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Executive Director: K. Cass
51, Boulevard de Montmorency
75016 Paris, France

E-mail: codata@dial.oleane.com
http://www.codata.org
M. Desaintfuscien

Data Processing in Precise Time and Frequency Applications

With 66 Figures and 15 Tables

Springer
The image on the front cover comes from an animation which shows worldwide Internet traffic. The color and height of the arcs between the countries encode the data-packet counts and destinations, while the “skyscraper” glyphs (or icons) encode total traffic volume at any site. This image was generated by Stephen G. Eick at the AT&T Bell Laboratories.
to my mother
CODATA is an interdisciplinary Scientific Committee of the International Council for Science (ICSU). The mission of CODATA is to strengthen international science for the benefit of society by promoting improved scientific and technical data management and use. It works to improve the quality, reliability, management and accessibility of data of importance to all fields of science and technology. CODATA is a resource that provides scientists and engineers with access to international data activities for increased awareness, direct cooperation and new knowledge. CODATA, established over 40 years ago by ICSU, promotes and encourages on a world-wide basis the compilation, evaluation and dissemination of reliable numerical data of importance to science and technology. This includes data initiatives and modeling of an interdisciplinary nature such as that encountered in far-reaching projects e.g. Global Change, various Genome projects, environmental and biodiversity issues, etc. Today 23 countries are members, 15 International Scientific Unions have assigned liaison delegates, there are 4 co-opted delegates and 20 supporting organizations from industry, government and academia.

CODATA is concerned with all types of data resulting from experimental measurements, observations and calculations in every field of science and technology, including the physical sciences, biology, geology, astronomy, engineering, environmental science, ecology and others. Particular emphasis is given to data management problems common to different disciplines and to data used outside the field in which they were generated.

CODATA’s primary purpose in launching the series “Data and Knowledge in a changing World” is to collect data and the wealth of information pertaining to the intelligent exploitation of data in the conduct of science and technology and to make these data and information available to a multidisciplinary community. This series in support of that goal provides a forum made up of many contributions which can be theoretical treatments, compilations or applied outlines. This includes computer related handling and visualization of data to the major scientific and technical fields.

To this end, the series on Data and Knowledge is open to contributions of various kinds, in particular:

- Fostering the improvement of the accessibility and quality of quantitative and qualitative data;
VIII  Introduction to the Series

- Treating classical and ground breaking methods by which numeric and symbolic data are acquired, analyzed and managed;
- Presenting new data and knowledge interfaces designed to optimize interoperability and thereby increase the potential for sharing data among databases and networks;
- Promoting international cooperation in communication and data sharing. This includes works dealing with standardization, data quality agreements and conceptual data descriptions (metadata, syntactic and semantic approaches) along with papers dealing with the evolution of internet based facilities, other forms of worldwide communications and electronic publishing;
- Providing new insights into, or interpretations of, processes leading to creative design in the field of concurrent and/or cooperative engineering, including cognitive aspects critical to data based decision making.

In the evolving information world we live in, where the traditional ways of transferring information as an essential resource are rapidly changing, this Series aims to identify emerging and innovative concepts for data compilation, handling, management, and evaluation. Its ambition is to be a catalyst for change while simultaneously nurturing a thought-provoking forum.
The main originality of this book lies in its presentation of an in-depth description of the metrological characterization of very stable frequency sources, such as atomic clocks, as well as the analysis of the principle of their most demanding applications, such as navigation, positioning and very long baseline interferometry.

All these cited fields of interest rely on the measurement of time intervals that necessarily give numerical data: one counts the number of time units occurring between the beginning and the end of an event. Consequently, the analysis of the metrological characterization of stable frequency sources and of the operation of the related scientific and technical applications rely on the treatment of numerical data that can be affected by random and systematic perturbations.

The author presents a rigorous, detailed and unified analysis of the specific signal processing of numerical data arising in that field. Although the subject might seem to be difficult at first sight, the pedagogical talents of Professor Desaintfuscin help one to easily follow the mathematical derivations and the discussion of their results.

Therefore, this book appears as a reference document for all the scientists and engineers involved in the design or daily use of the related equipment.

Claude Audoin
Directeur de recherche émérite,
Silver Medal (French National Research Center),
Rabi Award (Scientific Committee Annual Frequency Control Symposium)

Paris, January 2007
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