Part III
Intuitive Thinking and Visualization

Michael Neubrand

Introduction
Besides the concept of functional thinking, the idea of Anschauung is surely the other central aspect in the Legacy of Felix Klein for mathematics education. Anschauung—a term quite hard to translate—embraces many facets. It origins from Felix Klein’s basic thinking: The mathematics teacher should make things “anschaulich erfassbar”, i.e. intuitively comprehensible. Drawings, pictures, models, experiments, dynamic representations of any kind, etc., are among the various possibilities for intuitive thinking and visualization. In this Strand, five authors display the ideas of Felix Klein. The contributions start from the roots in Felix Klein’s work and the influences of these ideas, both in the national, and in the international context. Then, they move as far as to confront Felix Klein’s ideas to the recent possibilities of modern technological tools and dynamic geometry systems.

Martin Mattheis (Germany) reveals in his conceptual contribution the deeper intentions of Felix Klein behind his central term “Anschauung”. He sticks to different aspects, like sensate, idealizing, and abstract intuition and illustrates how Felix Klein dealt with intuition in the fields of numbers, functions, geometry, and spatial intuition.

However, Felix Klein was not alone in distributing ideas of intuition and visualization for the teaching of mathematics. Ysette Weiss (Germany) shows how Peter Treutlein, a German contemporary of Klein, used models in his teaching. Thoroughly esteemed by Felix Klein, Treutlein employed activities like paper folding or the construction of models to develop space intuition and to teach modern approaches to geometry.

Felix Klein had notable influence also in the international context. Masami Isoda (Japan) shows that the road how Felix Klein’s ideas came into the Japanese teaching was through the so-called Praktische Analysis. Even older roots which were also seen by Felix Klein play a role. Thinking in graphs and considering mechanical devices to foster geometric and functional intuition find their way in the Japanese mathematics textbooks.
Modern technology, however, brings new life into the area of intuition, and we should assume that Felix Klein would appreciate these new possibilities. Stefan Halverscheid and Oliver Labs (Germany) exhibit a lot of opportunities how technology can stimulate the interplay between abstraction and visualization. Their examples connect mathematical considerations about the surfaces of cubic and quartic polynomial functions (over the complex numbers) with the real production of models via 3-D-printers. Thus, the famous historical Göttingen collection of mathematical models becomes now vivid by the modern technology tools.

Maria Flavia Mammana (Italy), finally, shows how Felix Klein’s Meran Curriculum of 1905 can still be applied to the teaching of geometry in Grades 10–11 today. The intuitive approach to geometry is now facilitated using modern information technology. She presents activities with dynamic geometry software to intuitively set out geometric concepts between plane figures and spatial geometry.