Production and Inventory Management with Substitutions
Quantitative approaches for solving production planning and inventory management problems in industry have gained growing importance in the past years. Due to the increasing use of Advanced Planning Systems, a widespread practical application of the sophisticated optimization models and algorithms developed by the Production Management and Operations Research community now seem within reach.

The possibility that products can be replaced by certain substitute products exists in various application areas of production planning and inventory management. Substitutions can be useful for a number of reasons, among others to circumvent production and supply bottlenecks and disruptions, increase the service level, reduce setup costs and times, and lower inventories and thereby decrease capital lockup. Considering the current trend in industry towards shorter product life cycles and greater product variety, the importance of substitutions appears likely to grow. Closely related to substitutions are flexible bills-of-materials and recipes in multi-level production systems.

However, so far, the aspect of substitutions has not attracted much attention in academic literature. Existing lot-sizing models matching complex requirements of industrial optimization problems (e.g., constrained capacities, sequence-dependent setups, multiple resources) such as the Capacitated Lot-Sizing Problem with Sequence-Dependent Setups (CLSD) and the General Lot-Sizing and Scheduling Problem for Multiple Production Stages (GLSPMS) do not feature in substitution options.

This was the point where J. Christian Lang initiated his PhD project. In his project, he devised a graphical modeling framework for substitution options. Using this framework, he developed the following extensions of existing dynamic lot-sizing models to include product substitutions:

1. Uncapacitated and capacitated single-level dynamic lot-sizing models with substitutions
2. A capacitated single-level dynamic lot-sizing model with substitutions and sequence-dependent setups
3. An extension of the Multi-Level Capacitated Lot-Sizing Problem (MLCLSP) by flexible bills-of-materials
4. An extension of the General Lot-Sizing and Scheduling Problem for Multiple Production Stages (GLSPMS) to map substitutions and flexible production sequences

In addition, he developed exact and heuristic solution approaches for model categories 1 and 2 to solve problems with acceptable computational efforts, and analyzed these approaches in extensive computational experiments.

He also came up with a number of very interesting and illustrative examples of application areas where substitutions are of importance. He delved into one of these areas, blood transfusion inventory management, and developed a simulation-based optimization model for this complex stochastic optimization problem, demonstrating the potential of automated optimization approaches also for inventory management with substitutions.

J. Christian Lang’s PhD thesis is a very innovative piece of research. It contains a large number of new aspects and inventive ideas: It contributes a comprehensive classification scheme, extensive literature review, and a novel modeling approach for lot-sizing with substitutions. The new models and solution methods that his thesis provides have promising potential for successful use in practice. Hence, his work represents a significant scientific advance and will serve as a valuable resource for researchers and practitioners.

Darmstadt

Wolfgang Domschke

July 2009
Preface

This thesis is the result of three years of research conducted at the chair of Operations Research of the Department of Law, Business and Economics at Technische Universität Darmstadt, Germany. It would not have been possible without the backing of numerous people.

First of all, I deeply thank my advisor Wolfgang Domschke for his continuous support and mentorship. He has always granted my colleagues and me an extraordinarily high degree of freedom in pursuing our research interests. I learned a lot from his broad expertise, way of thinking and highly precise style of writing, and received helpful and constructive advice and feedback from him.

I am very grateful to Herbert Meyr for serving as co-advisor and the second referee of my thesis. In addition, I would like to thank him for the inspiring suggestions he made while I was working on my PhD project and his highly constructive and detailed feedback that helped to improve the content and presentation of my dissertation.

I thank Alvaro da Costa for coincidentally inspiring the idea for my PhD research. Also, I would like to thank Robert Klein and Uwe Probst for sparking my interest in research.

Furthermore, I thank my colleagues Gabriela Mayer, Anita Petrick, and Daniel Scholz at the chair of Operations Research, our teaching and research assistants, as well as all colleagues on our floor for creating a warm, familial atmosphere and enjoyable research environment. Especially, I would like to thank Florian Seeanner, Daniel Scholz, and Thomas Widjaja for fruitful discussions and feedback. I am particularly grateful for the ideas, contributions and feedback of Ole-Björn Baasch, Anna-Lena Beutel, Saskia Kauder, Baptiste Lebreton, Raissa Sachs, Florian Sahling, Björn-Ragnar Weber, and Wolfram Wiesemann. In addition, I would like to thank the entire QBWL research community for providing plenty of challenging and inspiring feedback.

I am further indebted to Kai Sachs and the other members of the TUD-Aktiv party – our joint dedication to academic politics was a welcome change in day-to-day research and indeed a valuable experience.

Yet there is a life besides academia. I am thankful to my parents for providing the educational background that enabled me to write this dissertation. Also, I am deeply grateful to my brother Benjamin for the many animating discussions and
joyful times. Finally, I would like to thank my parents, my brother, and my friends for enriching my life throughout times of joy and pain during this project.

Thank you all.

Darmstadt
July 2009

Jan Christian Lang
# Graphical Modeling of Substitutions and Flexible Bills-of-Materials

## Applications

## Modeling Approaches

### 3.2.1 Blending Models

### 3.2.2 Substitution Graphs

### 3.2.3 Substitution Hypergraphs

### 3.2.4 Task-Oriented Modeling

### 3.2.5 Comparison of Modeling Approaches

## Model Classification Criteria

### 3.3.1 Demand

### 3.3.2 Model Context

### 3.3.3 Substitution Characteristics

### 3.3.4 Conversions

### 3.3.5 Production

## Implementing Product Substitution

# Literature Review

## Assortment Problems

## Lot-Sizing with Substitutions

### 4.2.1 The Requirements Planning Problem with Substitutions

### 4.2.2 Substitution with/without Conversion Problems

### 4.2.3 The Multi-level Lot-Sizing Problem with Flexible Production Sequences

## Related Aspects

# Efficient Reformulations for Uncapacitated and Capacitated Lot-Sizing with Substitutions and Initial Inventories

## Introduction

## Outline

## The Lot-Sizing Problem with Substitution and Initial Inventory

### 5.3.1 Model Formulation

### 5.3.2 Facility Location Based Reformulation

### 5.3.3 Valid Inequalities

## The Multi-Resource Capacitated Lot-Sizing Problem with Substitution

## Computational Experiments

### 5.5.1 Problem Instances

### 5.5.2 Solution Approaches

### 5.5.3 Experimental Designs, Results and Interpretation

## Conclusions

## Transformation of LSP-SI into a CFLP

## Proof that LSP-SI is NP-Hard

## Proof for Product Substitution (1,S)-Cuts
6  MIP-Based Heuristics for Capacitated Lot-Sizing with Sequence-Dependent Setups and Substitutions .......................................................... 151
   6.1 Introduction ........................................................................ 151
   6.2 Outline ............................................................................ 152
   6.3 The Capacitated Lot-Sizing Problem with Sequence-Dependent Setups and Substitutions .......... 153
      6.3.1 Assumptions .......................................................... 153
      6.3.2 Formulation .......................................................... 154
   6.4 Determining Efficient or Good Sequences .............................. 157
   6.5 MIP-Based Heuristics ...................................................... 158
      6.5.1 Subproblems with Relaxed or Fixed Binary Variables ..... 158
      6.5.2 Decompositions ................................................... 160
      6.5.3 Relax&Fix ........................................................ 165
      6.5.4 Fix&Optimize .................................................... 166
      6.5.5 Two-Stage Relax&Fix/Optimize Algorithm .......... 169
      6.5.6 Time Limit and Stopping Criterion for Subproblems ...... 169
   6.6 Computational Experiments.............................................. 170
      6.6.1 Problem Instances ................................................ 170
      6.6.2 Experimental Design ............................................. 175
      6.6.3 Solution Approaches ............................................. 175
      6.6.4 Results and Interpretation ........................................ 177
   6.7 Summary and Conclusions ............................................... 182

7  Multi-Level Lot-Sizing Models with Flexible Bills-of-Materials ........ 185
   7.1 The Multi-Level Capacitated Lot-Sizing Problem with Substitutions ................................................. 185
      7.1.1 Assumptions .......................................................... 186
      7.1.2 Formulation .......................................................... 187
      7.1.3 Example of Substitution Hypergraph ....................... 189
      7.1.4 Transformation into Special Case of MLFP .......... 190
      7.1.5 Echelon Stocks and Flexible BOMs ............................. 191
   7.2 The General Lot-Sizing and Scheduling Problem for Multiple Production Stages with Flexible Production Sequences ........................................... 191
      7.2.1 Assumptions .......................................................... 191
      7.2.2 Formulation .......................................................... 193
      7.2.3 Ensuring Temporal Feasibility within Micro-Periods .......... 197
      7.2.4 Transformation of GLSPMS into Special Case of GLSPMS-FPS ...................................................... 203

8  Blood Inventory Control with Transshipments and Substitutions ...... 205
   8.1 Introduction ........................................................................ 205
      8.1.1 Analogy Between Transshipments and Substitutions ..... 205
      8.1.2 Outline ............................................................... 207
   8.2 Combining Transshipments and Substitutions ...................... 207
   8.3 Blood Bank Inventory Control ........................................... 208
8.4 Blood Bank Simulation Model ............................................ 211
  8.4.1 Assumptions ...................................................... 211
  8.4.2 Replenishment, Substitution and Transshipment Policies .... 212
8.5 Simulation-Based Optimization Approach .............................. 214
  8.5.1 Optimization Problem ............................................ 214
  8.5.2 Pattern Search Algorithm ......................................... 215
  8.5.3 Adaptation of PS Algorithm ................................. 217
8.6 Computational Experiments .............................................. 218
  8.6.1 Setup .............................................................. 218
  8.6.2 Experiment Designs, Results and Interpretation ............ 220
8.7 Conclusions ............................................................. 225
8.8 Limitations ............................................................... 225

9 Conclusions and Future Research ......................................... 227
  9.1 Conclusions ........................................................... 227
  9.2 Future Research ....................................................... 229
    9.2.1 Multi-Location Inventory Control with
          Transshipments and Substitutions .................. 229
    9.2.2 Production Planning with Substitution and
          Flexible BOMs ............................................... 230
    9.2.3 Substitutions and Flexible BOMs in Advanced
          Planning Software ............................................. 231

Appendix A Additional Related Literature ............................ 232

Bibliography ........................................................................ 237

Index .................................................................................. 255
List of Figures

1.1 Substitution and other flexibility instruments – classification ............................................................... 3

2.1 Supply Chain Planning (SCP) matrix (Fleischmann et al., 2005, p. 87) ............................................................... 10

2.2 Idealized APS software module architecture covering the SCP matrix (Meyr et al., 2005b, p. 109) ............................................................... 10

2.3 Classification criteria for production planning models – 1/4 ............................................................... 12

2.4 Classification criteria for production planning models – 2/4 ............................................................... 13

2.5 Classification criteria for production planning models – 3/4 ............................................................... 14

2.6 Classification criteria for production planning models – 4/4 ............................................................... 15

2.7 Example – stages, resources, tasks, and products in a flow production system ............................................................... 18

2.8 Example – resource sequences in a job-shop production environment ............................................................... 18

2.9 Interrelation of flexible BOMs and flexible production sequences ............................................................... 20

2.10 Example – serial resource structure vs. parallel machines ............................................................... 20

2.11 Example of a two-level time structure ............................................................... 27

2.12 Example showing necessity of (2.60), adapted from Meyr (2004a) ............................................................... 49

2.13 Example showing necessity of (2.61) adapted from Meyr (2004a) ............................................................... 51

2.14 Possible combinations of exact and heuristic algorithms (Puchinger and Raidl, 2005) ............................................................... 57

2.15 Example – Relax&Fix ............................................................... 58

2.16 Example – Relax&Fix with overlapping time windows ............................................................... 59

2.17 Example – Fix&Optimize with product-oriented decomposition ............................................................... 61

2.18 Example of a two-echelon transshipment network ............................................................... 63

2.19 Idealized SBO system architecture ............................................................... 69

2.20 Effect of objective function “noise” in SBO ............................................................... 74
List of Figures

2.21 Example of pattern search steps for an SBO problem with two variables ............................................................... 78

3.1 Substitution structures ................................................................. 86
3.2 Example – substitution graphs with demand classes and merging substitution graphs for two customers ....................... 87
3.3 Transitivity of substitution options ................................................ 88
3.4 Flattening transitive substitutability ........................................... 88
3.5 Substitution hypergraph vs. AND-XOR graph representation for multi-level models .............................................. 90
3.6 Example of a substitution hypergraph for a multi-level model ....... 91
3.7 Substitution of components only ................................................ 92
3.8 Interacting substitutions ............................................................. 93
3.9 Disassembly in substitution models ............................................. 94
3.10 STN task node with state nodes belonging to its input and output products ............................................................... 95
3.11 STN example with co-products ................................................ 95
3.12 STN example with product substitution ..................................... 95
3.13 RTN example ....................................................................... 96
3.14 RTN example with product substitution and flexible resource assignments .......................................................... 97
3.15 Classification criteria – substitutions and flexible BOMs/recipes – 1/3 ....................................................... 99
3.16 Classification criteria – substitutions and flexible BOMs/recipes – 2/3 ....................................................... 100
3.17 Classification criteria – substitutions and flexible BOMs/recipes – 3/3 ....................................................... 101
3.18 Partial vs. exclusive substitution ............................................ 103
3.19 Time of conversion ............................................................... 105

5.1 Example for the transformation of the LSP-SI into a CFLP ......... 148

6.1 Example – considering substitutions when choosing production sequences .......................................................... 152
6.2 Examples of possible “moves” in F&O using the different time-oriented decompositions ............................................. 161
6.3 Examples of possible “moves” in F&O using the different product-oriented decompositions ............................................. 163

7.1 Example of an AND-XOR substitution graph for the MLCLSP-S ............................................................... 190
7.2 Examples of possible STNs in GLSPMS-FPS .............................. 198
7.3 STN example where i is an indirect predecessor of k ............... 198
7.4 Illustration of recursion (7.38) for determining $g_{i,k}^{\text{min}}$ values ....... 200
7.5 Example showing necessity of (7.27) ..................................... 201
List of Figures

7.6  STN example where tasks $a$ and $b$ are directly linked by a product $i = k$ .............................................................. 202
7.7  Example showing necessity of (7.28) .............................................. 202

8.1  Combining substitutions and transshipments ................................. 208
8.2  Red blood cell product compatibility – transitive substitution graph (including blood group distribution in the USA) ......................................................... 209
8.3  The blood supply chain ................................................................. 210
8.4  SBO algorithm progress ............................................................... 222
## List of Tables

2.1 Notations for WWP ........................................................ 34  
2.2 Notations for CLSP ........................................................ 36  
2.3 Notations for CLSD ........................................................ 38  
2.4 Notations for GLSP ........................................................ 41  
2.5 Notations for MLCLSP .................................................... 43  
2.6 Notations for GLSPMS .................................................... 46  
2.7 Additional notations for SPL-based reformulation of CLSP ............. 56  

3.1 Comparison of substitution modeling approaches ........................ 97  

4.1 Notations for RPS .......................................................... 114  
4.2 Notations for MLFP ........................................................ 119  

5.1 Notations for LSP-SI model ............................................... 127  
5.2 Additional notations for SPL-based reformulation of LSP-SI ............ 129  
5.3 Notations for MR-CLSP-S ................................................. 132  
5.4 Instance generator settings ................................................. 135  
5.5 Median running times on LSP-SI instances in seconds (experiment 1) .............................................................. 137  
5.6 Percentage of LSP-SI instances solved within 10 min (experiment 1) ... 138  
5.7 Median running times on large general substitution LSP-SI instances in seconds (experiment 2) ................................................. 139  
5.8 Percentage of larger general substitution LSP-SI instances solved within 10 min (experiment 2) ................................. 140  
5.9 Median running times on MR-CLSP-S instances with lost sales / overtime in seconds (experiment 3) ................................. 141  
5.10 Percentage of MR-CLSP-S instances with lost sales / overtime solved within 10 min (experiment 3) ......................... 141  
5.11 Median running times on MR-CLSP-S instances with different capacity availability levels in seconds (experiment 4) ........... 142  
5.12 Percentage of MR-CLSP-S instances with different capacity availability levels solved within 10 min (experiment 4) ........... 143
5.13 Median running times on MR-CLSP-S instances with different numbers of resources in seconds (experiment 5) ............... 144
5.14 Percentage of MR-CLSP-S instances with different numbers of resources solved within 10 min (experiment 5) ......................... 144
5.15 Effect of MIP solver cut generation on tightness of lower bounds and running times of original and SPL formulation, examined on two instances with \( n_r = 1 \) and 3 ......................... 145
5.16 Notations for LSP-SI \( \rightarrow \) CFLP transformation ......................... 146
5.17 Analogies between LSP-SI and CFLP elements in the transformation .............................................................. 147

6.1 Notations for CLSD-S.................................................................. 155
6.2 Notations for CLSD-S subproblems ................................................ 159
6.3 Notations for CLSD-S instance generator ........................................ 172
6.4 Instance generator settings .......................................................... 173
6.5 Assumptions for distributions and characteristics of product features .................................................................................. 173
6.6 Algorithm variants (1/2) ................................................................. 176
6.7 Algorithm variants (2/2) ................................................................. 177
6.8 Notations for computational results .................................................... 178
6.9 Computational results – experiment 1 (1/2) ........................................ 179
6.10 Computational results – experiment 1 (2/2) ........................................ 180

7.1 Notations for MLCLSP-S ................................................................ 188
7.2 Notations for GLSPMS-FPS .......................................................... 193
7.2 (continued) .............................................................................. 194

8.1 Benefits of transshipments and substitutions ........................................ 206
8.2 Preventive vs. reactive transshipments and substitutions ......................... 206
8.3 Analogies between transshipment and substitution model entities .............. 206
8.4 Notations for simulation model ........................................................ 213
8.5 Notations for PS algorithm .............................................................. 216
8.6 Blood types preference order .......................................................... 219
8.7 Computational results – experiment 1 (simultaneous search pattern, order-up-to and critical levels) ........................................ 221
8.8 Computational results – experiment 2 (T&S) ........................................ 223
8.9 Computational results – SBO solution for configuration 8 (% change of average value of days of supply compared to initial solution) .............................................................. 224

A.1 Literature on aspects related to product substitution ......................... 233
### Acronyms

<table>
<thead>
<tr>
<th>Acronym</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSP-SI</td>
<td>Lot-sizing problem with substitution and initial inventory</td>
</tr>
<tr>
<td>BOM</td>
<td>Bill-of-materials</td>
</tr>
<tr>
<td>WWP</td>
<td>Wagner–Whitin problem</td>
</tr>
<tr>
<td>CLSP</td>
<td>Capacitated lot-sizing problem</td>
</tr>
<tr>
<td>CLSD</td>
<td>Capacitated lot-sizing problem with sequence-dependent setups</td>
</tr>
<tr>
<td>GLSP</td>
<td>General lot-sizing and scheduling problem</td>
</tr>
<tr>
<td>MLCLSP</td>
<td>Multi-level capacitated lot-sizing problem</td>
</tr>
<tr>
<td>GLSPMS</td>
<td>General lot-sizing and scheduling problem for multiple production stages</td>
</tr>
<tr>
<td>CLSP-S</td>
<td>Capacitated lot-sizing problem with substitution</td>
</tr>
<tr>
<td>CLSD-S</td>
<td>Capacitated lot-sizing problem with sequence-dependent setups and substitutions</td>
</tr>
<tr>
<td>RPS</td>
<td>Requirements planning problem with substitutions</td>
</tr>
<tr>
<td>MR-CLSP-S</td>
<td>Multi-resource capacitated lot-sizing problem with substitution</td>
</tr>
<tr>
<td>MILP</td>
<td>Mixed-integer linear programming</td>
</tr>
<tr>
<td>MIP</td>
<td>Mixed-integer programming</td>
</tr>
<tr>
<td>APS</td>
<td>Advanced planning system</td>
</tr>
<tr>
<td>SCP</td>
<td>Supply chain planning</td>
</tr>
<tr>
<td>ATP</td>
<td>Available-to-promise</td>
</tr>
<tr>
<td>CTP</td>
<td>Capable-to-promise</td>
</tr>
<tr>
<td>MTS</td>
<td>Make-to-stock</td>
</tr>
<tr>
<td>MTO</td>
<td>Make-to-order</td>
</tr>
<tr>
<td>STB</td>
<td>Small time bucket</td>
</tr>
<tr>
<td>LTB</td>
<td>Large time bucket</td>
</tr>
<tr>
<td>SP</td>
<td>Stochastic programming</td>
</tr>
<tr>
<td>RO</td>
<td>Robust optimization</td>
</tr>
<tr>
<td>DCF</td>
<td>Discounted cash flow</td>
</tr>
<tr>
<td>CLSPL</td>
<td>Capacitated lot-sizing problem with linked lot-sizes</td>
</tr>
<tr>
<td>SBO</td>
<td>Simulation-based optimization</td>
</tr>
</tbody>
</table>
NPV  Net present value
GPS  Generalized pattern search
RSM  Response surface methodology
GA  Genetic algorithm
PS  Pattern search
DS  Direct search
TS  Tabu search
MCNFP  Minimum cost network flow problem
CPU  Central processing unit
GHz  Gigahertz
CWH  Car wiring harness
SOA  Service-oriented architecture
STN  State-task network
RTN  Resource-task network
SPL  Simple plant location
ECU  Electronic control unit
WLP  Warehouse location problem
ISM  Integrated steel manufacturer
SWCP  Substitution with conversion problem
MLFP  Multi-level lot-sizing problem with flexible production sequences
EOQ  Economic-order-quantity
SWOP  Substitution without conversion problem
ZIP  Zero inventory-production
HPL  Homogeneous product lots
MRU  Most recent usage
IU  Immediate usage
MLCLSP-S  Multi-level capacitated lot-sizing problem with substitutions
GLSPMS-FPS  General lot-sizing and scheduling problem for multiple production stages with flexible production sequences
VMI  Vendor managed inventory
RBC  Regional blood center
PPM  Production process model
SNO  (Oracle®) Strategic Network Optimization
APO  (SAP®) Advanced Planner and Optimizer
ATO  Assemble-to-order
CRN  Common random numbers