Lecture Notes in Applied Mathematics and Mechanics

Volume 1

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About this Series

The Lecture Notes in Applied Mathematics and Mechanics LAMM are intended for an interdisciplinary readership in the fields of applied mathematics and mechanics. This series is published under the auspices of the International Association of Applied Mathematics and Mechanics (IAAMM; German GAMM).

Topics of interest include for example focus areas of the IAAMM such as: foundations of mechanics, thermodynamics, material theory and modeling, multibody dynamics, structural mechanics, solid mechanics, biomechanics, damage, fracture, multiscale modeling and homogenization, fluid mechanics, gas dynamics, laminar flows and transition, turbulence and reactive flows, interface flows, acoustics, waves, applied analysis, mathematical modeling, calculus of variations, variational principles applied operator theory, evolutionary equations, applied stochastics, systems with uncertainty, dynamical systems, control theory, optimization, applied and numerical linear algebra, analysis and numerics of ordinary and partial differential equations.

Each contribution to the series is intended to be accessible to researchers in mathematics and mechanics and is written in English. The aim of the series is to provide introductory texts for modern developments in applied mathematics and mechanics contributing to cross-fertilization. The Lecture Notes are aimed at researchers as well as advanced masters and PhD students in both mechanics and mathematics. Contributions to the series are self-contained and focused on a few central themes. The goal of each contribution is the communication of modern ideas and principles rather than on completeness or detailed proofs. Like lecture notes from a course, a well-chosen example is preferable to an abstract framework that cannot be comprehended without deeper involvement. The typical length of each contribution is between 100 and 300 pages. If the lecture notes represent the proceedings of a summer school with several contributors, a unified, consistent presentation and style are required (e.g., common notation). In exceptional cases, doctoral theses may be accepted, if they fulfill the above-mentioned criteria.

Potential contributors should contact the appropriate editor with a title, table of contents, and a sample chapter. Full manuscripts accepted by the editors will then be peer-reviewed.
The History of Theoretical, Material and Computational Mechanics - Mathematics Meets Mechanics and Engineering
In 2008 Professor Stein, the editor of this volume, applied for setting up a new section in the yearly GAMM conference related to the history of mechanics. This suggestion was approved by the Board of GAMM and the first session on history of mechanics started in 2010.

Lectures and contributions that were presented in these sessions are the backbone of this first volume of LAMM. There is no better way to start the series of GAMM lecture notes to reflect the history of the research field.

The contributions in this volume discuss different aspects of mechanics. They are related to solid and fluid mechanics in general and to specific problems in these areas including the development of numerical solution techniques. Thus this first addition of LAMM provides an overview on the field of mechanics and describes the wide area of applications within GAMM.

Finally I like to thank the editor, Professor Erwin Stein, for his continuous effort and his hard work to make this volume possible.

Hannover, May 2013

Peter Wriggers
Vice-President of GAMM
This collection of 23 articles is the output of lectures in special sessions on “The History of Theoretical, Material and Computational Mechanics” within the yearly conferences of the GAMM in the years 2010 in Karlsruhe, Germany, 2011 in Graz, Austria, and in 2012 in Darmstadt, Germany; GAMM is the “Association for Applied Mathematics and Mechanics”, founded in 1922 by Ludwig Prandtl and Richard von Mises.

Guiding topics for the yearly sections were proposed and leading scientists invited as keynote-lecturers. This is reflected in the four parts of this book. In their sequence and in the total concept the published articles provide a certain completeness and logical consistency within the selected topics of theoretical, material, applied and computational mechanics.

I am indebted to the co-chairmen of the sections, Professor Oskar Mahrenholtz in 2010 and 2011, and Professor Lothar Gaul in 2013. It should be mentioned that each of the three sections had two sessions, each with about 150 attendees which shows the great interest of the conference participants.

The success of the new historical sections motivated the other authors and me to publish them in a book, also stimulated by Professor Peter Wriggers, President of GAMM in the period from 2008 to 2010. I also thank him for writing a foreword.

The rich history of theoretical, material, applied and computational mechanics of solids, structures and fluids should be of vivid interest for the community of mechanicians working in science and technology as well as of applied mathematicians. This is important for the self-conception of students and practitioners in order to know and realize on which shoulders we stand and how long it often took to arrive at simple-looking formulas for describing dominant effects in loading and deformation processes of engineering structures and in fluid flow processes, and moreover to derive rather general mathematical models – despite the ambitions and efforts of eminent scientists over decades and even centuries.

Following, the four parts of the book are briefly commented.
In Part I, the origins and developments of conservation principles in mechanics and related variational methods are treated together with challenging applications from the 17th to the 20th century.

Part II treats general as well as more specific aspects of material theories of deforming solid continua and porous soils, e.g. the foundation of classical theories of elastoplastic deformations, the development of theories and analysis for contact with friction and plastic deformations, as well as the formation and progress of fracture in brittle and ductile solid materials.

Part III presents important theoretical and engineering developments in fluid mechanics, beginning with remarkable inventions in the old Egypt, the dominating role of the Navier-Stokes PDEs for fluid flows and their complex solutions for a wide field of parameters as well as the invention of pumps and turbines in the 19th and 20th century.

And finally, Part IV gives a survey on the development of direct variational (numerical) methods – the Finite Element Method – in the 20th century with many extensions and generalizations, requiring a strong coupling of engineering, mathematical and computer science aspects. These three articles are restricted to static and dynamic elastic continua, according to page limitations of the book.

One may ask whether the well-written historical essays on a period of about 3 1/2 centuries of research in mechanics can highlight overriding insight to the motivation, the connections, the progress and the setbacks of so many eminent scientists in the past. Additionally, it has to be regarded that a master plan for the contents of the book could only be realized roughly, viewing the open calls for contributions to the related historical sections of GAMM conferences.

Nevertheless, the structure and the contents of the book are above all characterized by the invited lectures (chapters) of well-known scientists in their fields.

However, in order to know the real genesis of the scientific truth, we would have to ask all those splendid researchers behind the huge work about their motivations and goals, which – of course – is not possible.

Instead, we reflected essential individual achievements as parts and driving forces of the integral subject “Mechanics” with their important and distinct positions in the whole framework of this discipline. Thus, each chapter can be widely understood independently from the others.

It is my pleasant duty to deeply thank all authors for elaborating their articles on a high standard and publishing them in this book. The friendly collaboration over nearly a year provided the nice feeling of partnership.


The authors and the editor appreciate the publication of the book as Volume 1 of the new series “Lecture Notes in Applied Mathematics and Mechanics (LNAMM)”. We thank Dr. Thomas Ditzinger, Springer-Verlag, for his advice and helpful collaboration.

Hannover, May 2013                                Erwin Stein, Editor
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