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An Irregular Mind

Szemerédi is 70
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Endre Szemerédi is a mathematician with truly exceptional research power. His influence on today’s mathematics is enormous. He solved several fundamental problems that had been raised decades earlier. Many of his results have generated research for the future, and have laid the foundation of new directions in mathematics. Some of his main achievements were born prematurely, their full power and significance became evident only decades later. Although Szemerédi’s research interest is combinatorics, number theory and computer science, his influence on other fields of mathematics, ergodic theory and analysis for instance, is remarkable.

Yet as a mathematician, Szemerédi started out late. He attended medical school for a year, and worked in a factory before studying mathematics. Paul Erdős soon discovered his extraordinary talents and expected great things from him.

Szemerédi lived up to these expectations by proving several fundamental results of tremendous importance. We only mention two of them in this short foreword. Szemerédi was quite young when he proved a central conjecture of Erdős and Turán from the 1930s stating that every sequence of integers with positive density contains arbitrarily long arithmetic progressions. While the significance of this result in combinatorial number theory is obvious, it has led to a new branch of Ramsey theory (called Szemerédi type Ramsey theorems) and of ergodic theory (through the work of Furstenberg and Katznelson). The recent burst of interest in additive number theory has attracted several outstanding mathematicians and produced spectacular results. But the starting point and also a major tool in almost all of these contributions is Szemerédi’s Theorem.

One of the key elements in Szemerédi’s solution is a lemma, now called Szemerédi’s Regularity Lemma, which is of independent interest, and has an influence that cannot be overestimated. This lemma asserts that every graph can be partitioned into equal parts, whose number only depends on an error bound, so that the bipartite graph between any two such parts is “essentially random” (with a small number of exceptional parts). This statement is counterintuitive since the graph is completely deterministic, and not
random. It shows that the randomness is everywhere and inevitably present. It is because of the genius of Szemerédi that the mathematical community (and humankind) has had the opportunity to discover, appreciate, and put to use this ubiquitous and unavoidable presence of randomness.

Szemerédi has an “irregular mind”, his brain is wired differently than for most mathematicians. Many of us admire his unique way of thinking, his extraordinary vision. His coauthors often mention that Szemerédi sees things differently, that he is able to find the hidden structure, or able to create one, out of thin air. His insistence that such a structure would work has often proved decisive.

This volume is a celebration of Szemerédi’s achievements and personality, on the occasion of his seventieth birthday. It exemplifies his extraordinary vision and unique way of thinking. A number of colleagues and friends, all top authorities in their fields, have contributed their latest research papers to this volume. The topics include extensions and applications of the regularity lemma, the existence of $k$-term arithmetic progressions in various subsets of the integers, extremal problems in hypergraph theory, and random graphs. All of them are beautiful, Szemerédi type mathematics. It also contains published accounts of the first two, very original and highly successful Polymath projects, one led by Tim Gowers and the other by Terry Tao, and a short and lovely article by András Hajnal on his early encounters with Szemerédi. It is a great shame that Erdős was not able to write one. We finish this foreword by quoting from Hajnal’s article:

“Endre has grown to be a great mathematician, one of the best our country has ever given to the world. That is what this conference and this book is about.”

Budapest Imre Bárány
June 2010 József Solymosi
LIST OF PUBLICATIONS OF ENDRÉ SZEMERÉDI


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