Part II

Temporal White Noise
The standard approach to constructing SPDE solvers starts with a space discretization of a SPDE, for which spectral methods (see, e.g., [78, 167, 249]), finite element methods (see, e.g., [6, 152, 491]) or spatial finite differences (see, e.g., [6, 189, 420, 495]) can be used. The result of such a space discretization is a large system of ordinary stochastic differential equations (SDEs), which requires time discretization to complete a numerical algorithm. In [101, 109] the SPDE is first discretized in time and then to this semi-discretization a finite-element or finite-difference method can be applied. Other numerical approaches include those making use of splitting techniques [33, 191, 293], quantization [161], or an approach based on the averaging-over-characteristic formula [361, 396]. In [315, 344] numerical algorithms based on the Wiener chaos expansion (WCE) were introduced for solving the nonlinear filtering problem for hidden Markov models. Since then, the WCE-based numerical methods have been successfully developed in a number of directions (see, e.g., [225, 489]).

In Part II of the book, we consider deterministic integration methods in random space for stochastic partial differential equations. In Chapter 6, we discuss Wiener chaos methods (WCE) and a multi-stage WCE for long time integration for linear advection-diffusion-reaction equations with multiplicative noise. In Chapter 7, we discuss stochastic collocation methods (precisely, sparse grid collocation methods) for linear parabolic equations with multiplicative noise. Subsequently, we compare the two methods for these linear equations in Chapter 8 while in Chapter 9 we apply stochastic collocation methods discussed in Chapters 7 and 8 to nonlinear equations, namely stochastic Euler equations for the one-dimensional piston problem.