


Stochastic Models for Spike Trains of Single Neurons
These notes are aimed at collecting in one volume the vast and scattered literature on stochastic models of spontaneous activity in single neurons. An attempt has been made to make the treatment self-contained by providing an introduction to neurophysiology as well as the mathematical background for each kind of model. The mathematical aspects of a model are stated as a series of lemmas and theorems; this gives the relative importance of the different results and also facilitates easy reference. However, the proofs are often sketchy and sometimes omitted. This has been done to make the notes compact and easier to read. While the coverage is fairly wide, not all studies in the literature are described herein, mainly because the differences are slight. This, however, should not be construed as an understatement of the relative importance of some of them; they have been included in a list of additional references. This list also includes studies in fields like operations research, inventory control and reliability theory. These have possible applications in neuron modelling though no direct reference is made to them in the notes.

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Madras
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S.K. Srinivasan

G. Sampath
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