision of a healthy standard of living. The last column in Table 10.2 is reserved for the overall improvement in the human well-being index computed using (10.10). A breakdown of human improvement index for subperiods is also provided in Table 10.3.

An analysis of the overall improvement rate combined with observations on a year-to-year variation of the relative rankings of countries reveals fairly consistent results. For example, France and Sweden owe their quite spectacular climb with regard to their rank among the 22 countries to their rather high improvement rates (25 per cent and 28 per cent, respectively). Spain, on the other hand, maintains its top position among the countries because of its high overall improvement rate, an achievement shared with Sweden.

We also observe that countries showing deterioration in performance (like Greece, Japan, Switzerland and Norway) have also experienced a fall in their relative ranking. It is also useful note that a comparison of quantity indexes across time will not reveal much about overall improvement. With such a comparison, one would incorrectly conclude, for example, that Spain shows no improvement, whereas with its highest growth performance, it is, in fact, the country that boosts the distribution of human well-being index over time.[4]

Conclusions

This chapter, relying on an economic-theoretical approach to index numbers, proposes a framework for incorporating environmental indicators to the measurement of human well-being. Furthermore, this study also introduces an improvement index which alleviates the well-known deficiency of across-time comparison of deprivation indexes. The benefit of the proposed index is that it does not require normative judgements in the selection of weights to aggregate over constituent indices. Instead, optimally chosen weights, within an activity analysis framework, are determined by the data. In developing the index which incorporates environmental indicators, due emphasis is put on production with negative externalities and directional distance functions – a very recent analytical device – are employed as a major tool to construct quantity indexes and improvement indexes. The improvement index is well grounded in the theory of productivity growth. The chapter also provides a numerical example of computations over 22 high income OECD countries to show that the indexes proposed are capable of differentiating variations in human well-being where more traditional indexes fail.

Notes

1. See Ivanova *et al.* (1999), Anand and Ravallion (1993) and McGillivray (1991).
2. Shephard (1970) introduced the notion of weak disposability of outputs.
3. Shephard and Färe (1974) introduced this property.
4. See Zaim *et al.* (2001) for details on the superiority of the improvement index in this study versus across-time comparison of deprivation indexes.

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11

Measuring Non-economic Well-being Achievement

Mark McGillivray

Introduction

It is common to treat human well-being as a multidimensional concept, enveloping diverse, separable or behaviourally distinct components, domains or dimensions (Finnis 1980; Nussbaum 1988; Sen 1990, 1993; UNDP 1990–2003; Doyal and Gough 1993; Galtung 1994; Cummins 1996; Qizilbash 1996; Stewart 1996; Narayan 2000; Alkire 2002, among many other studies).[1] It is in particular thought to be a much richer or vital concept than economic well-being: much of the literature is justifiably emphatic about this point. Accordingly, there is a long history of efforts both to refocus attention away from the established, although invariably far less than perfect, monetary measures of national economic well-being achievement and to capture better non-economic well-being achievement. A plethora of indicators of national well-being achievement has been proposed for these purposes. Indicators of health and educational status are most widely-used in inter-country ordinal and cardinal assessments of national well-being achievement, and are now available for diverse samples of 160 or more countries (see UNDP 2003). Multidimensional indicators are also available for similar samples, based either solely or predominantly on these indicators, and include the Physical Quality of Life Index (PQLI) and the very well-known Human Development Index (HDI).

As valid as their conceptual justifications might be, these standard indicators are often highly correlated, both ordinally and cardinally, among countries with income per capita, the most accepted measure of economic well-being achievement (Hicks and Streeten 1979; Larson and Wilford 1979; McGillivray 1991; McGillivray and White 1993; Srinivasan 1994; Noorbakhsh 1998; Cahill 2005). This is especially the case for large, diverse samples of countries, much to the frustration or disappointment of some proponents of these indicators.[2] Inter-country variation in non- or non-exclusively economic well-being achievement, measured using these standard measures is, therefore, well-predicted by variation in economic well-being. An implication of this relationship is that the standard non-economic or non-exclusively economic measures might not capture the richness or vitality of the well-being concept, giving an incomplete picture of it, or at least the part of it that they are intended to capture. The contribution of the standard non-economic measures has been questioned on these grounds, with some commentators going so far as to claim they are empirically redundant *vis-à-vis* income per capita.[3]

Yet a simple and instructive point has been given insufficient attention in the literature. While there is a high correlation between income per capita and the standard non- or non-exclusively economic indicators in large and diverse samples of countries, some countries perform better in the latter than predicted by the former and some countries perform worse. What would seem, therefore, to be more interesting and informative than correlations between indicators, is that variation in measures of standard non- or non-exclusively economic well-being achievement not accounted for by income per capita. Amartya Sen, in various publications, and the UNDP, in its *Human Development Reports*, address this variation, but stop short of providing a formal analysis of it.[4] A formal measure of this well-being achievement, on which international comparisons might be based, would thus appear to be warranted. Among the insights provided by such a measure is the systematic identification of those countries that have better non-economic well-being achievement than their economic achievement predicts. This information is important if we accept that there is more to well-being achievement than what has been achieved in its economic sphere. It also allows us to begin to ask why some countries do better in this regard than others.

This chapter commences by extracting, using principal components analysis, the maximum possible information from various standard national non-economic well-being achievement measures. It then empirically identifies the variation in this extraction not accounted for by variation in income per capita, in the form of a variable called μ_i. This variable is the residual yielded by a cross-country regression of the extraction on the logarithm of PPP GDP per capita. μ_i is interpreted *inter alia* as a measure of non-economic human well-being achievement *per se*, in the sense that it captures well-being achieved independently of income. Given that μ_i is purely a statistical construct, obtained econometrically, the chapter then looks at correlations between this measure and variants of it and other well-being or well-being related indicators, in an attempt to find the variable or group of variables that best captures non-economic well-being achievement. It should be emphasized that this is a pure measurement exercise, in that inferences regarding causality are not drawn explicitly. It is of potential practical benefit, however, as it provides a case for allocating more resources to the collection and reporting of the variables, especially if the variable or variables are available or reported for relatively small samples of countries. Alternatively, it provides a case for more use of the variables in well-being assessments if they are available for reasonably large samples of countries. Among the measures not as widely reported or available across countries or not as widely used as those mentioned above, two variables perform best in this regard. One is a measure of gender empowerment and the other is a measure of educational attainment. It is though found that none of these measures perform consistently better than a very widely used one, that measure being adult literacy.

Non-economic well-being achievement

Let us commence with the following composite, 'standard' index of non-economic well-being for country i:

$$W_i = \sum_{k=1}^{m} \Phi_k x_{k,i}^t \quad i = 1, \ldots, n \tag{11.1}$$

where $x_{k,i}^t$ are appropriately transformed values of the well-being indicators $x_{k,i}$ and the Φ_k are weights. The $x_{k,i}$ are 'standard' non-economic well-being indicators. Characterized above, these indicators are those commonly used and reported, available for a large number of countries and typically highly correlated with income per capita. W_i captures that maximum obtainable information from the $x_{i,k}$ subject to an appropriate condition. This is achieved by choosing the Φ_k that maximize the variance of W_i subject to a normalization condition. Φ_ks are therefore obtained by principal components analysis, with W_i being the first principal component extracted from the $x_{k,i}^t$ and Φ_k being an $(m \times 1)$ eigenvector. The corresponding eigenvalue is λ_k and the normalization condition is that Φ_k^2 equals λ_k.[5]

W_i as a standard non-economic measure will be highly correlated with income per capita. Our task is to extract from it that information not predicted by economic well-being, as captured by some measure of income per capita. The following regression equation is therefore estimated:

$$W_i = \alpha + \beta \ln y_i + \mu_i \tag{11.2}$$

where $\ln y_i$ is the logarithm of income per capita. The logarithm is used to reflect diminishing returns to the conversion of income into economic well-being. The use of logarithmic values is consistent with the well-known Atkinson formula for the utility or well-being derived from income. This formula is written as follows:

$$W(y_i) = \frac{1}{1-\varepsilon} y_i^{1-\varepsilon} \tag{11.3}$$

where $W(y_i)$ is the utility or well-being derived from income and ε measures the extent of diminishing returns. As ε approaches one $W(Y_i)$ becomes the logarithm of y_i.[6]

The error term from (11.2), μ_i, is central to my analysis. It is by definition orthogonal with respect to $\ln y_i$, and as such is not subject to the criticism that it reveals disappointingly little additional information in inter-country well-being than income per capita. More pointedly, it is interpreted as a measure of non-economic or income-independent human well-being achievement. It is also interpreted, possibly contentiously, as a measure both of the success in converting economic well-being into non-economic well-being and of the non-economic well-being component, dimension or domain within the space of W_i.

Estimating μ_i: data and results

The chosen components of index W_i prior to transformations are years of life expectancy ($x_{1,i}$), the adult literacy rate ($x_{2,i}$) and the gross school enrolments ratio ($x_{3,i}$). The measure of income is PPP GDP per capita. Data on these variables are taken from the UNDP's *Human Development Report 2002* (UNDP 2002). These variables are the components of the HDI. W_i shares some similarities with the HDI, therefore.[7] They are available for a sample of 173 countries and are very widely used. Moreover, as Tables 11.1 and 11.2 show, they are quite highly correlated among each other, with PPP GDP per capita and the HDI as a whole. The Pearson (zero-order) coefficients between these variables and the logarithm of PPP GDP per capita in Table 11.1 range from 0.701 to 0.794 and the corresponding Spearman (rank-order) coefficients in Table 11.2 range from 0.695 to 0.840.

Results of the principal components analysis, which is based on the transformed components, $x^t_{k,i}$, are shown in Table 11.3.[8] W_i, the first principal

Table 11.1 Zero-order (Pearson) correlation coefficients between commonly-used well-being indicators ($n = 173$)

		Life expectancy ($x_{1,i}$)	**Adult literacy ($x_{2,i}$)**	**Gross enrolment ($x_{3,i}$)**	**HDI**	**PPP GDP per capita (log)($\ln y_i$)**
Life expectancy	($x_{1,i}$)	1.000				
Adult literacy	($x_{2,i}$)	0.726	1.000			
Gross enrolment	($x_{3,i}$)	0.736	0.803	1.000		
HDI		0.925	0.870	0.881	1.000	
PPP GDP per capita (log)	($\ln y_i$)	0.794	0.701	0.792	0.923	1.000

Table 11.2 Rank-order (Spearman) correlation coefficients between commonly used well-being indicators ($n = 173$)

		Life expectancy ($x_{1,i}$)	**Adult literacy ($x_{2,i}$)**	**Gross enrolment ($x_{3,i}$)**	**HDI**	**PPP GDP per capita ($\ln y_i$)**
Life expectancy	($x_{1,i}$)	1.000				
Adult literacy	($x_{2,i}$)	0.724	1.000			
Gross enrolment	($x_{3,i}$)	0.715	0.773	1.000		
HDI		0.938	0.841	0.833	1.000	
PPP GDP per capita (log)	($\ln y_i$)	0.840	0.695	0.780	0.938	1.000

Table 11.3 Principal components analysis results

		Principal components First ($PC_{1,i} = W_i$)	**Second ($PC_{2,i}$)**	**Third ($PC_{3,i}$)**
Eigenvalue		2.510	0.293	0.197
Cumulative percentage of eigenvalues		83.654	93.424	100.000
Component weight (Φ_k):	Life expectancy ($x_{1,i}$)	0.565	−0.824	−0.051
	Adult literacy ($x_{2,i}$)	0.582	0.441	−0.683
	Gross enrolment ($x_{3,i}$)	0.585	0.356	0.729

component performs very well in extracting information from the three component variables, capturing 84 per cent of the eigenvalues. The component variable weights Φ_k are very similar, varying from 0.565 to 0.585. Correlation coefficients between W_i, and its component variables, shown in Table 11.4, are all very high, ranging from 0.895 to 0.927 and 0.894 to 0.908 for the zero- and rank-order coefficients, respectively. Each of the preceding results are consistent with the rather high correlations between the three component variables reported above. W_i is also very highly correlated with the HDI and, pertinently, with $\ln y_i$. The zero-order and rank-order coefficients between W_i and the HDI are 0.976 and 0.956, respectively. The corresponding coefficients between W_i and $\ln y_i$ are 0.833 and 0.838, respectively. A scatter plot of W_i and PPP GDP per capita are shown in Figure 11.1.

Regressing W_i, on $\ln y_i$ yielded the following equation:

$$\hat{W}_i = -0.755 + 0.089 \ln y_i$$
$$(-19.50) \quad (19.67)$$

Table 11.4 Correlation coefficients between well-being indicators

		Well-being index ($W_i = PC_{1,i}$) Zero order	Rank order
Life expectancy	($x_{1,i}$)	0.895	0.894
Adult literacy	($x_{2,i}$)	0.923	0.908
Gross enrolment	($x_{3,i}$)	0.927	0.905
HDI		0.976	0.956
PPP GDP per capita (log)	($\ln y_i$)	0.833	0.838

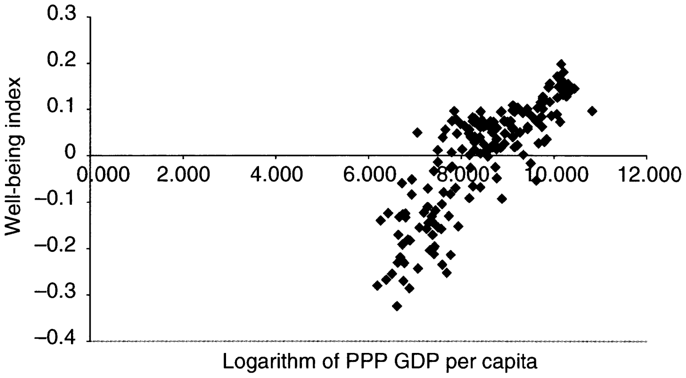

Figure 11.1 Scatter plot of well-being index and income per capita

The numbers in parentheses are *t* ratios. The R^2 and $\bar{R}^2$ are 0.694 and 0.692, respectively. Estimates of μ_i, are shown, along with values of W_i and all other variables mentioned above in Appendix Table 11A.1. Correlation coefficients between μ_i and the standard non- or non-exclusively economic indicators are shown in Table 11.5. Of the latter variables, that variable most highly correlated with μ_i is adult literacy. Those countries with the 15 highest and 15 lowest residual values are shown in Table 11.6. High residual values indicate that countries do better in terms of non-economic, or non-income predicted, well-being achievement. The group of countries that does best in terms of this well-being is dominated by those that either still have, or in their recent pasts have had, non-market, centrally-planned economies. Eleven of the top 15 and each of the top ten countries in terms of this well-being fall into this category. More generally, most of these 15 countries have moderately low incomes per capita and, albeit to a lesser extent, HDI values. These are characteristics of all but three of the 30 countries listed in

Table 11.5 Correlations between μ_i and well-being indicators

Variables	Zero order	Rank order	*n*
HDI	0.373*	0.242*	173
Life expectancy ($x_{1,i}$)	0.421*	0.262*	173
Adult literacy ($x_{2,i}$)	0.612*	0.513*	173
Gross enrolment ($x_{3,i}$)	0.482*	0.398*	173
Well-being index (W_i)	0.554*	0.438*	173

Note: *Significantly different from zero at the 90 per cent confidence level or greater.

Table 11.6. Twelve of the 15 bottom ranked countries are countries located in sub-Saharan Africa. Most are ranked very low in terms of each of W_i, the HDI and PPP GDP per capita. There are some stark exceptions to this, however. The bottom 15 groups includes Luxembourg, Oman and Equatorial Guinea, which are ranked among the top 25 per cent of the 173 country sample in terms of income per capita. Luxembourg has by far the highest PPP GDP per capita of this sample, but its ranking in terms of μ is 163, the 11th lowest in the sample. The bottom 15 countries also include Botswana, a middle ranked country in terms of income per capita. Botswana is ranked low in the first two, but not third, of these variables.

Correlates with μ_i: data and results

μ_i is a purely statistical construct. Policy-makers might be reluctant to, for example, monitor a residual obtained from a linear regression of a principal component on the logarithm of income per capita. A key question, therefore, concerns that variable which best individually accounts for the variation in μ_i across countries. From Table 11.6 one might conclude that countries that do well in terms of μ_i might be those that have devoted larger shares of state funds to social sectors given the appearance of so many former or current centrally-planned economies in the top 15 group. One might also conclude that the countries that do poorly in terms of μ_i might be those that have high rates of HIV/AIDS infection due to the fact that sub-Saharan African countries dominate the bottom 15 group. But the interest here is the single variable which alone accounts for most cross-country variation in μ_i. Of particular interest is whether less widely available, reported or used well-being or well-being related indicators perform better than the standard indicators, the $x^t_{k,i}$ and the HDI.[9] If so, then this would appear to be an *a priori* case for the relevant bodies to further develop and report these indicators, including expanding their country coverage. It could also provide a

Table 11.6 Well-being data, selected countries

	PPP GDP per capita					Well-being index		Residual	
Country	Value (y_i)	Value ($\ln y_i$)	Rank	HDI Value	Rank	Value (W_i)	Rank	Value (μ_i)	Rank
Tajikistan	1152	7.049	151	0.667	112	0.050	81	0.177	1
Armenia	2559	7.847	117	0.754	77	0.096	33	0.152	2
Uzbekistan	2441	7.800	119	0.727	95	0.075	50	0.135	3
Georgia	2664	7.888	115	0.748	81	0.079	46	0.131	4
Moldova, Rep.	2109	7.654	126	0.701	105	0.056	78	0.130	5
Viet Nam	1996	7.599	128	0.688	109	0.040	89	0.118	6
Azerbaijan	2936	7.985	112	0.741	89	0.069	61	0.113	7
Suriname	3799	8.242	103	0.756	74	0.083	44	0.103	8
Cuba	4519	8.416	90	0.795	55	0.095	35	0.101	9
Mongolia	1783	7.486	134	0.655	113	0.012	106	0.100	10
Ecuador	3203	8.072	110	0.732	93	0.064	64	0.100	11
Kyrgyzstan	2711	7.905	114	0.712	102	0.048	84	0.099	12
Congo	825	6.715	163	0.512	136	−0.059	123	0.098	13
Philippines	3971	8.287	97	0.754	76	0.081	45	0.097	14
Ukraine	3816	8.247	102	0.748	80	0.074	52	0.095	15
Mauritania	1677	7.425	136	0.438	152	−0.196	157	−0.102	159
Côte d'Ivoire	1630	7.396	139	0.428	156	−0.200	158	−0.104	160
Vanuatu	2802	7.938	113	0.542	131	−0.152	147	−0.104	161
Oman	13356	9.500	40	0.751	78	−0.016	114	−0.108	162
Luxembourg	50061	10.821	1	0.925	16	0.097	32	−0.112	163
Mozambique	854	6.750	160	0.322	170	−0.270	170	−0.117	164
Gambia	1649	7.408	137	0.405	160	−0.213	160	−0.118	165
Central African Rep.	1172	7.066	150	0.375	165	−0.244	166	−0.118	166
Botswana	7184	8.880	64	0.572	126	−0.093	132	−0.129	167
Burkina Faso	976	6.883	155	0.325	169	−0.286	172	−0.144	168
Djibouti	2377	7.774	121	0.445	149	−0.214	161	−0.151	169
Equatorial Guinea	15073	9.621	38	0.679	111	−0.053	122	−0.155	170
Guinea	1982	7.592	129	0.414	159	−0.235	165	−0.157	171
Niger	746	6.615	168	0.277	172	−0.324	173	−0.158	172
Angola	2187	7.690	125	0.403	161	−0.253	167	−0.183	173

case for greater use of the available data on them in reporting and analyzing well-being achievement. The following simple hypotheses were therefore evaluated:

$$H_0 : |\varrho_{ns,j}| \leq |\varrho_s^{max}|$$

$$H_1 : |\varrho_{ns,j}| > |\varrho_s^{max}|$$

where $\varrho_{ns,j}$ is the correlation coefficient between μ_i and the jth less widely available, reported or used indicator and ϱ_s^{max} is the highest correlation coefficient between μ_i and the non-economic standard indicators, respectively, for the sample of countries under consideration. I shall for convenience label the former as non-standard indicators.[10] The null hypothesis is that the non-standard indicator under consideration accounts for no more of the variation in μ_i than the standard one that does best in this regard. The alternative hypothesis is that the former does better than the latter in capturing this variation empirically. Both zero-order (Pearson) and rank-order (Spearman) coefficients are reported. All coefficients are also subjected to the standard hypothesis test, that being whether they are significantly different from zero.[11]

Two issues need to be addressed prior to conducting the hypothesis tests. The first is measurement error. While few if any well-being indicators considered thus far are free of measurement error, arguably those subject to greatest error are the standard non-economic indicators, as defined. This is of relevance to the above hypothesis tests given its implications for W_i, as can now be demonstrated. Let the true, unobservable and measurement error free variable be W_i^*. Its relationship with W_i is

$$W_i = W_i^* + \mu_i^* \tag{11.4}$$

where μ_i^* is the error in measuring W_i^*. It follows from (11.4) that μ_i is a composite variable, defined as

$$\mu_i = \nu_i + \mu_i^* \tag{11.5}$$

where ν_i is the true measure of non-economic well-being achievement, as defined above.

Given (11.1), μ_i^* is defined as

$$\mu_i^* = \sum_{k=1}^{m} \Phi_k \mu_{k,i}^{t,*} \tag{11.6}$$

where $\mu_{k,i}^{t,*}$ are the errors in measuring $x_{k,i}^{t,*}$. μ_i^* is thus a composite error term, with the same general structure as the well-being indicator W_i. It follows from (11.1), (11.5) and (11.6) that regressing μ_i on $x_{1,i}^t$, $x_{2,i}^t$ or $x_{3,i}^t$, $x_{1,i}^t$, $x_{2,i}^t$ or $x_{3,i}^t$ is the equivalent of regressing $(\nu_i + \mu_i^*)$ on $(x_{1,i}^{t,*} + \Phi_1\mu_{1,i}^{t,*})$, $(x_{2,i}^{t,*} + \Phi_2\mu_{2,i}^{t,*})$ or $(x_{3,i}^{t,*} + \Phi_3\mu_{3,i}^{t,*})$, respectively. A regression of μ_i on the HDI also involves regressing of μ_i^* on itself given that the HDI shares variables with W_i. The resulting correlation coefficients will therefore be distorted upwards, in absolute terms, in the sense that each regression involves regressing μ_i^* on itself or on one of its components. This in turn means that ϱ_s^{max} will be distorted upwards, therefore, possibly leading to the erroneous rejection of the null hypothesis outlined above.[12]

Addressing this issue is less than straightforward as we are required to speculate as to likely values of μ_i^* to obtain v_i. v_i can then be regressed on $x_{1,i}^t$, $x_{2,i}^t$, $x_{3,i}^t$ and the HDI to obtain a less distorted ϱ_s^{max}. The issue was addressed as follows. Given (11.4) and (11.5), we can after some algebraic manipulation write the following equation:

$$W_i = \alpha + \beta \ln y_i + \gamma_q \pi_{q,i} + v_{q,i} \tag{11.7}$$

where $\gamma_q \pi_{q,i}$ are alternative estimates of μ_i^*. $\pi_{q,i}$ is one of q variables and γ_q are the corresponding parameters. A number of different formulations of $\pi_{q,i}$ and values of γ_q were considered. Three formulations and values were, in the final analysis, adopted. These formulations are, of course, necessarily no more than informed guesses as to the likely values of μ_i^*. No attempt was made to guestimate the $\mu_{k,i}^{t,*}$, and as such each of the $x_{k,i}^t$ are assumed to be approximately equally erroneously measured.

It is reasonable to assume that error in measuring W_i will be subject to a random process but also be a decreasing function of the resources a country allocates to the collection and reporting of aggregate well-being data and the effectiveness with which these resources have been allocated. Moreover, it is also reasonable to posit that both of the second of these factors will be an increasing function of the income per capita. The formulations of $\pi_{q,i}$ are based on these assumptions. The first, $\pi_{1,i}$, was defined as a standard random variable with a mean of zero and variance of 1, expressed as a ratio of the reciprocal of $\ln y_i$. For a given random value, therefore, $\pi_{1,i}$ will be smaller the larger is a country's income per capita and vice versa. In estimating (11.7) with $\pi_{1,i}$, the value of γ_1 was unrestricted, being determined purely by the data. This is appropriate as the resulting estimate of μ_i^* will be scaled in proportion to W_i. $\pi_{2,i}$ was defined as a random normal variable but with a mean, standard deviation and variance differing according to country group. For low- and middle-income countries, the standard deviation was four and two times that of the high income countries, respectively. γ_2 was determined by the data to ensure that the corresponding estimate of μ_i^* is in proportion to W_i. Finally, $\pi_{3,i}$ was defined as a uniform random number, but with its range being set according to some fraction of W_i. This fraction was set at 0.025, 0.05 and 0.20 for high, middle and low income countries, respectively. γ_3 was restricted to 1 in estimating (11.7) with $\pi_{3,i}$.

The second issue also relates to ϱ_s^{max} and the possible erroneous rejection of the null hypothesis outlined above. It is obvious from (11.1) and (11.2) that

$$\mu_i = \sum_{k=1}^{m} \Phi_k x_{k,i}^t - (\alpha + \beta \ln y_i) \tag{11.8}$$

It follows from (11.8) that regressing μ_i on $x^t_{1,i}$, $x^t_{2,i}$ or $x^t_{3,i}$ to obtain ϱ^{max}_s is the equivalent of regressing μ_i partly on itself. This also applies to regressing μ_i on the HDI. As is the case with measurement error, this in turn means that ϱ^{max}_s will be pushed upwards, purely by construction. It might hardly be surprising, therefore, if the null is rarely rejected. This issue was addressed by first subtracting each $\Phi_k x^t_{k,i}$ from W_i prior to regressing the latter on $\ln y_i$ and $\gamma_q \pi_{q,i}$ to obtain adjusted estimates of $\nu_{q,i}$, denoted as $\nu'_{q,k,i}$.[13] The residuals obtained from these processes were then regressed separately on $x^t_{k,i}$ to obtain adjusted correlation coefficients, from which ϱ^{max}_s is ultimately selected.[14]

The non-standard variables were taken from or constructed using data in the *Human Development Report 2002* (UNDP 2002) and the *World Happiness Database* (Veenhoven 2002a, 2002b). The variables are categorized as follows: Human Poverty, Health Services Provision, Health Status, Survival, Education Status, Gender Bias, Gender Empowerment, Income Inequality, Governance and Happiness. There is, of course, overlap between these categories. The governance indicators are subjective and relate to well-being derived from civil liberties, political rights, non-violence and the like. The happiness variables are intended to measure subjective, self-assessed well-being. A full list of variables and their definitions is provided in Appendix Table 11A.2.

Results are reported in Table 11.7.[15] Fifty-six zero- and rank-order coefficients between the non-standard indicators and μ_i are reported (see the second and seventh columns of Table 11.7, headed $\varrho_{ns,j}$). Thirty-five of the former and 30 of the latter are significantly different from zero. Those with the highest correlations with μ_i, are the contraceptive prevalence, youth literacy, and women professionals and technicians variables. The zero-order coefficients between these variables and μ_i are 0.535, 0.581 and 0.569, respectively. The corresponding rank-order coefficients are 0.538, 0.559 and 0.374. Only two of the variables in the income inequality, governance and happiness groups – life enjoyment and happy life years – are significantly correlated with μ_i.[16]

Evaluation of the hypotheses relating to whether the non-standard indicators perform better than their standard counterparts in accounting for the variation in estimates of μ_i and its variants, $\nu'_{q,k,i}$, produced interesting results. The above-outlined null hypothesis, that $|\varrho_{ns,j}| \leq |\varrho^{max}_s|$, cannot be rejected in favour of the alternative in almost all cases if former coefficients are obtained using estimates of μ_i. As is shown in Table 11.7, the estimates of ϱ^{max}_s obtained using μ_i are larger in absolute value than the corresponding $\varrho_{ns,j}$ in all samples. These estimates are shown in the third and eighth columns of Table 11.7, headed μ_i. Moreover, in almost all cases the standard variable that was most correlated with μ_i was adult literacy ($x^t_{2,i}$) (see the fourth and ninth columns of Table 11.7).

Table 11.7 Correlation coefficients between estimates of μ_i and well-being indicators

	Zero order ϱ_s^{max}					Rank order ϱ_s^{max}					
Variables	$\varrho_{ns,j}$	μ_i	**Variable**	$v'_{q,k,i}$	**Variable**	$\varrho_{ns,j}$	μ_i	**Variable**	$v'_{q,k,i}$	**Variable**	n
Human poverty											
Human poverty index	−0.483*	0.629*	$x^t_{2,i}$	0.374*	$x^t_{3,i}$	−0.470*	0.627*	$x^t_{3,i}$	0.381*	$x^t_{3,i}$	87
Survival to 40	−0.428*	0.615*	$x^t_{2,i}$	0.390*	$x^t_{2,i}$	−0.342*	0.595*	$x^t_{2,i}$	0.410*	$x^t_{2,i}$	116
Water usage	−0.182	0.636*	$x^t_{2,i}$	0.393*	$x^t_{2,i}$	−0.221*	0.623*	$x^t_{2,i}$	0.411*	$x^t_{2,i}$	108
Poverty headcount ($1)	−0.278*	0.586*	$x^t_{2,i}$	0.275*	$x^t_{2,i}$	−0.215	0.546*	$x^t_{2,i}$	0.336*	$x^t_{2,i}$	60
Poverty headcount ($2)	−0.200	0.588*	$x^t_{2,i}$	0.276*	$x^t_{2,i}$	−0.196	0.546*	$x^t_{2,i}$	0.336*	$x^t_{2,i}$	60
Health services											
Sanitation facilities	0.199*	0.615*	$x^t_{2,i}$	0.357*	$x^t_{2,i}$	0.139	0.512*	$x^t_{2,i}$	0.328*	$x^t_{2,i}$	123
Drug access	−0.042	0.610*	$x^t_{2,i}$	0.433*	$x^t_{2,i}$	−0.094	0.510*	$x^t_{2,i}$	0.380*	$x^t_{2,i}$	170
Water services	0.185*	0.572*	$x^t_{2,i}$	0.352*	$x^t_{2,i}$	0.076	0.497*	$x^t_{2,i}$	0.320*	$x^t_{2,i}$	165
Measles immunization	0.456*	0.609*	$x^t_{2,i}$	0.459*	$x^t_{2,i}$	0.416*	0.593*	$x^t_{2,i}$	0.439*	$x^t_{2,i}$	165
Tuberculosis immunization	0.394*	0.636*	$x^t_{2,i}$	0.431*	$x^t_{2,i}$	0.398*	0.514*	$x^t_{2,i}$	0.385*	$x^t_{2,i}$	140
Oral rehydration	−0.205	0.769*	$x^t_{2,i}$	0.615*	$x^t_{2,i}$	−0.015	0.784*	$x^t_{2,i}$	0.666*	$x^t_{2,i}$	56
Contraceptive prevalence	0.535*	0.682*	$x^t_{2,i}$	0.483*	$x^t_{2,i}$	0.538*	0.629*	$x^t_{2,i}$	0.442*	$x^t_{2,i}$	91
Birth attendance	0.371*	0.651*	$x^t_{2,i}$	0.460*	$x^t_{2,i}$	0.327*	0.610*	$x^t_{2,i}$	0.452*	$x^t_{2,i}$	122
Physicians	0.389*	0.632*	$x^t_{2,i}$	0.445*	$x^t_{2,i}$	0.413*	0.516*	$x^t_{2,i}$	0.394*	$x^t_{2,i}$	165
Health status											
Undernourishment	−0.132	0.671*	$x^t_{2,i}$	0.486*	$x^t_{2,i}$	−0.120	0.678*	$x^t_{2,i}$	0.517*	$x^t_{2,i}$	101
Underweight children	−0.257*	0.662*	$x^t_{2,i}$	0.456*	$x^t_{2,i}$	−0.286*	0.634*	$x^t_{2,i}$	0.452*	$x^t_{2,i}$	124
Underheight children	−0.186*	0.667*	$x^t_{2,i}$	0.454*	$x^t_{2,i}$	−0.186*	0.639*	$x^t_{2,i}$	0.443*	$x^t_{2,i}$	118
Underweight infants	−0.281*	0.619*	$x^t_{2,i}$	0.448*	$x^t_{2,i}$	−0.286*	0.474*	$x^t_{2,i}$	0.381*	$x^t_{2,i}$	150
Adults with HIV/AIDS	−0.290*	0.587*	$x^t_{2,i}$	0.408*	$x^t_{2,i}$	−0.325*	0.485*	$x^t_{2,i}$	0.392*	$x^t_{2,i}$	144

Continued

Table 11.7 Continued

Variables	Zero order ϱ_s^{max}					Rank order ϱ_s^{max}					
	$\varrho_{ns,j}$	μ_i	Variable	$v'_{q,k,i}$	Variable	$\varrho_{ns,j}$	μ_i	Variable	$v'_{q,k,i}$	Variable	n
Women with HIV/AIDS	−0.213*	0.717*	$x_{2,i}^t$	0.505*	$x_{2,i}^t$	−0.197*	0.461*	$x_{2,i}^t$	0.348*	$x_{2,i}^t$	73
Malaria cases	−0.346*	0.697*	$x_{2,i}^t$	0.514*	$x_{2,i}^t$	−0.342*	0.723*	$x_{2,i}^t$	0.494*	$x_{2,i}^t$	84
Tuberculosis cases	−0.205*	0.617*	$x_{2,i}^t$	0.437*	$x_{2,i}^t$	−0.038	0.516*	$x_{2,i}^t$	0.384*	$x_{2,i}^t$	170
Cigarette consumption	0.132	0.569*	$x_{2,i}^t$	0.359*	$x_{2,i}^t$	0.143	0.358*	$x_{2,i}^t$	0.216*	$x_{2,i}^t$	110
Survival											
Infant mortality rate	−0.393*	0.612*	$x_{2,i}^t$	0.429*	$x_{2,i}^t$	−0.203*	0.509*	$x_{2,i}^t$	0.376*	$x_{2,i}^t$	172
Child mortality rate	−0.419*	0.513*	$x_{2,i}^t$	0.429*	$x_{2,i}^t$	−0.204*	0.509*	$x_{2,i}^t$	0.376*	$x_{2,i}^t$	172
Survival to 65 (females)	0.425*	0.613*	$x_{2,i}^t$	0.434*	$x_{2,i}^t$	0.273*	0.517*	$x_{2,i}^t$	0.391*	$x_{2,i}^t$	166
Survival to 65 (males)	0.347*	0.613*	$x_{2,i}^t$	0.434*	$x_{2,i}^t$	0.233*	0.517*	$x_{2,i}^t$	0.391*	$x_{2,i}^t$	166
Maternal mortality rate	−0.416*	0.640*	$x_{2,i}^t$	0.446*	$x_{2,i}^t$	−0.174*	0.571*	$x_{2,i}^t$	0.343*	$x_{2,i}^t$	144
Education status											
Youth literacy rate	0.581*#	0.630*	$x_{2,i}^t$	0.426*	$x_{2,i}^t$	0.559*#	0.611*	$x_{2,i}^t$	0.428*	$x_{2,i}^t$	128
Primary school enrolment	0.445*	0.548*	$x_{2,i}^t$	0.368*	$x_{2,i}^t$	0.349*	0.451*	$x_{2,i}^t$	0.345*	$x_{2,i}^t$	122
Secondary school enrolment	0.317*	0.550*	$x_{2,i}^t$	0.394*	$x_{2,i}^t$	0.186	0.369*	$x_{2,i}^t$	0.279*	$x_{2,i}^t$	95
Children grade 5	0.062	0.507*	$x_{2,i}^t$	0.397*	$x_{2,i}^t$	0.092	0.482*	$x_{2,i}^t$	0.289*	$x_{2,i}^t$	48
Gender bias											
Gender-related development index	0.357*	0.587*	$x_{2,i}^t$	0.389*	$x_{2,i}^t$	0.243*	0.495*	$x_{2,i}^t$	0.355*	$x_{2,i}^t$	146
Human development disparity	−0.390*	0.587*	$x_{2,i}^t$	0.389*	$x_{2,i}^t$	−0.436*	0.495*	$x_{2,i}^t$	0.355*	$x_{2,i}^t$	146
Life expectancy ratio	0.340*	0.613*	$x_{2,i}^t$	0.434*	$x_{2,i}^t$	0.380*	0.517*	$x_{2,i}^t$	0.390*	$x_{2,i}^t$	166
Adult literacy ratio	0.456*	0.583*	$x_{2,i}^t$	0.387*	$x_{2,i}^t$	0.358*	0.490*	$x_{2,i}^t$	0.351*	$x_{2,i}^t$	149
School enrolment ratio	0.460*	0.609*	$x_{2,i}^t$	0.435*	$x_{2,i}^t$	0.372*	0.509*	$x_{2,i}^t$	0.382*	$x_{2,i}^t$	162
Earned income ratio	0.130	0.659*	$x_{2,i}^t$	0.418*	$x_{2,i}^t$	0.115	0.662*	$x_{2,i}^t$	0.450*	$x_{2,i}^t$	90

Gender empowerment										
Gender empowerment measure	0.265*	0.629*	$x^t_{3,i}$	0.306*	$x^t_{2,i}$	0.127	0.478*	$x^t_{2,i}$	0.141	$x^t_{2,i}$	66
Women in parliament	0.113	0.594*	$x^t_{2,i}$	0.411*	$x^t_{2,i}$	0.127	0.458*	$x^t_{2,i}$	0.338*	$x^t_{2,i}$	170
Women in senior positions	0.457*	0.680*	$x^t_{3,i}$	0.409*	$x^t_{2,i}$	0.364*	0.483*	$x^t_{2,i}$	0.225*	$x^t_{2,i}$	77
Women professionals and technicians	0.569*#	0.680*	$x^t_{3,i}$	0.409*	$x^t_{2,i}$	0.374*	0.480*	$x^t_{2,i}$	0.218*	$x^t_{2,i}$	78
Income inequality											
Gini coefficient	−0.117	0.609*	$x^t_{2,i}$	0.420*	$x^t_{2,i}$	−0.048	0.404*	$x^t_{2,i}$	0.301*	$x^t_{2,i}$	116
Income share ratio (20 per cent)	−0.154	0.609*	$x^t_{2,i}$	0.420*	$x^t_{2,i}$	−0.040	0.404*	$x^t_{2,i}$	0.301*	$x^t_{2,i}$	116
Income share ratio (10 per cent)	−0.128	0.609*	$x^t_{2,i}$	0.420*	$x^t_{2,i}$	−0.049	0.404*	$x^t_{2,i}$	0.301*	$x^t_{2,i}$	116
Governance											
Polity score	0.144	0.614*	$x^t_{2,i}$	0.439*	$x^t_{2,i}$	0.111	0.625	$x^t_{2,i}$	0.395*	$x^t_{2,i}$	147
Civil liberties	−0.100	0.612*	$x^t_{2,i}$	0.429*	$x^t_{2,i}$	−0.107	0.513	$x^t_{2,i}$	0.381*	$x^t_{2,i}$	173
Political rights	−0.113	0.612*	$x^t_{2,i}$	0.429*	$x^t_{2,i}$	−0.103	0.513	$x^t_{2,i}$	0.381*	$x^t_{2,i}$	173
Press freedom	−0.067	0.613*	$x^t_{2,i}$	0.429*	$x^t_{2,i}$	−0.078	0.509	$x^t_{2,i}$	0.376*	$x^t_{2,i}$	173
Voice and accountability	0.058	0.621*	$x^t_{2,i}$	0.453*	$x^t_{2,i}$	0.064	0.497	$x^t_{2,i}$	0.367*	$x^t_{2,i}$	156
Political stability and non-violence	−0.046	0.628*	$x^t_{2,i}$	0.455*	$x^t_{2,i}$	−0.074	0.509	$x^t_{2,i}$	0.373*	$x^t_{2,i}$	151
Law and order	−0.087	0.611*	$x^t_{2,i}$	0.432*	$x^t_{2,i}$	−0.117	0.510	$x^t_{2,i}$	0.380*	$x^t_{2,i}$	159
Rule of law	−0.046	0.628*	$x^t_{2,i}$	0.455*	$x^t_{2,i}$	−0.074	0.509	$x^t_{2,i}$	0.373*	$x^t_{2,i}$	151
Happiness											
Life enjoyment	−0.410*	0.653*	$x^t_{2,i}$	0.453*	$x^t_{2,i}$	−0.361*	0.317	$x^t_{2,i}$	0.203	$x^t_{2,i}$	66
Happy life years	−0.209*	0.653*	$x^t_{2,i}$	0.433*	$x^t_{2,i}$	−0.228*	0.317	$x^t_{2,i}$	0.203	$x^t_{2,i}$	66
Life enjoyment inequality	−0.036	0.691*	$x^t_{2,i}$	0.507*	$x^t_{2,i}$	−0.030	0.319	$x^t_{2,i}$	0.273*	$x^t_{2,i}$	55

Notes: * = significantly different from zero at the 90 per cent confidence level or greater; # = significantly greater than adult ϱ_s^{max} at the 90 per cent confidence level or greater; $x^t_{2,i}$ is transformed adult literacy and $x^t_{3,i}$ is transformed gross school enrolment.

That the null hypothesis cannot be rejected is not surprising, given the measurement error and construction issues and resultant inflation of ϱ_s^{max}, as discussed above. Much lower values of these coefficients were obtained from regressing $v'_{q,k,i}$ on the standard indicators. These coefficients are shown in the fifth and tenth columns of Table 11.7, headed $v'_{q,k,i}$.[17] The null hypothesis still cannot be rejected in almost all cases. The only sample for which adult literacy was not the most highly correlated variable with these adjusted residuals was that determined by the availability of the Human Poverty Index. For that sample, school enrolment ($x^t_{3,i}$) was the standard indicator most highly correlated cardinally and ordinally with the chosen $v'_{q,k,i}$. It should be noted, however, that these coefficients were not significantly higher those that between adult literacy and this residual for the same sample.[18]

The null hypothesis, that $|\varrho_{ns,j}| \leq |\varrho_s^{max}|$ was ultimately rejected for two variables only: youth literacy, and women professionals and technicians. This was the case for both the zero- and rank-order correlation coefficients for the former, but for the zero-order correlation for the latter indicator. There would appear, therefore, to be case for further development and use of these indicators in the ways mentioned above.

Conclusion

A range of indicators has been used over recent decades in an attempt to capture empirically non-economic dimensions of human well-being. Most of the commonly used indicators, available for large country samples, are very highly correlated with various measures of income per capita. Given this they have been criticized for not being able to tell us much more than income per capita alone and, as a consequence, for not sufficiently capturing non-economic dimensions of cross-country well-being achievement. This chapter has responded to this criticism. It identified the variation in a composite of the most widely used non-economic well-being indicators not accounted for by income per capita. It did this by regressing this composite on the logarithm of PPP GDP per capita, observing the values of the residual term of the regression. This residual was interpreted as an income-independent, or non-economic, measure of national well-being achievement. Estimates of this residual were provided for 173 countries. An interesting result is that the top-ranked countries, in terms of non-economic well-being achieved measured according to this residual, were dominated by those that either still have, or in their recent pasts have had, non-market, centrally planned economies. The bottom-ranked countries were far more diverse, seemingly without a unifying, common characteristic.

The chapter then looked at correlations between its measure and other less widely used well-being indicators in an attempt to find the indicator that best captures non-economic well-being achievement. The rationale for this is that the above-mentioned residual is a purely statistical construct, derived from a series of econometric procedures. It is not what might be described as a direct measure of well-being, therefore. As it turned out, only two of the less widely used indicators perform better in this regard than a standard indicator. Those variables were youth literacy and a gender empowerment variable, the female share of professional and technical employment. In all other cases a standard, widely used measure performed best in this regard. That variable was the adult literacy rate. This was a particularly robust result, which was obtained consistently across different samples of countries and under different assumed error measurement scenarios.

What are the implications of these results? Most obviously, it suggests that if we wish to use a measure of well-being, in the sense defined above, that best captures this chapter's notion of non-economic well-being achievement, across different samples of countries, we should be using the adult literacy rate. This is an interesting finding, to the extent that the adult literacy rate is subject to the above-mentioned criticism regarding correlations with income. It is also disappointing, on the one hand, that there have been many attempts to shift focus away from the standard measures, including adult literacy, towards newer, hopefully more enlightening indicators. On the other hand, it is not disappointing, given that such a widely used measure performs so consistently well in capturing non-economic well-being achievement. With regard to the female share of technical and professional employment and youth literacy variables, there would appear to be a case for expanding the coverage, reporting and usage of these indicators if one is comprehensively to measure non-economic well-being achievement with a variable other than one obtained by construction, using econometric techniques. Greater coverage of the former variable would appear to be especially warranted, given that it is available for a relatively small sample of countries. A message for policy from this result is that if we want to promote non-economic well-being, as defined in this chapter, we should continue to strive for improvements in adult literacy. This message is made stronger given the result for youth literacy.

Finally, let us consider some possible directions for future research. First, while this chapter has made some attempt to account for measurement error in the standard indicators, further work on this is clearly required both at a conceptual level, involving further consideration of the source of measurement error, and at the purely empirical level. The nature of the errors might be different or more complicated than envisaged in this chapter. As such, it is not beyond the bounds of imagination to speculate the correlation between the variants of μ_i and adult literacy is due to errors in measurement not captured in this chapter. Further tests for the sensitivity of this result to possible measurement error would appear to be warranted, therefore.

Second, there is far from universal acceptance that a logarithmic transformation of income per capita, used in this chapter, is appropriate. Alternative transformations could be investigated.

Third, non-economic achievement could be measured using period averages of the relevant data instead of data for a single year. This might better capture long-run relationships between income and the non-economic indicators.

Fourth, one could account for possible endogeneity between income and the non-economic indicators in estimating the residual between them.

Fifth, rather than seeking to correlate this chapter's measure of non-economic well-being achievement on a single variable, one could look at correlating it against a composite of a number of indicators, thereby providing a multidimensional non-economic well-being achievement indicator.

Finally, rather than seeking a variable or variables that are merely associated with the chapter's constructed measure of well-being achievement, one could undertake a far more sophisticated analysis that looks for causal relationships.

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Appendix: Detailed results and variable definitions

Table 11A.1 Well-being data

Country	Life expectancy	Adult literacy	Gross enrolment	PPP GDP per capita			HDI		Well-being index		Residual	
	($x_{1,i}$)	($x_{2,i}$)	($x_{3,i}$)	Value (y_i)	Value ($\ln y_i$)	Rank	Value	Rank	Value (W_i)	Rank	Value (μ_i)	Rank
Tajikistan	67.6	99.0	67	1152	7.049	151	0.667	112	0.050	81	0.177	1
Armenia	72.9	98.4	80	2559	7.847	117	0.754	77	0.096	33	0.152	2
Uzbekistan	69.0	99.0	76	2441	7.800	119	0.727	95	0.075	50	0.135	3
Georgia	73.2	99.0	70	2664	7.888	115	0.748	81	0.079	46	0.131	4
Moldova, Rep.	66.6	98.9	72	2109	7.654	126	0.701	105	0.056	78	0.130	5
Viet Nam	68.2	93.4	67	1996	7.599	128	0.688	109	0.040	89	0.118	6
Azerbaijan	71.6	97.0	71	2936	7.985	112	0.741	89	0.069	61	0.113	7
Suriname	70.6	94.0	82	3799	8.242	103	0.756	74	0.083	44	0.103	8
Cuba	76.0	96.7	76	4519	8.416	90	0.795	55	0.095	35	0.101	9
Mongolia	62.9	98.9	58	1783	7.486	134	0.655	113	0.012	106	0.100	10
Ecuador	70.0	91.6	77	3203	8.072	110	0.732	93	0.064	64	0.100	11
Kyrgyzstan	67.8	97.0	68	2711	7.905	114	0.712	102	0.048	84	0.099	12
Congo	51.3	80.7	63	825	6.715	163	0.512	136	−0.059	123	0.098	13
Philippines	69.3	95.3	82	3971	8.287	97	0.754	76	0.081	45	0.097	14
Ukraine	68.1	99.0	77	3816	8.247	102	0.748	80	0.074	52	0.095	15
Turkmenistan	66.2	98.0	81	3956	8.283	100	0.741	87	0.073	58	0.090	16
Myanmar	56.0	84.7	55	1027	6.934	152	0.552	127	−0.051	121	0.087	17
Sri Lanka	72.1	91.6	70	3530	8.169	108	0.741	88	0.057	77	0.084	18
Fiji	69.1	92.9	83	4668	8.448	89	0.758	72	0.077	47	0.079	19
Albania	73.2	84.7	71	3506	8.162	109	0.733	92	0.048	83	0.076	20
Lebanon	73.1	86.0	78	4308	8.368	95	0.755	75	0.066	62	0.076	21
São Tomé and Príncipe	65.1	83.1	58	1792	7.491	133	0.632	119	−0.014	113	0.074	22
Bolivia	62.4	85.5	70	2424	7.793	120	0.653	114	0.008	108	0.068	23
Maldives	66.5	96.7	77	4485	8.408	93	0.743	84	0.062	68	0.068	24
Jamaica	75.3	86.9	62	3639	8.199	104	0.742	86	0.041	86	0.066	25
Peru	68.8	89.9	80	4799	8.476	88	0.747	83	0.063	66	0.063	26

Continued

Table 11A.1 Continued

Country	Life expectancy ($x_{1,i}$)	Adult literacy ($x_{2,i}$)	Gross enrolment ($x_{3,i}$)	PPP GDP per capita Value (y_i)	PPP GDP per capita Value ($\ln y_i$)	PPP GDP per capita Rank	HDI Value	HDI Rank	Well-being index Value (W_i)	Well-being index Rank	Residual Value (μ_i)	Residual Rank
Solomon Islands	68.3	76.6	50	1648	7.407	138	0.622	121	−0.033	119	0.062	27
Lithuania	72.1	99.0	80	7106	8.869	65	0.808	49	0.096	34	0.061	28
Macedonia, FYR	73.1	94.0	70	5086	8.534	85	0.772	68	0.066	63	0.061	29
Latvia	70.4	99.0	82	7045	8.860	66	0.800	53	0.094	38	0.060	30
Belize	74.0	93.2	73	5606	8.632	82	0.784	58	0.074	53	0.060	31
Malawi	40.0	60.1	73	615	6.422	170	0.400	163	−0.124	137	0.059	32
China	70.5	84.1	73	3976	8.288	96	0.726	96	0.041	85	0.058	33
Tanzania, U. Rep.	51.1	75.1	32	523	6.260	172	0.440	151	−0.140	144	0.058	34
Bulgaria	70.8	98.4	72	5710	8.650	80	0.779	62	0.071	59	0.056	35
Indonesia	66.2	86.9	65	3043	8.021	111	0.684	110	0.014	104	0.054	36
Kenya	50.8	82.4	51	1022	6.930	153	0.513	134	−0.084	130	0.054	37
Panama	74.0	91.9	74	6000	8.700	75	0.787	57	0.073	55	0.053	38
Poland	73.3	99.0	84	9051	9.111	53	0.833	37	0.109	26	0.053	39
Australia	78.9	99.0	116	25693	10.154	12	0.939	6	0.198	1	0.049	40
Paraguay	70.1	93.3	64	4426	8.395	94	0.740	90	0.040	88	0.047	41
Guyana	63.0	98.5	66	3963	8.285	99	0.708	103	0.029	93	0.046	42
Saint Lucia	73.4	90.2	70	5703	8.649	81	0.772	67	0.059	74	0.043	43
Uruguay	74.4	97.7	79	9035	9.109	54	0.831	40	0.098	31	0.042	44
Dominica	72.9	96.4	65	5880	8.679	77	0.779	61	0.059	72	0.041	45
Kazakhstan	64.6	98.0	77	5871	8.678	78	0.750	79	0.058	75	0.040	46
Estonia	70.6	99.0	86	10066	9.217	48	0.826	42	0.104	27	0.038	47
Colombia	71.2	91.7	73	6248	8.740	72	0.772	66	0.060	70	0.037	48
Nicaragua	68.4	66.5	63	2366	7.769	122	0.635	118	−0.026	118	0.037	49
Honduras	65.7	74.6	61	2453	7.805	118	0.638	116	−0.023	115	0.037	50
Cambodia	56.4	67.8	62	1446	7.277	145	0.543	130	−0.070	127	0.036	51
Belarus	68.5	99.0	77	7544	8.929	63	0.788	56	0.076	48	0.036	52
Chile	75.3	95.8	78	9417	9.150	50	0.831	39	0.095	36	0.035	53

Venezuela	72.9	92.6	65	5794	8.665	79	0.770	69	0.051	79	0.034	54
Romania	69.8	98.1	69	6423	8.768	69	0.775	63	0.060	71	0.034	55
Libyan Arab Jamahiriya	70.5	80.0	92	7570	8.932	62	0.773	64	0.074	54	0.033	56
United Kingdom	77.7	99.0	106	23509	10.065	20	0.928	13	0.172	3	0.031	57
Jordan	70.3	89.7	55	3966	8.286	98	0.717	99	0.013	105	0.030	58
Zambia	41.4	78.1	49	780	6.659	165	0.433	153	−0.132	142	0.030	59
New Zealand	77.6	99.0	99	20070	9.907	24	0.917	19	0.157	8	0.029	60
Madagascar	52.6	66.5	44	840	6.733	161	0.469	147	−0.127	139	0.029	61
Syrian Arab Republic	71.2	74.4	63	3556	8.176	106	0.691	108	0.001	111	0.028	62
Belgium	78.4	99.0	109	27178	10.210	9	0.939	5	0.181	2	0.027	63
Croatia	73.8	98.3	68	8091	8.999	59	0.809	48	0.073	56	0.027	64
Yemen	60.6	46.3	51	893	6.795	158	0.479	144	−0.125	138	0.025	65
Sweden	79.7	99.0	101	24277	10.097	17	0.941	2	0.169	4	0.024	66
Cape Verde	69.7	73.8	77	4863	8.489	87	0.715	101	0.025	97	0.024	67
Spain	78.5	97.6	95	19472	9.877	25	0.913	21	0.148	11	0.023	68
Costa Rica	76.4	95.6	67	8650	9.065	57	0.820	43	0.075	49	0.022	69
Russian Federation	66.1	99.0	78	8377	9.033	58	0.781	60	0.069	60	0.020	70
Finland	77.6	99.0	103	24996	10.126	16	0.930	10	0.165	5	0.018	71
Argentina	73.4	96.8	83	12377	9.424	44	0.844	34	0.101	30	0.017	72
Slovakia	73.3	99.0	76	11243	9.328	46	0.835	36	0.093	39	0.017	73
Nigeria	51.7	63.9	45	896	6.798	157	0.462	148	−0.133	143	0.016	74
Netherlands	78.1	99.0	102	25657	10.153	13	0.935	8	0.165	6	0.015	75
Portugal	75.7	92.2	96	17290	9.758	30	0.880	28	0.128	20	0.014	76
Thailand	70.2	95.5	60	6402	8.764	70	0.762	70	0.036	90	0.011	77
El Salvador	69.7	78.7	63	4497	8.411	91	0.706	104	0.005	109	0.011	78
Hungary	71.3	99.0	81	12416	9.427	43	0.835	35	0.094	37	0.010	79
Korea, Rep. of	74.9	97.8	90	17380	9.763	28	0.882	27	0.124	22	0.010	80
Brazil	67.7	85.2	80	7625	8.939	60	0.757	73	0.048	82	0.007	81
Samoa (Western)	69.2	80.2	65	5041	8.525	86	0.715	100	0.011	107	0.006	82
Greece	78.2	97.2	81	16501	9.711	34	0.885	25	0.116	24	0.006	83
France	78.6	99.0	94	24223	10.095	18	0.928	12	0.149	10	0.005	84
Dominican Republic	67.1	83.6	72	6033	8.705	74	0.727	94	0.025	96	0.005	85

Continued

Table 11A.1 Continued

Country	Life expectancy ($x_{1,i}$)	Adult literacy ($x_{2,i}$)	Gross enrolment ($x_{3,i}$)	PPP GDP per capita Value (y_i)	Value ($\ln y_i$)	Rank	HDI Value	Rank	Well-being index Value (W_i)	Rank	Residual Value (μ_i)	Rank
Mexico	72.6	91.4	71	9023	9.108	55	0.796	54	0.060	69	0.004	86
Trinidad and Tobago	74.3	93.8	65	8964	9.101	56	0.805	50	0.059	73	0.003	87
St Vincent and the Grenadines	69.6	88.9	58	5555	8.622	83	0.733	91	0.016	102	0.003	88
Slovenia	75.5	99.0	83	17367	9.762	29	0.879	29	0.115	25	0.000	89
Iran, Islamic Rep.	68.9	76.3	73	5884	8.680	76	0.721	98	0.019	100	0.000	90
Canada	78.8	99.0	97	27840	10.234	7	0.940	3	0.157	7	0.000	91
Barbados	76.8	98.0	77	15494	9.648	36	0.871	31	0.103	28	−0.001	92
Egypt	67.3	55.3	76	3635	8.198	105	0.642	115	−0.026	117	−0.001	93
Germany	77.7	99.0	94	25103	10.131	15	0.925	18	0.146	13	−0.002	94
Lesotho	45.7	83.4	61	2031	7.616	127	0.535	133	−0.079	128	−0.002	95
Togo	51.8	57.1	62	1442	7.274	146	0.493	141	−0.111	134	−0.003	96
Congo, Dem. Rep.	51.3	61.4	31	765	6.640	166	0.431	155	−0.171	152	−0.007	97
Norway	78.5	99.0	97	29918	10.306	3	0.942	1	0.156	9	−0.008	98
Nepal	58.6	41.8	60	1327	7.191	148	0.490	142	−0.123	136	−0.008	99
Denmark	76.2	99.0	97	27627	10.227	8	0.926	14	0.147	12	−0.009	100
Algeria	69.6	66.7	72	5308	8.577	84	0.697	106	−0.002	112	−0.011	101
Tunisia	70.2	71.0	74	6363	8.758	71	0.722	97	0.014	103	−0.011	102
Israel	78.7	94.6	83	20131	9.910	23	0.896	22	0.116	23	−0.012	103
Czech Republic	74.9	99.0	70	13991	9.546	39	0.849	33	0.083	43	−0.012	104
Malta	78.0	92.0	80	17273	9.757	31	0.875	30	0.101	29	−0.013	105
Grenada	65.3	94.4	65	7580	8.933	61	0.747	82	0.027	95	−0.014	106
Austria	78.1	99.0	90	26765	10.195	10	0.926	15	0.139	16	−0.014	107
Zimbabwe	42.9	88.7	65	2635	7.877	116	0.551	128	−0.069	126	−0.016	108
Malaysia	72.5	87.5	66	9068	9.113	52	0.782	59	0.041	87	−0.016	109
Italy	78.5	98.4	84	23626	10.070	19	0.913	20	0.126	21	−0.016	110
Turkey	69.8	85.1	62	6974	8.850	67	0.742	85	0.017	101	−0.017	111
India	63.3	57.2	55	2358	7.766	123	0.577	125	−0.083	129	−0.020	112

Antigua and Barbuda	73.9	86.6	69	10541	9.263	47	0.800	52	0.051	80	−0.020	113
Japan	81.0	99.0	82	26755	10.194	11	0.933	9	0.132	18	−0.021	114
Iceland	79.2	99.0	89	29581	10.295	5	0.936	7	0.141	15	−0.021	115
Saint Kitts and Nevis	70.0	97.8	70	12510	9.434	41	0.814	44	0.062	67	−0.023	116
Ghana	56.8	71.5	42	1964	7.583	130	0.548	129	−0.104	133	−0.025	117
Cameroon	50.0	75.8	43	1703	7.440	135	0.512	135	−0.118	135	−0.026	118
Seychelles	72.7	88.0	73	12508	9.434	42	0.811	47	0.058	76	−0.028	119
Bahrain	73.3	87.6	80	15084	9.621	37	0.831	38	0.075	51	−0.028	120
Ireland	76.6	99.0	91	29866	10.304	4	0.925	17	0.135	17	−0.028	121
Brunei Darussalam	75.9	91.5	76	16779	9.728	33	0.856	32	0.084	42	−0.028	122
United States	77.0	99.0	95	34142	10.438	2	0.939	4	0.146	14	−0.029	123
Switzerland	78.9	99.0	84	28769	10.267	6	0.928	11	0.129	19	−0.031	124
Lao PDR	53.5	48.7	58	1575	7.362	142	0.485	143	−0.131	141	−0.032	125
Uganda	44.0	67.1	45	1208	7.097	149	0.444	150	−0.155	149	−0.032	126
Eritrea	52.0	55.7	26	837	6.730	162	0.421	157	−0.191	156	−0.036	127
Rwanda	40.2	66.8	40	943	6.849	156	0.403	162	−0.181	154	−0.036	128
Haiti	52.6	49.8	52	1467	7.291	144	0.471	146	−0.145	146	−0.040	129
South Africa, Rep.	52.1	85.3	93	9401	9.149	51	0.695	107	0.019	99	−0.041	130
Benin	53.8	37.4	45	990	6.898	154	0.420	158	−0.183	155	−0.042	131
Mauritius	71.3	84.5	63	10017	9.212	49	0.772	65	0.023	98	−0.042	132
Comoros	59.8	55.9	35	1588	7.370	141	0.511	137	−0.142	145	−0.044	133
Cyprus	78.0	97.1	68	20824	9.944	22	0.883	26	0.086	41	−0.045	134
Guatemala	64.8	68.6	49	3821	8.248	101	0.631	120	−0.066	124	−0.045	135
Gabon	52.7	71.0	86	6237	8.738	73	0.637	117	−0.025	116	−0.048	136
Bhutan	62.0	47.0	33	1412	7.253	147	0.494	140	−0.158	150	−0.049	137
Bahamas	69.2	95.4	74	17012	9.742	32	0.826	41	0.063	65	−0.050	138
Singapore	77.6	92.3	75	23356	10.059	21	0.885	24	0.090	40	−0.051	139
Mali	51.5	41.5	28	797	6.681	164	0.386	164	−0.219	162	−0.059	140
Swaziland	44.4	79.6	72	4492	8.410	92	0.577	124	−0.068	125	−0.062	141
Papua New Guinea	56.7	63.9	38	2280	7.732	124	0.535	132	−0.130	140	−0.064	142
Morocco	67.6	48.9	52	3546	8.174	107	0.602	123	−0.091	131	−0.064	143
Guinea-Bissau	44.8	38.5	37	755	6.627	167	0.349	167	−0.231	163	−0.066	144

Continued

Table 11A.1 Continued

Country	Life expectancy ($x_{1,i}$)	Adult literacy ($x_{2,i}$)	Gross enrolment ($x_{3,i}$)	PPP GDP per capita			HDI		Well-being index		Residual	
				Value (y_i)	Value ($\ln y_i$)	Rank	Value	Rank	Value (W_i)	Rank	Value (μ_i)	Rank
Sudan	56.0	57.8	34	1797	7.494	132	0.499	139	−0.154	148	−0.067	145
Bangladesh	59.4	41.3	37	1602	7.379	140	0.478	145	−0.171	153	−0.073	146
Hong Kong, China (SAR)	79.5	93.5	63	25153	10.133	14	0.888	23	0.073	57	−0.074	147
Saudi Arabia	71.6	76.3	61	11367	9.338	45	0.759	71	0.002	110	−0.074	148
Namibia	44.7	82.0	78	6431	8.769	68	0.610	122	−0.048	120	−0.075	149
Pakistan	60.0	43.2	40	1928	7.564	131	0.499	138	−0.158	151	−0.077	150
Sierra Leone	38.9	36.0	27	490	6.194	173	0.275	173	−0.280	171	−0.077	151
Kuwait	76.2	82.0	59	15799	9.668	35	0.813	45	0.027	94	−0.079	152
Ethiopia	43.9	39.1	27	668	6.504	169	0.327	168	−0.255	168	−0.079	153
Chad	45.7	42.6	31	871	6.770	159	0.365	166	−0.232	164	−0.080	154
Burundi	40.6	48.0	18	591	6.382	171	0.313	171	−0.268	169	−0.081	155
Qatar	69.6	81.2	75	18789	9.841	26	0.803	51	0.036	91	−0.086	156
United Arab Emirates	75.0	76.3	68	17935	9.795	27	0.812	46	0.030	92	−0.087	157
Senegal	53.3	37.3	36	1510	7.320	143	0.431	154	−0.204	159	−0.101	158
Mauritania	51.5	40.2	40	1677	7.425	136	0.438	152	−0.196	157	−0.102	159
Côte d'Ivoire	47.8	46.8	38	1630	7.396	139	0.428	156	−0.200	158	−0.104	160
Vanuatu	68.0	34.0	38	2802	7.938	113	0.542	131	−0.152	147	−0.104	161
Oman	71.0	71.7	58	13356	9.500	40	0.751	78	−0.016	114	−0.108	162
Luxembourg	77.4	99.0	72	50061	10.821	1	0.925	16	0.097	32	−0.112	163
Mozambique	39.3	44.0	23	854	6.750	160	0.322	170	−0.270	170	−0.117	164
Gambia	46.2	36.6	45	1649	7.408	137	0.405	160	−0.213	160	−0.118	165
Central African Republic	44.3	46.7	24	1172	7.066	150	0.375	165	−0.244	166	−0.118	166
Botswana	40.3	77.2	70	7184	8.880	64	0.572	126	−0.093	132	−0.129	167
Burkina Faso	46.7	23.9	23	976	6.883	155	0.325	169	−0.286	172	−0.144	168
Djibouti	43.1	64.6	22	2377	7.774	121	0.445	149	−0.214	161	−0.151	169
Equatorial Guinea	51.0	83.2	64	15073	9.621	38	0.679	111	−0.053	122	−0.155	170
Guinea	47.5	41.0	28	1982	7.592	129	0.414	159	−0.235	165	−0.157	171
Niger	45.2	15.9	16	746	6.615	168	0.277	172	−0.324	173	−0.158	172
Angola	45.2	42.0	23	2187	7.690	125	0.403	161	−0.253	167	−0.183	173

Table 11A.2 Variable definitions

Variable	Year and definition
Human development index	2000 Human development index value: a composite index combining measures of life expectancy, adult literacy, school enrolment and PPP GDP per capita.
Life expectancy	2000 Life expectancy at birth (years): the number of years a newborn infant would live if prevailing patterns of age-specific mortality rates at the time of birth were to stay the same throughout the child's life.
Adult literacy	2000 Adult literacy rate: the percentage of people aged 15 and above who can, with understanding, both read and write a short, simple statement on their everyday life.
Gross enrolment	1999 Combined primary, secondary and tertiary gross enrolment ratio (%): the number of students enrolled in a level of education, regardless of age, as a percentage of the population of official school age for that level.
Human poverty index (HPI-1)	2000 Human poverty index value: a composite index combining measures of lack of access to improved water services, probability of not surviving to age 40, underweight children and adult illiteracy.
Survival to 40	1995–2000 Probability at birth of not surviving to age 40 (% of cohort): calculated as 1 minus the probability of surviving to a specified age for a given cohort.
Water usage	2000 Population not using improved drinking water sources (%): calculated as 100 minus the percentage of the population using any of the following types of water supply for drinking: piped water, a public tap, a borehole with a pump, a protected well, a protected spring or rainwater.
Poverty headcount ($1)	1983–2000 Percentage of the population living below income poverty line set at $1 a day in 1985 prices ($1.08 in 1993 prices) adjusted for purchasing power parity.
Poverty headcount ($2)	1983–2000 Percentage of the population living below income poverty line set at $2 a day in 1985 prices ($2.16 in 1993 prices) adjusted for purchasing power parity.
Sanitation facilities	2000 Population using adequate sanitation facilities (%): the percentage of the population using adequate sanitation facilities, such as a connection to a sewer or septic tank system, a pour-flush latrine, a simple pit latrine or a ventilated improved pit latrine. An excreta disposal system is considered adequate if it is private or shared (but not public) and if it hygienically separates human excreta from human contact.
Drug access	1999 Population with access to essential drugs (%): the percentage of the population for whom a minimum of 20 of the most essential drugs are continuously and affordably available at public or private health facilities or drug outlets within one hour's travel from home.

Continued

Table 11A.2 Continued

Variable	Year and definition
Water services	2000 Population using improved water services (%): the proportion of the population using piped water, water from a public tap, water from a borehole with a pump, water from a protected well or protected spring or rainwater for drinking.
Measles immunization	1999 One-year-olds fully immunized against measles (%).
Tuberculosis immunization	1999 One-year-olds fully immunized against tuberculosis (%).
Oral rehydration	1994–2000 Oral rehydration therapy use rate (%): the percentage of all cases of diarrhoea in children under age five treated with oral rehydration salts or recommended home fluids, or both.
Contraceptive prevalence	1995–2000 Contraceptive prevalence (%): the percentage of married women aged 15–49 who are using, or whose partners are using, any form of contraception, whether modern or traditional.
Birth attendance	1994–2000 Births attended by skilled health staff (%): the percentage of deliveries attended by a doctor, nurse or midwife or trained traditional birth attendant.
Physicians	1990–99 Physicians (per 100,000 people): includes graduates of a faculty or school of medicine who are working in any medical field (including teaching, research and administration).
Undernourishment	1997–99 Undernourished people (as percentage of total population): people whose food intake is insufficient to meet their minimum energy requirements on a chronic basis.
Underweight children	1995–2000 Underweight children under age-five (%): includes moderate and severe underweight, which is defined as below two standard deviations from the median weight for age of the reference population.
Under height children	1995–2000 Children under height for age (% under age 5): includes moderate and severe stunting, which is defined as below two standard deviations from the median height for age of the reference population.
Underweight infants	1995–2000 Infants with low birth weight (%): the percentage of infants with a birth weight of less than 2,500 grams.
Adults with HIV/AIDS	2001 People living with HIV/AIDS, adults (% age 15–49): the estimated number of people living with HIV/AIDS at the end of the year specified.
Women with HIV/AIDS	2001 People living with HIV/AIDS, women (% age 15-49): the estimated number of people living with HIV/AIDS at the end of the year specified.

Continued

Table 11A.2 Continued

Variable	Year and definition
Malaria cases	2000 Malaria cases (per 100,000 people): the total number of malaria cases reported to the World Health Organization by countries in which malaria is endemic.
Tuberculosis cases	1999 Tuberculosis cases (per 100,000 people): the total number of tuberculosis cases reported to the World Health Organization. A tuberculosis case is defined as a patient in whom tuberculosis has been bacteriologically confirmed or diagnosed by a clinician.
Cigarette consumption	1999–2000 Cigarette consumption per adult (annual average): the sum of production and imports minus exports of cigarettes divided by the population aged 15 and above.
Infant mortality rate	2000 Infant mortality rate (per 1,000 live births): the probability of dying between birth and exactly one year of age expressed per 1,000 live births.
Child mortality rate	2000 Under-five mortality rate (per 1,000 live births): the probability of dying between birth and exactly five years of age expressed per 1,000 live births.
Survival to 65 (females)	1995–2000 Probability at birth of surviving to age 65, female (% of cohort): the probability of a newborn infant surviving to a specified age if subject to prevailing patterns of age-specific mortality rates.
(Males)	1995–2000 Probability at birth of surviving to age 65, male (% of cohort): the probability of a newborn infant surviving to a specified age if subject to prevailing patterns of age-specific mortality rates.
Maternal mortality rate	1985–99 Maternal mortality ratio reported (per 100,000 live births): reported annual number of deaths of women from pregnancy-related causes per 100,000 live births, not adjusted for the well-documented problems of underreporting and misclassification.
Youth literacy rate	2000 Youth literacy rate (% age 15–24): the percentage of people aged 15-24 who can, with understanding, both read and write a short, simple statement on their everyday life.
Primary school enrolment	1998 Net primary enrolment ratio (%): the number of students enrolled in a level of education who are of official school age for that level, as a percentage of the population of official school age for that level.
Secondary school enrolment	1998 Net secondary enrolment ratio (%): the number of students enrolled in a level of education who are of official school age for that level, as a percentage of the population of official school age for that level.
Children grade 5	1995–97 Children reaching grade 5 (%): the percentage of children starting primary school who eventually attain grade 5 (grade 4 if the duration of primary school is four years). The estimates are based on the reconstructed cohort method, which uses data on enrolment and repeaters for two consecutive years.

Continued

Table 11A.2 Continued

Variable	Year and definition
Gender-related development index	2000 Gender-related development index (GDI) value: the HDI but with its components adjusted for inequalities between men and women.
Human development disparity	2000 Ratio of the human development index to the gender-related development index.
Life expectancy ratio	2000 Ratio female to male life expectancy at birth.
Adult literacy ratio	2000 Ratio of female to male adult literacy rate.
School enrolment ratio	2000 Ratio of female to male combined primary, secondary and tertiary gross enrolment ratio.
Earned income ratio	2000 Ratio of female to male estimated earned income: each income is roughly derived on the basis of the ratio of the female non-agricultural wage to the male non-agricultural wage, the female and male shares of the economically active population, total female and male population and GDP per capita (PPP US$).
Gender empowerment measure	1991–2002 Gender empowerment measure (GEM) value: a composite index combining measures in gender inequality in parliamentary seats, legislative, senior official and managerial positions, professional and technical employment and earned income.
Women in parliament	2002 Seats in parliament held by women (as percentage of total): refers to seats held by women in a lower or single house or an upper house or senate, where relevant.
Women in senior positions	1991–2000 Female legislators, senior officials and managers (as percentage of total): women's share of positions defined according to the International Standard Classification of Occupations (ISCO-88).
Women professionals and technicians	1991–2000 Female professional and technical workers (as percentage of total): women's share of positions defined according to the International Standard Classification of Occupations (ISCO-88).
Gini coefficient	Various years Gini coefficient values expressed as percentages.
Income share ratio (20 per cent)	Various years Ratio of income or consumption share of the richest 20 per cent of the population to that of the poorest 20 per cent, expressed as a percentage.
Income share ratio (10 per cent)	Various years Ratio of income or consumption share of the richest ten per cent of the population to that of the poorest ten per cent, expressed as a percentage.
Polity score	2000 A subjective measure of the extent to which laws and institutions which allow for democratic participation are present.
Civil liberties	2000 A subjective, Freedom House assessment of nations based upon the observance of civil liberties.

Continued

Variable	**Year and definition**
Political rights	2000 A subjective, Freedom House assessment of nations based upon the observance of political rights.
Press freedom	2000 A subjective, Freedom House assessment of whether nations have a free press.
Voice and accountability	2000–01 A subjective assessment, based on surveys of public perception regarding the quality of national governance, taking into account political process, civil liberties, political rights and press freedom and independence.
Political stability and non-violence	2000–01 A subjective assessment, based on surveys of public perception regarding the quality of national governance.
Law and order	2001 Subjective law and order measure from the International Country Risk Guide.
Rule of law	2000–01 A subjective assessment, based on surveys of public perception regarding the quality of national governance.
Life enjoyment	1990s Self-assessed subjective enjoyment of life, based on information obtained from surveys. Respondents are asked to assess their life satisfaction on scale of one to ten, and a national average is derived from these individual assessments.
Happy life years	1990s Happiness adjusted life years. National life enjoyment multiplied by years of life expectancy at birth.
Life enjoyment inequality	1990s Inequality in happiness among nations. Obtained by taking the standard deviation of national life enjoyment.

Sources: Governance variables – UNDP (2002); happiness variables: Veenhoven (2002a, 2002b).

Table 11A.3 Correlations between PPP GDP per capita (log) and well-being indicators

Variables	**Zero order**	**Rank order**	*n*
Human development			
Human development index	0.923	0.938	173
Life expectancy	0.794	0.840	173
Adult illiteracy	0.701	0.705	173
Gross enrolment	0.792	0.780	173
Well-being index (W_i)	0.833	0.838	173
Human poverty			
Human poverty index (HPI-1)	−0.816	−0.829	87
Survival to 40	−0.733	−0.773	116
Water usage	−0.676	−0.719	108
Poverty headcount ($1)	−0.700	−0.709	60
Poverty headcount ($2)	−0.790	−0.790	60

Continued

Table 11A.3 Continued

Variables	Zero order	Rank order	*n*
Health services			
Sanitation facilities	0.643	0.674	123
Drug access	0.626	0.675	170
Water services	0.676	0.699	122
Measles immunization	0.315	0.445	165
Tuberculosis immunization	0.524	0.482	140
Oral rehydration	0.161	−0.017	56
Contraceptive prevalence	0.678	0.698	91
Birth attendance	0.768	0.789	122
Physicians	0.607	0.696	165
Health status			
Undernourishment	−0.706	−0.714	101
Underweight children	−0.681	−0.713	124
Underheight children	−0.761	−0.774	118
Underweight infants	−0.593	−0.623	150
Adults with HIV/AIDS	−0.292	0.447	144
Women with HIV/AIDS	−0.054	−0.033	73
Malaria cases	−0.379	−0.463	84
Tuberculosis cases	−0.328	−0.602	170
Cigarette consumption	0.693	0.728	110
Survival			
Infant mortality rate	−0.823	−0.892	172
Child mortality rate	−0.800	−0.896	172
Survival to 65 (females)	0.797	0.851	166
Survival to 65 (males)	0.756	0.846	166
Maternal mortality rate	−0.756	−0.847	144
Education status			
Youth literacy rate	0.649	0.665	128
Primary school enrolment	0.655	0.573	122
Secondary school enrolment	0.871	0.849	95
Children grade 5	0.716	0.826	48
Gender bias			
Gender-related development index	0.932	0.944	146
Human development disparity	−0.513	−0.582	146
Life expectancy ratio	0.347	0.407	166
Adult literacy ratio	0.643	0.673	149
School enrolment ratio	0.340	0.395	162
Earned income ratio	0.347	0.322	90
Gender empowerment			
Gender empowerment measure	0.806	0.826	66
Women in parliament	0.403	0.391	170
Women in senior positions	0.058	−0.068	77
Women professionals and Technicians	−0.002	−0.023	78
Income inequality			
Gini coefficient	−0.434	−0.438	116
Income share ratio (20 per cent)	−0.324	−0.375	116
Income share ratio (10 per cent)	−0.300	−0.356	116

Continued

Table 11A.3 Continued

Variables	**Zero order**	**Rank order**	*n*
Governance			
Polity score	0.394	0.527	147
Civil liberties	−0.540	−0.575	173
Political rights	−0.522	−0.579	173
Press freedom	−0.530	−0.545	173
Voice and accountability	0.676	0.662	156
Political stability and non-violence	0.748	0.772	151
Law and order	0.809	0.784	159
Rule of law	0.784	0.772	151
Happiness			
Life enjoyment	0.419	−0.115	66
Happy life years	0.656	0.663	66
Life enjoyment inequality	−0.556	−0.667	55

Notes

1. For the purposes of this chapter, notions such as human well-being, quality of human life, human development, basic human needs fulfilment are treated as synonymous.
2. One can speculate why this might be so, but it is entirely reasonable to posit that higher per capita incomes facilitate private and public expenditure on goods relevant to higher non-economic well-being achievement. Smaller country samples yield much lower correlation coefficients, although in most cases these coefficients are statistically significant. Larger correlations do not necessarily hold for samples of individuals or households at the sub-national level, however (see, for example, Klasen 2000). As such it must be emphasized that the context referred to in this chapter is for countries, not individuals or households.
3. See Larson and Wilford 1979; McGillivray 1991; McGillivray and White 1993; and Cahill 2005. The redundancy label has been assigned on the basis of correlation coefficients between the non-economic indicators and per capita income typically ranging from the low 0.70s upwards. Larson and Wilford (1979), for example, considered the PQLI to be empirically redundant based on the correlation between it and GNP per capita of 0.776. McGillivray (1991) draw this conclusion for the HDI based on a correlation coefficient between it and GNP per capita of 0.889. More generally, it is not uncommon for correlations between non-economic or non-exclusively economic indicators to range from 0.70 to 0.90 or higher.
4. See, for example, Drèze and Sen (1991). The UNDP examines this variation by reporting the difference between each country's GDP per capita and HDI rankings (see, for example, UNDP 2004: 139–42).
5. Ram (1982), Ogwang (1994) and Lai (2000) also use the principal components technique to derive well-being measures.
6. For our current purposes, income is seen as a well-being or welfare indicator in its own right, hence the use of the Atkinson formula. But it is also seen as a means for converting economic well-being into non-economic well-being. Allowing for diminishing returns is justified given the boundedness of many non-economic indicators and the increasing costs associated with greater achievement in others (such as life expectancy). It is recognized that selecting values for ε can be contentious, and for this reason alternative transformations of y_i, obtained from (11.3) but with different values of ε, are also used later in this chapter. Anand and Sen (2000) provide a detailed discussion of this issue in the context of the HDI.

7. The HDI is a weighted average of life expectancy, adult literacy, gross school enrolment and the logarithm of PPP GDP per capita, each scaled within theoretical ranges of zero and one-hundred. The first and fourth of these variables are assigned weights of one third, while the second and third variables are assigned weights of two ninths and one ninth, respectively. It follows that W_i, differs from the HDI in that it assigns different weights to each variable (income per capita receives a weight of zero through its exclusion) and that the variables are transformed using a different procedure, outlined below. Ranis *et al.* (2000) use a similar index, identical to the HDI in all respects other than assigning a zero weighting to income per capita. W_i, is preferred here mainly because it captures greater variation in the component variables but also because its weights are less arbitrary (although of ambiguous theoretical interpretation).
8. The principal components analysis was conducted using the computer program SHAZAM, which allows the analysis to be done on a number of alternative matrices. The correlation matrix was chosen, which is appropriate when the original variables are measured in different units, as is the case with the $x_{k,i}$. This dictated that the $x_{k,i}$, in equation (11.1) above, from which W_i were extracted, were obtained through the following transformation of the $x_{k,i}$:

$$x_{k,i}^t = \frac{x_{k,i} - \bar{x}_{k,i}}{\left[\sum_{i=1}^n \left(x_{k,i} - \bar{x}_{k,i}\right)^2\right]^{\frac{1}{2}}}$$

   where the bar denotes a mean value. This is a linear transformation. For further details, see Whistler *et al.* (2001).
9. Note that it makes no difference whether one uses $x_{k,i}^t$ or $x_{k,i}$ (the non-transformed variables), given the nature of the transformation.
10. It is acknowledged that this term is used quite loosely, as the distinction between non-standard and standard indicators is not always clear. In particular, a number of the non-standard indicators have been used for some time, and are available for large samples of countries. In this case, an indicator is in effect deemed 'standard' if it has been used to form the HDI. Similarly, the term non-economic indicator, used throughout this chapter, is used to simply describe an indicator that is not based on some measure of income per capita. Likewise, a non-exclusively economic indicator is one that has been partly obtained using a measure of income per capita.
11. μ_i and its variants were re-estimated for each of the samples for which data the non-standard indicators were available. This is necessary to ensure that they are orthogonal with respect to $\ln y_i$.
12. Note that the nature of this measurement error problem is different from that usually discussed in econometrics textbooks, as it involves coefficients that are pushed away from zero rather than being biased towards them.
13. That is, $W_i - \Phi_1 x_{1,i}^t$ was regressed on $\ln y_i$ and $\gamma_1 \pi_{1,i}$ to obtain $\nu'_{1,1,i}$. This was repeated, subtracting $\Phi_2 x_{2,i}^t$ and then $\Phi_3 x_{3,i}^t$ from W_i to eventually obtain $\nu'_{1,2,i}$ through to $\nu'_{3,3,i}$. Given that $k = 1, 2, 3$ and $q = 1, 2, 3$, this resulted in nine residuals and in turn nine zero-order correlation coefficients and nine rank-order coefficients, for each sample, from which the ϱ_s^{max} were obtained.
14. No attempt was made to obtain adjusted correlation coefficient between μ_i and the HDI. This was of no practical consequence, given that the unadjusted coefficients

between these variables did not qualify as ϱ_s^{max}. Note also that another method of addressing this issue is to re-estimate (11.1), successfully dropping each of the component variables, one at a time. This method was also used, but produced very similar results to that described above.

15. Estimates of the residuals were obtained using different, non-logarithmic transformations of y_i consistent with various alternative values of ε in equation (11.3). Broadly similar results were obtained. These details are also available, on request, from the author.
16. Appendix Table 11A.3 reports correlation coefficients between $\ln y_i$ and the variables listed in Table 11.7. It has been suggested that the correlations between these variables and μ_i will be a decreasing function of their correlations with $\ln y_i$ with, in particular, the indicator being most highly correlated with μ_i being that which is the lowest when correlated with $\ln y_i$. A comparison of the coefficients in Tables 11.6 and 11A.3 shows that this is not the case. It is true that variables highly correlated with $\ln y_i$ tend to be lowly correlated with μ_i, but the relationship is not a systematic one in the sense suggested.
17. Columns 5 and 10 of Table 11.7 report the largest correlation coefficients obtained regressing each $v'_{q,k,i}$ on each $x^t_{k,i}$. Details of all correlation coefficients are available from the author.
18. Full details of these results are available from the author.

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Notes: f = figure; n = note; t = table; **bold** = extended discussion or heading emphasized in main text.