From Text to ‘Lived’ Resources
MATHEMATICS TEACHER EDUCATION

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Ghislaine Gueudet · Birgit Pepin · Luc Trouche
Editors

From Text to ‘Lived’ Resources

Mathematics Curriculum Materials and Teacher Development

Springer
‘Mathematics Curriculum Material and Teacher Development’ can be read as the title of a dull book on an old fashioned topic from the era of curriculum development in the 1980s, perhaps ‘jazzed up’ by the catchword ‘teacher development’ from the 1990s. So, one might expect a latecomer to research in Mathematics Education. A simple cursory look over the content of the table of contents of this book shows that this is a false assumption. There are at least three major issues investigated in this book, which make it an up-to-date and fascinating contribution to research in Mathematics Education (or Didactics of Mathematics as I would prefer to call it):

– ‘Curriculum material’ has definitely not been perceived in the restricted way it had been discussed two or three decades ago. The fact that the authors use the concept ‘curriculum resources’ highlights that beside the traditional curriculum materials, like textbooks and other curricular documents, a whole range of texts and other resources have been taken into account, including software, electronic resources and the Internet. All these resources seem to become increasingly important in expressing and sharing ideas not only on curriculum materials themselves, but also in terms of curriculum development. They also help in terms of teacher education and everyday practice. The inclusion of more modern resources does not deny the most important teacher resource – the textbook. A main message of this book is to place the artefact ‘mathematics textbook’ in a wider, systematic perspective of material resources available for (mathematics) teachers and students. The book also shows that this broadening of the concept of teacher resources is helpful for understanding practices in various contexts. In selected countries, and communities of mathematics teachers, it is a fact that a wide range of ‘resources’, apart from textbooks and traditional curriculum documents, is present and relevant for teachers’ daily practice. Teachers’ professional knowledge, practical constraints (like money and other classroom arrangements) and cultural resources like language, collegiality, organisation and time, amongst others, have to be analysed to comprehensively understand the processes involved in teacher use of resources. In fact, this book opens a perspective on resources, which is not necessarily material.
The book supports recent trends in research on teaching and learning mathematics with the help of artefacts: to fully understand the role of curriculum material, it is not sufficient to simply analyse the artefact as such. A comprehensive content analysis of an artefact used by teachers can help to develop deeper knowledge of its functions in mathematics education. Nevertheless, it is only by analysing the use of the artefact that one may be able to adequately judge upon the affordances and constraints of a given artefact. For example ‘instrumental genesis’ (initiated and introduced to Didactics of Mathematics by Rabardel) analyses how an artefact is turned into an ‘instrument’ via the genesis of individual or social utilisation schemes. The research literature claims that a curriculum resource can only be judged by an analysis of its inherent features in addition to an analysis of the ways in which the different agents of the educational process use these resources. In an instrumental genesis approach, this is condensed in the concept of ‘utilisation scheme’, which is also fundamental to the documentation approach described in this book. As a consequence, the documentation approach conveys the notion of an agent having created the ‘document’ for a specific purpose.

In the book the word artefact is used in a broad sense, leaning on Wartofsky’s (1979) notion (XIII: ‘anything which human beings create by the transformation of nature and of themselves’) which differs from the traditional understanding of curriculum resources. The texts in this book are not only analysing material resources, but pay due attention to immaterial sources available to (mathematics) teachers. Beside material resources, a comprehensive analysis of teachers’ resources must also take into account immaterial resources like colleagues and communities of teaching practices. The book discusses ‘collaborative use’, and selected chapters explore the relations between teacher communities of practice, the documents shared in these communities and the consequences for the professional development of teachers from this collaboration. Here, the individual use of resources is adequately complemented by using resources in an environment shared by a community of teachers. Moreover, the book shows under which conditions such collaboration can empower teachers to become active instructional designers.

With the broadening of the view from material to immaterial resources, from individual to collective use of resources, methodologies investigating documentation and professional interaction (sharing of knowledge) of teachers also have to be extended beyond the ‘standard’ features of classroom and school research (often done by video-taping and consecutive case study analysis) or large scale statistical research using questionnaires (maybe complemented by interviews and the like). A reader sensitive to research methodology will find a whole range of research methods to explore the diverse phenomena – with various foci according to the different theoretical stances taken by the authors. As a consequence of the innovative character of the book, no consensus on research methodology has been reached yet – and this heterogeneity seems to be appropriate for a newly developed approach and the explorative character of the investigation of resources used by mathematics teachers.
Having stated this, one characteristic nevertheless stands out, and for the majority of the book’s chapters: nearly all texts heavily rely on case studies. The empirical results point to the necessity of a mix of research methods to better understand teachers’ use of resources.

Although the last paragraph typically puts forward an argument, which shows the value and importance of the book for researchers in Mathematics Education, I would like to highlight that the texts in this book can also be very helpful for practising teachers, who could learn about the wide range of resources available for enhancing their teaching practice. Curriculum developers and policy makers may benefit from the book’s reports of investigations, which show once again that implementing change in education and educational reform is not a straightforward, top-down process. Researchers are reminded that having the best available ideas and concepts for change does not imply factual change of teaching. The book shows that sharing artefacts and collectively developing utilisation schemes in collaborative groups of teachers and researchers can be a more effective means to curriculum change. Cooperation around appropriately designed resources – be they material and/or conceptual – can be a way to develop teaching and learning mathematics.

Giessen, Germany

Rudolph Straesser

Reference

Introduction

Ghislaine Gueudet, Birgit Pepin, and Luc Trouche

The teachers, in their professional activity, interact with a wide range of resources; these interactions and their consequences hold a central place in teachers’ professional development. The purpose of this book is to develop this perspective and to explore it in the field of mathematics education.

We consider on the one hand curriculum material. Traditionally, textbooks remain central resources for the teaching of mathematics in most countries. Nevertheless, other kinds of resources, in particular digital resources, and amongst them resources accessible via the Internet, are increasingly used. Understanding the evolutions brought by digital material is a central motivation of our work.

On the other hand, the reason for introducing the term ‘resource’ instead of ‘material’ is to broaden the perspective on the elements available for the teachers’ work, and to include in particular interactions with a variety of agents:

– Interactions between the teacher and her students constitute central resources for this teacher. Digitisation creates new forms of students’ productions and new modes of communication between students and teachers; but even an expression on a student’s face in class can constitute a resource for the teacher.
– Interactions between the teacher and her colleagues seem to hold an increasing place. Teachers can collectively design curriculum plans, lessons, and once again the digital means convey new forms of communication, networking and association.

Teachers collect resources, select, transform, share, implement and revise them. Drawing from the French term ‘ingénierie documentaire’, we call these processes ‘documentation’. The literal English translation is ‘to work with documents’, but the meaning it carries is richer. Documentation refers to the complex and interactive
ways that teachers work with resources; in-class and out-of-class, individually, but also collectively.

We propose a new perspective, considering teachers not as passive users, but as designers, creative ‘users’ and ‘sharers’ of their own resources, and viewing these resources as ‘lived’ resources. Teachers’ professional knowledge influences this design; at the same time, the documentation work extends existing-and generates new-professional knowledge.

Working in 12 different countries, the authors develop a variety of perspectives on teacher resources, on their use and on the associated teachers’ professional development, with different foci and theoretical frameworks.

The book is organised in four parts. Each is complemented by a reaction, presenting an expert’s view of the whole section.

The first part focuses on the different kinds, and nature of, curriculum resources for mathematics teachers from a practical, methodological and theoretical point of view. It examines what is, or is not, available for teachers’ professional activity. It also introduces the question of what kinds of changes are afforded by digital resources:

- Jill Adler introduces a conceptualisation of resources as re-sourcing teachers’ professional activity. She focuses on teacher professional knowledge, and provides evidence of different uses, by teachers in class, of knowledge resources.
- Ghislaine Gueudet and Luc Trouche propose what they coin as documentational approach of didactics for the study of the teacher’s documentational work. This new theoretical approach emphasises that geneses, documentational geneses as well as professional geneses, are strongly intertwined. The authors also expose a specific methodology for the study of these geneses: the reflexive investigation of teachers’ documentation work;
- Maria Alessandra Mariotti and Mirko Marracci consider the question of semiotic mediation initiated by the development of the available digital resources. They explore the semiotic potential of an artefact for teacher use in their classrooms.
- Gérard Sensevy focuses on didactical intentions, for individual teachers and for different kinds of teacher groups. He studies the influence of resources on teachers’ pedagogical intentions. Furthermore, he considers teacher action in class in terms of joint actions, and which include student actions, where the students’ contributions constitute a major resource for teachers.

The Reaction to Part I is written by Bill Barton.

The second part of the book focuses on the characteristics of curriculum material. The articles raise questions about the design of curriculum materials, and about their integration, appropriation and transformation by teachers in and for their everyday teaching. Is the teacher use of curriculum materials aligned with the use envisioned by curriculum designers? What are the consequences of teacher transactions with resources for teacher professional development? The various factors shaping the nature of the resources, their design and their use, are examined here, with a
specific focus on sociocultural factors and how these influence the development of curriculum materials.

– Kenneth Ruthven investigates the use and integration of technology in mathematics classroom practice. He identifies five structuring features, of different natures, that shape the incorporation of new technologies into teachers’ practice: working environment, resource system, activity format, curriculum script and time economy;

– Janine Remillard considers different modes of engagement which teachers develop vis-a-vis curriculum resources and how they develop. She argues that teachers are often positioned, or position themselves, as passive users; and her overarching aim is to reframe the teacher–curriculum relationship such that teachers are positioned as partners and collaborators with curriculum resources.

– Birgit Pepin investigates the role of resources, more precisely a task analysis schedule, as catalyst for teacher learning. She explores the different forms of feedback resulting from developing and working with a ‘tool’ designed to analyse mathematical tasks/curriculum materials for instruction. Her results provide deeper insights, at one level, into the processes of teacher learning with the help of analytic tools and the feedback these may afford, and at another, how a tool or artefact may change into a catalytic tool at the interface between task design and enactment.

– William Schmidt describes the development of a textbook content metric that can be used in longitudinal studies to map and measure the curricular experience of individual students. Teachers and schools, sometimes districts, choose textbooks, and teachers in turn decide on the ‘coverage’ of those textbooks. This, in turn, has implications for student exposure to these curriculum materials, and the ways of working with them.

– Christine Proust proposes a historical perspective on the nature of ‘school documentation’, in terms of design and use of mathematical texts in the scribal schools of Mesopotamia about 4,000 years ago. She observes patterns of this documentation across different schools indicating strong institutional conditioning.

The Reaction to Part II is proposed by Malcolm Swan.

The third part focuses on the use of resources by teachers and students, in-class and out-of-class, and includes studies that explore the influence of the resources’ characteristics on teacher and student activity. Furthermore, the articles in this part consider the interactions between the various educational agents, and the effects of these interactions on the development and design of resources:

– Carolyn Kieran, Denis Tanguay and Armando Solares study the ‘how’ and the ‘why’ of teachers adapting researcher-designed resources, and in the context of integration of computer algebra system (CAS) technologies. They claim that the whole adaptation process, from its beginning (how teacher engage with a resource designed by researchers) to the changes made in class during the implementation, rests on teacher knowledge and beliefs.
– Using classroom videos, Dominique Forest and Alain Mercier analyse how teachers can organise their pedagogic practice and student interventions drawing on language and gestures. They show how classroom videos can become resources for teacher professional development and research.

– Sebastian Rezat focuses on textbooks, considering teachers’ and students’ use of textbooks. He establishes links between teacher’s use of mathematics textbooks effecting students, and vice versa, and argues that students’ use of resources must be considered as an important aspect within teachers’ documentation work.

– Maria Trigueros and Maria-Dolores Lozano study documentational geneses of teachers working within Enciclomedia, a national project in Mexico that offers a particular online resource. They identify developments in terms of teacher documentation systems and of teacher pedagogic practice, which includes the use of the digital means offered and supported by traditional textbooks.

– Paul Drijvers uses and further develops the concept of orchestration. He argues for a specific focus on what happens in class, the didactical performance, and identifies types of orchestrations. Survey results suggest that teachers’ intentions may differ from their actual teaching. He investigates factors leading teachers to retain a given type, and conditions for evolutions and development.

The Reaction to Part III is proposed by Luis Radford.

The fourth part of the book focuses on the collaborative aspects of teacher documentation, considering that teachers are in contact, and work, with various groups and communities in their professional lives. In this part concepts are introduced that illuminate the influence of the nature of groups and communities, the particularities of the processes of documentation within groups, and individual–collective relationships. The articles in this part identify various potential roles and interventions of collaborative teacher documentation in mathematics teacher education.

– Carl Winsløw proposes a comparative study of two kinds of teacher collectives: lesson studies as a means for professional development of mathematics teachers in Japan; and Danish high-school teachers’ collaboration in the setting of multidisciplinary modules. He introduces, and provides evidence for, the importance of didactic infrastructures, their constraints and affordances in terms of teachers’ collaborative work in preparing, observing and evaluating their teaching.

– Ghislaine Gueudet and Luc Trouche extend the documentational approach of mathematics didactics presented in chapter two by emphasising the importance of collective aspects in teachers’ documentation work. Drawing on the notion of ‘communities of practice’, they introduce the notions of community genesis and community documentation genesis, and study the relationships between the different kinds of geneses.

– Jana Visnovska, Chrystal Dean and Paul Cobb problematise the rhetoric of teachers as instructional designers. They argue that all teachers engage in documentation work; but the ability of designing coherent instructional sequences requires specific support and appropriate professional development structures.
The Reaction to Part IV is proposed by Barbara Jaworski. Deborah Ball offers a general view on all contributions; the conclusion section presents a synthesis of the book’s main results.

The authors in this book provide different lenses to view the interactions between teachers and teaching resources, and the implications for teacher professional development. These different views come together in the book, resulting in the emergence of a new theorisation of teacher documentation work, and a new perspective on teachers’ resources.
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