Computational Space Flight Mechanics
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The mechanics of space flight is an old discipline. Its topic originally was the motion of planets, moons and other celestial bodies in gravitational fields. Kepler's (1571 - 1630) observations and measurements have led to probably the first mathematical description of planet's motion. Newton (1642 - 1727) gave then, with the development of his principles of mechanics, the physical explanation of these motions.

Since then man has started in the second half of the 20th century to capture physically the Space in the sense that he did develop artificial celestial bodies, which he brought into Earth's orbits, like satellites or space stations, or which he did send to planets or moons of our planetary system, like probes, or by which people were brought to the moon and back, like capsules. Further he developed an advanced space transportation system, the U.S. Space Shuttle Orbiter, which is the only winged space vehicle ever in operation.

In the last two and a half decades there were several activities in the world in order to succeed the U.S. Orbiter, like the HERMES project in Europe, the HOPE project in Japan, the X-33, X-34 and X-37 studies and demonstrators in the United States and the joint U.S. - European project X-38. However, all these projects were cancelled.

The motion of these vehicles can be described by Newton's equation of motion. The problem is complicated firstly by the translational and rotational movements of the various needed non-inertial frames, defined for example for the proper physical description of the motion of celestial bodies as well as space vehicles. Secondly, during atmospheric flight of space vehicles the description of the aerodynamic forces and moments is a challenging task, which is also true for the propulsion and reaction control forces.

Today it is no problem to solve the governing equations in the most general form using discrete numerical methods. The numerical approximation schemes, the computer power and the modern storage capacity are in such an advanced state, that solutions with high degree of accuracy can be obtained in a few seconds. Therefore the general practice in this book is to provide numerical solutions for all discussed topics and problems. This could be the orbit determination by the orbital elements, Lagrange's perturbation equations for disturbed Earth's orbits, the flight of a mass point in flight path coordinates (three degree of freedom), and the flight of a controlled space vehicle in body fixed coordinates (six degree of freedom).
This book has been written not only for graduate and doctoral students but also for non-specialists who may be interested in this subject or concerned with space flight mechanics.

The author has worked for many years in the field of fluid dynamics at research institutes and in industry. In the mid of the 1990s his responsibility expanded to re-entry and landing systems, which of course had included the disciplines of guidance and navigation as well as trajectory determination, the classical operational area of space flight mechanics. At that time he evolved besides his management obligations a personal interest for the discipline of space flight mechanics in particular with view to numerical solution methods. He does not regard himself as a specialist in this field and it is with humility that he has contributed to the subject of this book to which so many have contributed so much more and where a lot of others are much more proficient.

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Claus Weiland
Acknowledgements

When I talked to my colleague E.H. Hirschel, who authored books on aerodynamics/aerothermodynamics and on the history of aeronautical research in Germany, about the idea of writing a book on computational space flight mechanics, I got the answer that this is a good but also a very challenging idea.

Therefore I started with some trial chapters. After in-depth discussions of content and design of these chapters and the cordial encouragement and support by him, I came to the decision to commence with the writing of such a book.

So, I am much indebted to him, also that he read several times all the chapters of the book and that he provided me with a lot of critical and constructive comments.

Many thanks are due also to O. Wagner, who reads parts of the book.

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