Fred Kröger · Stephan Merz

Temporal Logic and State Systems

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Preface

Following the Stanford Encyclopedia of Philosophy,

“the term temporal logic has been broadly used to cover all approaches to the representation of temporal information within a logical framework”.

Applications of temporal logic include philosophical issues about time, the semantics of tenses in natural languages, and its use as a formal framework for the treatment of behavioural aspects of computerized systems.

In a more narrow sense, temporal logic is understood as a modal-logic type of approach: temporal relationships between different assertions are expressed by applying particular temporal logic operators to them. This book focuses on this type of temporal logic and we will study computer science applications to what we call state systems: systems which involve “states” and exhibit “behaviours” by “running” through sequences of such states.

One of the most challenging problems facing today’s software engineers and computer scientists is to find ways and establish techniques to reduce the number of errors in the systems they build. It is widely acknowledged that formal methods may contribute to solving this challenge with significant success. In particular, temporal logic is a well-established and successfully used formal tool for the specification and verification of state systems. Its formulas are interpreted over “runs” of such systems and can thus express their behavioural properties. The means induced by the (semantical and deductive) logical apparatus provide methods to formally prove such properties.

This monograph is written in the tradition of the first author’s textbook [83] Temporal Logic of Programs

and the two volumes

*The Temporal Logic of Reactive and Concurrent Systems – Specification* and
*The Temporal Logic of Reactive and Concurrent Systems – Safety*

of Manna and Pnueli [102, 104]. This means that we will present the “mathematics” of temporal logic in considerable detail and we will then systematically study
specification and verification methods, which will be illustrated by fully elaborated examples.

Compared with those books, however, the topics and their presentation are rearranged and we have included significant new material and approaches. In particular, branching time logics, expressiveness issues of temporal logic, aspects related to Lamport’s *Temporal Logic of Actions* (TLA), and model checking methods are additionally presented.

There is a wealth of relevant and interesting material in the field. The “main text” of this book presents topics that – in our opinion – constitute a “canonical” exposition of the field. In additional *Second Reading* paragraphs we have occasionally inserted short “excursions” that expand on related or advanced themes or that present interesting complements. These paragraphs can be skipped without loss of continuity in the main presentation.

The first chapter of this book gives a short overview of basic concepts and notions of (mathematical) logic. This is not only to introduce the reader not familiar with logic into that world, it also defines basic terminology and notation that we use throughout the remaining text.

Chapters 2–5 and 10 form the purely logical part of the book. Even when restricted to the modal-logic type as mentioned above, there are many different versions and variants of temporal logic. We start in Chap. 2 with the basic propositional linear temporal logic and study in Chap. 3 some important propositional extensions. It should be mentioned that even the borderline between temporal logic(s) and modal logics is not really well defined. Some relationships concerning this are briefly discussed in Second Reading paragraphs.

Chapter 4 is devoted to the expressiveness of propositional linear temporal logics. In particular, the logics are compared with other description formalisms: classical predicate logic and \( \omega \)-automata.

Chapter 5 introduces first-order linear temporal logic together with some additional useful extensions. Chapter 10 discusses some other temporal logics and, particularly, introduces branching time logics.

The remaining Chaps. 6–9 and 11 deal with applications of temporal logics to state systems. Various versions of *transition systems* – as formal representations of such systems – are introduced in Chap. 6, and Chap. 7 gives a general systematic presentation of (deductive) temporal logic verification methods for them. Chapter 8 applies the methods to the special (“classical”) case of the verification of concurrent programs.

Chapter 9 addresses aspects that arise when system specifications are “structured”. Particularly, the refinement of specifications is considered and we study how this can be described in the logic TLA.

The “semantical” model checking approach to system verification offers an alternative to deductive methods. It has attracted much interest, largely because it can be fully automated in a way that scales to systems of interesting complexity. Chapter 11 presents the essential concepts and techniques underlying this approach.
Every chapter ends with some bibliographical notes referring to the relevant literature. After the last chapter we include an extensive list of formal laws of the various temporal logics studied in this book.

We have used drafts of this book as supports for courses at the advanced undergraduate and the graduate level. Different selections of the material are possible, depending on the audience and the orientation of the course. The book is also intended as an introduction and reference for scientists and practicing software engineers who want to familiarize themselves with the field. We have aimed to make the presentation as self-contained as possible.

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Fred Kröger
Stephan Merz
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