Springer Series in Optical Sciences

Volume 111

Founded by
H.K.V. Lotsch

Editor-in-Chief
William T. Rhodes, Georgia Institute of Technology, Atlanta, USA

Editorial Board
Ali Adibi, Georgia Institute of Technology, Atlanta, USA
Theodor W. Hänsch, Max-Planck-Institut für Quantenoptik, Garching, Germany
Ferenc Krausz, Ludwig-Maximilians-Universität München, Garching, Germany
Barry R. Masters, Cambridge, USA
Katsumi Midorikawa, Saitama, Japan
Herbert Venghaus, Fraunhofer Institut für Nachrichtentechnik, Berlin, Germany
Horst Weber, Technische Universität Berlin, Berlin, Germany
Harald Weinfurter, Ludwig-Maximilians-Universität München, Munchen, Germany
The Springer Series in Optical Sciences, under the leadership of Editor-in-Chief William T. Rhodes, Georgia Institute of Technology, USA, provides an expanding selection of research monographs in all major areas of optics: lasers and quantum optics, ultrafast phenomena, optical spectroscopy techniques, optoelectronics, quantum information, information optics, applied laser technology, industrial applications, and other topics of contemporary interest.

With this broad coverage of topics, the series is of use to all research scientists and engineers who need up-to-date reference books.

The editors encourage prospective authors to correspond with them in advance of submitting a manuscript. Submission of manuscripts should be made to the Editor-in-Chief or one of the Editors. See also www.springer.com/series/624

**Editor-in-Chief**

William T. Rhodes  
School of Electrical and Computer Engineering  
Georgia Institute of Technology  
Atlanta, GA 30332-0250  
USA  
e-mail: bill.rhodes@ece.gatech.edu

**Editorial Board**

Ali Adibi  
School of Electrical and Computer Engineering  
Georgia Institute of Technology  
Atlanta, GA 30332-0250  
USA  
e-mail: adibi@ee.gatech.edu

Theodor W. Hänsch  
Max-Planck-Institut für Quantenoptik  
Hans-Kopfermann-Straße 1  
85748 Garching, Germany  
e-mail: t.w.haensch@physik.uni-muenchen.de

Ferenc Krausz  
Ludwig-Maximilians-Universität München  
Lehrstuhl für Experimentelle Physik  
Am Coulombwall 1  
85748 Garching, Germany and  
Max-Planck-Institut für Quantenoptik  
Hans-Kopfermann-Straße 1  
85748 Garching, Germany  
e-mail: ferenc.krausz@mpq.mpg.de

Barry R. Masters  
Cambridge  
USA

Katsumi Midorikawa  
Saitama  
Japan

Herbert Venghaus  
Fraunhofer Institut für Nachrichtentechnik  
Heinrich-Hertz-Institut  
Einsteinufer 37  
10587 Berlin, Germany  
e-mail: venghaus@hhi.de

Horst Weber  
Optisches Institut  
Technische Universität Berlin  
Straße des 17. Juni 135  
10623 Berlin, Germany  
e-mail: weber@physik.tu-berlin.de

Harald Weinfurter  
Sektion Physik  
Ludwig-Maximilians-Universität München  
Schellingstraße 4/III  
80799 München, Germany  
e-mail: harald.weinfurter@physik.uni-muenchen.de

More information about this series at http://www.springer.com/series/624
Preface to the Fourth Edition

In the fourth edition, I include further progress of the recent study of fundamental dynamics and applications in chaotic semiconductor lasers. Semiconductor lasers with extra device structures are essentially unstable lasers that exhibit chaos even without external perturbations as discussed in the previous editions. The new results for the stability and instability in such devices, e.g., vertical-cavity surface-emitting lasers, broad-area semiconductor lasers, quantum-dot semiconductor lasers, and quantum-cascade lasers, are treated. New developments are also achieved in the control and applications of instability and chaos in semiconductor lasers. For example, the method of self-mixing interferometry in quantum-cascade lasers treated in the current edition is indispensable in practical applications, since fast external photodetector is not available in those THz optical wavelength regions. One of the new topics in this edition is the consistency and synchronization property of many coupled semiconductor lasers in connection with the analogy of the dynamics in synaptic neurons. In particular, zero-lag synchronization is very important in nonlinear networks, since zero-lag synchronization between distant neurons plays a crucial role for the information processing in the human brain. The consistency and synchronization properties in chaotic networks consisting of many coupled semiconductor lasers are discussed as a new chapter in this edition. Also, as an application of the consistency and synchronization in chaotic semiconductor lasers, neuro-inspired information processing, which is called reservoir computing, is presented.

Shizuoka, Japan

Junji Ohtsubo
Preface to the Third Edition

After the publication of the second edition of this book in 2008, further significant advance has been made in chaos research in semiconductor lasers. One of the topics that is worth treating in this book is the method of ultrafast physical random number generations using chaotic semiconductor lasers, which is suitable for random key distributions in modern cryptographic applications. Based on the method, we can generate true physical random numbers that are hundred times or even thousand times faster than those in existing methods. In conjunction with the method, photonic integrated circuits for chaotic light generators have recently been developed. Thus, chaos, especially chaos in semiconductor lasers, is now not only an interesting issue from the viewpoint of fundamental research, but also an important tool for engineering applications. These topics are treated as a new chapter in this book. In parallel with these topics, great advance has been made for the study of the dynamics in various types of semiconductor lasers with new device structures. I have already treated the dynamics in vertical-cavity surface-emitting lasers and broad-area semiconductor lasers in the first and second editions. Further advance has been brought in the dynamics of these lasers and they have been added in the third edition. Other examples of newly developed lasers are quantum-dot and quantum-cascade semiconductor lasers and they show interesting dynamics. I also discuss these new topics in Chap. 8. At the same time, several subjects are appropriately revised and a number of misprints in the second edition have been corrected. For the second edition, I have received several advices and comments for the improvements of the book, although I would not mention each of them. I have taken some of them into account for the new edition. I would like to thank those persons.

Junji Ohtsubo
Chaos research in laser physics, especially in semiconductor lasers, has developed further even after completion of the first edition of this book in the late summer of 2004, and it is still growing rapidly. For example, various forms of chaotic dynamics have been applied in newly developed semiconductor lasers, such as in vertical-cavity surface-emitting semiconductor lasers and broad-area semiconductor lasers. Chaotic dynamics plays an important role in these new lasers, even for their solitary oscillations, and control of the dynamics is currently an important issue for practical applications. Another significant advance has been made in the area of chaotic optical secure communications. Chaotic secure communications using existing public optical communications links have been tested, and successful results have been obtained. In this second edition, I have filled in the gaps in the explanation of chaotic laser dynamics in the previous edition, and I have also added several important topics that have been developed recently. In particular, a new chapter on laser stabilizations has been added, and a number of misprints in the first edition have been corrected. I believe this book will be of interest not only to researchers in the field of laser chaos, but also to those working in nonlinear science and technology.

Hamamatsu
Spring 2007

Junji Ohtsubo
The aim of this book is the description of the state of the art of chaos research in semiconductor lasers and their applications, and the future perspective of this field. However, for the beginner, including graduates who intend to participate newly in this field, the book starts with an introduction and explanation of chaos in laser systems and the derivation of semiconductor laser rate equations assuming two-level systems. I discuss stabilities, instabilities, and various chaotic dynamics in semiconductor lasers induced by optical and optoelectronic feedback, optical injection, and injection current modulation. As optical feedback, the effects of the conventional reflector, the grating feedback mirror, and the phase-conjugate mirror are considered. Recent results both for theoretical and experimental investigations are presented. Instabilities and chaotic dynamics for novel laser structures (self-pulsating semiconductor lasers, vertical-cavity surface-emitting semiconductor lasers (VCSELs), broad-area semiconductor lasers, and semiconductor laser arrays) are also discussed not only for solitary operations but also in the presence of external perturbations.

As applications of semiconductor laser chaos, control and noise suppression of lasers based on chaos control algorithm are presented. Externally controlled lasers are also interesting for applications of new laser systems with high coherent light sources or tunable light sources. The self-mixing interferometer in semiconductor lasers is an attractive application based on dynamic properties using bistable states in optical feedback effects. I also discuss these subjects. As another application of chaos, several methods of data encryption into the chaotic carrier and its decryption are introduced for secure data transmissions and communications based on chaos synchronization in semiconductor laser systems. This book is focused on the dynamic characteristics of semiconductor lasers and their applications. Therefore, the detailed descriptions for materials and structures of semiconductor lasers are beyond the scope of this book. Of course, such characteristics are closely related to chaotic phenomena in semiconductor lasers. The interested reader is referred to the related books. For those who are interested in optics but not familiar with nonlinear systems and chaos, I have attached an appendix to describe the phenomena of chaotic dynamics and to accustom the reader to the common tools for chaos
analyses in nonlinear systems. Chaos research, especially in semiconductor laser systems, is still developing rapidly and is expected to produce fruitful results not only for the fundamental research of chaos but also for applications as dynamic engineering.

Chapters 1, 2, 3 and 4 are devoted to the basics and the introduction of laser chaos and chaotic dynamics in semiconductor lasers, so that readers who want to know what laser chaos is and how it behaves in semiconductor lasers can follow them. Chapters 5, 6, 7, 8, 9, 10, 11 and 12 discuss the topics of chaos in semiconductor lasers and readers may skip to each topic according to their interest. Expected readers of this book are as follows; first, I assume those researchers who have already been involved in this field to gain an overview of the state of the art of their research. The next group is the graduate students and researchers who intend to participate in this field. For them, I have derived and explained most equations in the text from first principles as far as possible. Those readers who are familiar with electromagnetic theory and have some fundamental knowledge of optics and lasers will be able to follow the book. Finally, this book is devoted to all other researchers and engineers who are interested in dynamics in nonlinear systems and laser instabilities and applications. Since the laser is a very excellent model of a nonlinear system that shows chaotic dynamics, I believe that this book will provide useful information for readers not only in the field of optics but also in other related areas. Moreover, I hope that the ideas and techniques discussed here will give rise to a new paradigm of nonlinear systems such as chaos engineering or dynamic engineering.

For the publication of this book, I am indebted to many people. Here, I will not be able to express thanks to all those people, but, at first, I would like to thank colleagues and some previous students in my laboratory, Drs. Yun Liu, Atsushi Murakami, Keizo Nakayama, Yoshio Takiguchi, Shuying Ye, Hong Yu, for their many discussions and support. I also extend my thanks to many other researchers at various institutions and universities who gave me fruitful discussions and advice. They are Prof. Wolfgang Elsäßer, Dr. Peter Davis, Dr. Ingo Fischer, Prof. Jia-Ming Liu, Dr. Cristina Masoller, Dr. Claudio Mirasso, Prof. Rajarshi Roy, Prof. Kevin Alan Shore, and Dr. Atsushi Uchida. I also owe thanks to many other people with whom I had useful discussions. Finally, I express sincere thanks to Prof. Toshimitsu Asakura who gave me the opportunity to write this book and also encouraged me in various stages of the research.

Hamamatsu
Junji Ohtsubo
April 2005
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.3</td>
<td>Derivation of Rate Equations</td>
<td>38</td>
</tr>
<tr>
<td>3.3.1</td>
<td>Gain at Laser Oscillation</td>
<td>38</td>
</tr>
<tr>
<td>3.3.2</td>
<td>Rate Equation for the Field</td>
<td>39</td>
</tr>
<tr>
<td>3.3.3</td>
<td>Linewidth Enhancement Factor</td>
<td>41</td>
</tr>
<tr>
<td>3.3.4</td>
<td>Laser Rate Equations</td>
<td>43</td>
</tr>
<tr>
<td>3.4</td>
<td>Optical Gain</td>
<td>47</td>
</tr>
<tr>
<td>3.5</td>
<td>Linear Stability Analysis and Relaxation Oscillation</td>
<td>51</td>
</tr>
<tr>
<td>3.5.1</td>
<td>Linear Stability Analysis</td>
<td>51</td>
</tr>
<tr>
<td>3.5.2</td>
<td>Relaxation Oscillation</td>
<td>53</td>
</tr>
<tr>
<td>3.6</td>
<td>Langevin Noises</td>
<td>56</td>
</tr>
<tr>
<td>3.6.1</td>
<td>Rate Equations Including Langevin Noises</td>
<td>56</td>
</tr>
<tr>
<td>3.6.2</td>
<td>Langevin Noises</td>
<td>56</td>
</tr>
<tr>
<td>3.6.3</td>
<td>Implementation of Noise Terms for Numerical Calculations</td>
<td>58</td>
</tr>
<tr>
<td>3.6.4</td>
<td>Noise Spectrum</td>
<td>60</td>
</tr>
<tr>
<td>3.6.5</td>
<td>Relative Intensity Noise</td>
<td>61</td>
</tr>
<tr>
<td>3.6.6</td>
<td>Phase Noise and Spectral Linewidth</td>
<td>62</td>
</tr>
<tr>
<td>3.7</td>
<td>Modulation Characteristics</td>
<td>65</td>
</tr>
<tr>
<td>3.7.1</td>
<td>Injection Current Modulation</td>
<td>65</td>
</tr>
<tr>
<td>3.7.2</td>
<td>Intensity Modulation Characteristics</td>
<td>67</td>
</tr>
<tr>
<td>3.7.3</td>
<td>Phase Modulation Characteristics</td>
<td>68</td>
</tr>
<tr>
<td>3.8</td>
<td>Waveguide Models of Semiconductor Lasers</td>
<td>70</td>
</tr>
<tr>
<td>3.8.1</td>
<td>Index- and Gain-Guided Structures</td>
<td>70</td>
</tr>
<tr>
<td>3.8.2</td>
<td>Waveguide Models</td>
<td>71</td>
</tr>
<tr>
<td>3.8.3</td>
<td>Spatial Modes of Gain- and Index-Guided Lasers</td>
<td>73</td>
</tr>
<tr>
<td>3.8.4</td>
<td>Effects of Spontaneous Emission in Gain- and Index-Guided Lasers</td>
<td>75</td>
</tr>
<tr>
<td>3.8.5</td>
<td>Laser Types</td>
<td>76</td>
</tr>
<tr>
<td>References</td>
<td></td>
<td>80</td>
</tr>
</tbody>
</table>

4 Theory of Optical Feedback in Semiconductor Lasers 83

4.1 Theory of Optical Feedback 83

4.1.1 Optical Feedback Effects and Classifications of Optical Feedback Phenomena 83

4.1.2 Theoretical Model 86

4.2 Linear Stability Analysis for Optical Feedback Systems 88

4.2.1 Steady-State Solutions 88

4.2.2 Linear Stability Analysis 91

4.2.3 Linear Mode, and Stability and Instability in Semiconductor Lasers 93

4.2.4 Gain Reduction Due to Optical Feedback 95

4.2.5 Linewidth in the Presence of Optical Feedback 96
5.6.3 Dynamics in the Presence of Frequency Detuning ........................................ 161
5.6.4 Finite and Slow Response Phase-Conjugate Feedback ................................ 162
5.7 Dynamics of Incoherent Optical Feedback ................................................. 164
5.8 Dynamics of Polarization-Rotated Optical Feedback ............................... 167
  5.8.1 TE–TM Mode Dynamics in Polarization-Rotated Optical Feedback ........ 167
  5.8.2 Square-Wave Generation in Polarization-Rotated Optical Feedback ........ 170
5.9 Dynamics of Filtered Optical Feedback ...................................................... 172
  5.9.1 Filtered Optical Feedback ................................................................. 172
  5.9.2 External Cavity Modes ................................................................. 174
  5.9.3 Frequency Oscillations and Chaotic Dynamics ..................................... 176
References ........................................................................................................ 178

6 Dynamics in Semiconductor Lasers with Optical Injection ....................... 183
  6.1 Optical Injection .................................................................................... 183
    6.1.1 Optical Injection Locking .............................................................. 183
    6.1.2 Injection Locking Condition ......................................................... 186
  6.2 Stability and Instability in Optical Injection Systems ............................ 192
    6.2.1 Rate Equations ............................................................................. 192
    6.2.2 Chaotic Bifurcations by Optical Injection ..................................... 194
    6.2.3 Chaos Map in the Phase Space of Frequency Detuning and Injection ... 199
    6.2.4 Coexistence of Chaotic Attractors in Optically Injected Semiconductor Lasers ..................... 201
  6.3 Enhancement of Modulation Bandwidth and Generation of High Frequency Chaotic Oscillation by Strong Optical Injection ........................................................ 204
    6.3.1 Enhancement of Modulation Bandwidth by Strong Optical Injection ..................... 204
    6.3.2 Origin of Modulation Bandwidth Enhancement ............................. 210
    6.3.3 Visual Solution for Resonance Frequency ....................................... 214
    6.3.4 Modulation Response by Strong Optical Injection ......................... 217
    6.3.5 Suppression of Frequency Chirping by Strong Optical Injection .................. 218
    6.3.6 Generation of High-Frequency Chaotic Oscillation by Strong Optical Injection ..................... 220
References ........................................................................................................ 223
7 Dynamics of Semiconductor Lasers with Optoelectronic Feedback and Modulation .................... 227
  7.1 Theory of Optoelectronic Feedback .................................................. 227
    7.1.1 Optoelectronic Feedback Systems ............................................. 227
    7.1.2 Pulsation Oscillations in Optoelectronic Feedback Systems ................. 229
  7.2 Linear Stability Analysis for Optoelectronic Feedback Systems ......................... 232
    7.2.1 Linear Stability Analysis ...................................................... 232
    7.2.2 Characteristics of Semiconductor Lasers with Optoelectronic Feedback .......... 235
  7.3 Dynamics and Chaos in Semiconductor Lasers with Optoelectronic Feedback ................. 237
    7.3.1 Chaotic Dynamics in Negative Optoelectronic Feedback .......................... 237
    7.3.2 Chaotic Dynamics in Positive Optoelectronic Feedback .......................... 240
  7.4 Optoelectronic Feedback with Wavelength Filter ........................................ 244
    7.4.1 System of Optoelectronic Feedback with Wavelength Filter ...................... 244
    7.4.2 Dynamics of Optoelectronic Feedback with Wavelength Filter ..................... 246
  7.5 Chaotic Dynamics of Semiconductor Lasers Induced by Injection Current Modulation ........................ 248
    7.5.1 Instabilities of a Modulated Semiconductor Laser .................................. 248
    7.5.2 Linear Stability Analysis ...................................................... 250
    7.5.3 Chaotic Dynamics in Modulated Semiconductor Lasers .................................. 252
  7.6 Nonlinear Dynamics of Various Combinations of External Perturbations .................... 254
    7.6.1 Optically Injected Semiconductor Laser Subject to Optoelectronic Feedback ........ 254
    7.6.2 Semiconductor Lasers with Optical Feedback and Modulation ........................ 257
References .......................................................................................... 259

8 Instability and Chaos in Various Laser Structures .............................................. 263
  8.1 Multimode Lasers ............................................................................ 263
    8.1.1 Multimode Operation of Semiconductor Lasers ..................................... 263
    8.1.2 Theoretical Model of Multimode Lasers ........................................... 265
    8.1.3 Dynamics of Multimode Semiconductor Lasers with Optical Feedback ............. 268
8.2 Self-Pulsating Lasers ................................ 270
  8.2.1 Theory of Self-Pulsating Lasers ................ 270
  8.2.2 Instabilities at Solitary Oscillations .......... 273
  8.2.3 Instability and Chaos by Optical Feedback .... 277
  8.2.4 Instability and Chaos by Injection Current
       Modulation .................................. 280

8.3 Vertical-Cavity Surface-Emitting Lasers .............. 282
  8.3.1 Structure of Vertical-Cavity Surface-Emitting
       Lasers ..................................... 282
  8.3.2 Spatial-Mode Expansion Model .................. 284
  8.3.3 Spin-Flip Model ................................ 286
  8.3.4 Two-Gain Model ................................ 291
  8.3.5 Characteristics of VCSELs in Solitary
       Oscillations .................................. 293
  8.3.6 Spatio-Temporal Dynamics in VCSELs .......... 297
  8.3.7 Optical Feedback Effects in VCSELs .......... 300
  8.3.8 Square-Wave Generation in VCSELs ............. 304
  8.3.9 Short Optical Feedback in VCSELs ............. 305
  8.3.10 Optical Injection Dynamics in VCSEL ...... 307

8.4 Broad-Area Semiconductor Lasers ...................... 311
  8.4.1 Theoretical Model of Broad-Area Semiconductor
       Lasers ....................................... 311
  8.4.2 Dynamics of Broad-Area Semiconductor Lasers
       at Solitary Oscillations ..................... 316
  8.4.3 Optical Feedback Effects in Broad-Area
       Semiconductor Lasers ....................... 324
  8.4.4 Effects of Optical Injection in Broad-Area
       Semiconductor Lasers ....................... 327

8.5 Laser Arrays ....................................... 329

8.6 Quantum-Dot Semiconductor Lasers .................. 331
  8.6.1 Theory of Quantum-Dot Semiconductor Lasers ... 331
  8.6.2 Quantum-Dot Semiconductor Lasers in Solitary
       Oscillations .................................. 336
  8.6.3 Optical Feedback Effects in Quantum-Dot
       Semiconductor Lasers ....................... 338
  8.6.4 Optical Injection Effects in Quantum-Dot
       Semiconductor Lasers ....................... 339
  8.6.5 Bandwidth Enhancement in Optically Injected
       Quantum-Dot Semiconductor Lasers ............. 342

8.7 Quantum-Cascade Semiconductor Lasers ................. 344
  8.7.1 Principle of Quantum-Cascade Semiconductor
       Lasers ....................................... 344
  8.7.2 Linewidth Enhancement Factor
       of Quantum-Cascade Semiconductor Lasers .... 346
8.7.3 Rate Equations of Quantum Cascade Semiconductor Lasers .................. 348
8.7.4 Nonlinear Interactions in Quantum-Cascade Semiconductor Lasers ............ 350
8.7.5 Effects of Optical Feedback in Quantum-Cascade Semiconductor Lasers ........... 352
8.7.6 Enhancement of Modulation Bandwidth in Quantum-Cascade Semiconductor Lasers .......................... 353

References .................................................................................................................. 356

9 Chaos Control and Applications ................................................................. 363
9.1 General Methods of Chaos Control ......................................................... 363
9.1.1 OGY Method ......................................................... 363
9.1.2 Continuous Control Method ......................................................... 364
9.1.3 Occasional Proportional Method .................................................. 365
9.1.4 Sinusoidal Modulation Method .................................................. 366
9.2 Chaos Control in Semiconductor Lasers .................................................. 367
9.2.1 Continuous Control ......................................................... 367
9.2.2 Occasional Proportional Feedback Control .................................. 369
9.2.3 Sinusoidal Modulation Control .................................................. 370
9.2.4 Optical Control ..................................................................................... 372
9.3 Controlling Chaos and Noise Suppression ............................................... 376
9.3.1 Noise Suppression by Sinusoidal Modulation .................................. 376
9.3.2 Stability and Instability of LFFs by Injection Current Modulation .................. 380
9.3.3 Chaos Targeting ..................................................................................... 382

References .................................................................................................................. 383

10 Stabilization of Semiconductor Lasers .................................................. 385
10.1 Linewidth Narrowing by Optical Feedback ........................................... 385
10.1.1 Linewidth Narrowing by Strong Optical Feedback .................................. 385
10.1.2 Linewidth Narrowing by Grating Feedback .................................. 388
10.1.3 Linewidth Narrowing by Phase-Conjugate Optical Feedback .................. 389
10.1.4 Linewidth Narrowing by Resonant Optical Feedback .......................... 393
10.2 Linewidth Narrowing by Optoelectronic Feedback .................................. 396
10.3 Stabilization in Lasers with Various Structures ........................................... 398
10.3.1 Noise Suppression in Self-pulsation Semiconductor Laser .................. 398
10.3.2 Stabilization of VCSELs ........................................................................ 399
10.3.3 Stabilization of Broad-Area Semiconductor Lasers .......................... 402
10.3.4 Stabilization of Laser Arrays .................................................................. 407
10.4 Controls in Nobel Structure Lasers .......................... 408
10.4.1 Photonic VCSELs .................................. 408
10.4.2 Control of Mode Distribution Through Etched
Microstructure in Broad-Area Semiconductor
Lasers .................................................................. 410
10.4.3 Quantum-Dot Broad-Area Semiconductor Lasers ... 411
References .......................................................... 413

11 Metrology Based on Chaotic Semiconductor Lasers ........ 419
11.1 Optical Feedback Interferometers .......................... 419
11.1.1 Bistability and Multistability in Feedback
Interferometers ................................................. 419
11.1.2 Interferometric Measurement in Self-mixing
Semiconductor Lasers .................................. 423
11.2 Applications in Feedback Interferometer ............... 425
11.2.1 Displacement and Vibration Measurement ........ 425
11.2.2 Absolute Position Measurement .................. 428
11.2.3 Angle Measurement .................................. 429
11.2.4 Measurement of the Linewidth Enhancement
Factor .......................................................... 431
11.3 Sensing of Self-mixing Signal Across Laser Diode Voltage... 433
11.3.1 Detection Principles .................................. 433
11.3.2 Self-mixing Interferometry in VCSELs ......... 435
11.3.3 Self-mixing Interferometry in QCLs ............. 438
11.4 Self-mixing Doppler Velocimetry ....................... 441
11.4.1 Velocity Measurement ................................ 441
11.4.2 Rigorous Rate Equations of Self-mixing
Doppler Effects .............................................. 442
11.5 Chaotic Lidar ............................................... 445
11.6 Active Feedback Interferometer and Applications ...... 448
11.6.1 Stability and Bistability in Active Feedback
Interferometer .................................................. 448
11.6.2 Chaos Control in Active Feedback
Interferometers .............................................. 452
References .......................................................... 455

12 Chaos Synchronization in Semiconductor Lasers .......... 459
12.1 Concept of Chaos Synchronization ....................... 459
12.1.1 Chaos Synchronization ............................... 459
12.1.2 Generalized and Complete Chaos
Synchronization ............................................... 463
12.2 Theory of Chaos Synchronization in Semiconductor
Lasers with Optical Feedback .............................. 466
12.2.1 Model of Synchronization Systems ................. 466
12.2.2 Rate Equations in Unidirectional Coupling Systems .................................................. 468
12.2.3 Generalized Chaos Synchronization ................................................................. 469
12.2.4 Complete Chaos Synchronization ................................................................. 469
12.2.5 Mutual Coupling Systems ............................................................................. 470
12.3 Chaos Synchronization in Semiconductor Lasers with an Optical Feedback System ................................................. 472
12.3.1 Chaos Synchronization—Numerical Examples ......................................... 472
12.3.2 Chaos Synchronization—Experimental Examples ........................................ 476
12.3.3 Anticipating Chaos Synchronization ............................................................. 478
12.3.4 Bandwidth Enhanced Chaos Synchronization .............................................. 480
12.3.5 Incoherent Synchronization Systems .......................................................... 481
12.3.6 Polarization Rotated Chaos Synchronization ................................................. 483
12.3.7 Synchronization in Broad-Area Semiconductor Lasers ......................................... 487
12.4 Chaos Synchronization in Optically Injected Lasers ................................................. 490
12.4.1 Theory of Chaos Synchronization in Optically Injected Lasers ......................... 490
12.4.2 Examples of Chaos Synchronization in Optically Injected Lasers ....................... 492
12.5 Chaos Synchronization in Optoelectronic Feedback Systems ................................. 494
12.5.1 Theory of Chaos Synchronization in Optoelectronic Feedback Systems ............... 494
12.5.2 Examples of Chaos Synchronization in Optoelectronic Feedback Systems ............... 495
12.6 Chaos Synchronization in Injection Current Modulated Systems ................................. 496
12.7 Chaos Synchronization in Mutually Coupled Lasers ................................................. 497
12.7.1 Mutually Coupled Edge-Emitting Semiconductor Lasers ........................................ 497
12.7.2 Mutually Coupled VCSELs ........................................................................ 500
12.7.3 Optoelectronic Mutually Coupled Semiconductor Lasers ....................................... 501
12.8 Common-Chaotic-Signal Induced Synchronization in Semiconductor Lasers ................ 504
References ............................................................................................................... 507

13 Chaotic Communications in Semiconductor Lasers ................................................. 511
13.1 Message Encryption in a Chaotic Carrier and Its Decryption .................................. 511
13.1.1 Chaotic Communications ........................................................................ 511
13.1.2 Chaos Masking ......................................................................................... 513
13.1.3 Chaos Modulation ..................................................................................... 515
13.1.4 Chaos Shift Keying ........................................ 515
13.1.5 Chaotic Data Communications in Laser Systems ... 516
13.2 Cryptographic Applications in Optical Feedback Systems .... 517
  13.2.1 Chaotic Communications in Optical Feedback Systems .......... 517
  13.2.2 Chaos Masking in Optical Feedback Systems ............. 520
  13.2.3 Chaos Modulation in Optical Feedback Systems .......... 525
  13.2.4 Chaos Shift Keying in Optical Feedback Systems .......... 527
  13.2.5 Chaotic Communications in Incoherent Optical Feedback Systems .......... 528
  13.2.6 Chaos Pass Filtering Effects ........................... 529
13.3 Cryptographic Applications in Optical Injection Systems .... 532
13.4 Cryptographic Applications in Optoelectronic Systems ...... 534
13.5 Chaotic Communications in Mutually Coupled Semiconductor Lasers ......................... 535
  13.5.1 Communications in Mutually Coupled Semiconductor Lasers .......... 535
  13.5.2 Chaos-Pass Filtering Effects in Mutually Coupled Systems .......... 538
13.6 Chaotic Communications in Drive-Response Systems .... 539
13.7 Performance of Chaotic Communications .................... 541
13.8 Security of Chaotic Communications ....................... 544
13.9 Chaotic Carrier and Bandwidth of Communications .......... 548
13.10 Chaos Communications in the Real World .................. 549
  13.10.1 Chaos Masking Video Signal Transmissions ............. 549
  13.10.2 Chaotic Signal Transmissions Through Public Data Link  ................. 552
References .................................................. 555

14 Semiconductor Laser Networks: Synchrony, Consistency, and Analogy of Synaptic Neurons ......................... 559
  14.1 Nonlinear Networks and Coupled Semiconductor Lasers .......... 559
    14.1.1 Coupled Nonlinear Networks .......................... 559
    14.1.2 Chaotic Semiconductor Lasers and Neurons .......... 560
    14.1.3 Semiconductor Lasers as Chaotic Elements .......... 561
  14.2 Two Coupled Semiconductor Lasers ....................... 563
    14.2.1 Two Unidirectionally Coupled Semiconductor Lasers .......... 563
    14.2.2 Two Mutually Coupled Semiconductor Lasers .......... 563
  14.3 Three Coupled Semiconductor Lasers ..................... 566
    14.3.1 Three Coupled Lasers ................................ 566
    14.3.2 Three Chain-Coupled Semiconductor Lasers .......... 567
    14.3.3 Three Ring-Coupled Semiconductor Lasers .......... 570
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.4</td>
<td>Four Coupled Semiconductor Lasers</td>
<td>572</td>
</tr>
<tr>
<td>14.4.1</td>
<td>Four Chain-Coupled Semiconductor Lasers</td>
<td>572</td>
</tr>
<tr>
<td>14.4.2</td>
<td>Four Ring-Coupled Semiconductor Lasers</td>
<td>572</td>
</tr>
<tr>
<td>14.5</td>
<td>Star-Coupled Semiconductor Lasers</td>
<td>575</td>
</tr>
<tr>
<td>14.5.1</td>
<td>Unidirectional Coupled Hub-Star Systems</td>
<td>575</td>
</tr>
<tr>
<td>14.5.2</td>
<td>Mutually Coupled Hub-Star Systems</td>
<td>576</td>
</tr>
<tr>
<td>14.5.3</td>
<td>Group Synchrony in Hub-Star Laser Systems</td>
<td>579</td>
</tr>
<tr>
<td>14.6</td>
<td>Many Coupled Semiconductor Lasers</td>
<td>580</td>
</tr>
<tr>
<td>14.6.1</td>
<td>Examples of Many Coupled Semiconductor Lasers</td>
<td>580</td>
</tr>
<tr>
<td>14.6.2</td>
<td>Synchronization Properties Depending on Topological Configurations</td>
<td>582</td>
</tr>
<tr>
<td>14.6.3</td>
<td>Greatest Common Divisor Rule in Coupled Semiconductor Laser Networks</td>
<td>584</td>
</tr>
<tr>
<td>14.7</td>
<td>Reservoir Computing and Application</td>
<td>585</td>
</tr>
<tr>
<td>14.7.1</td>
<td>Neuro-Inspired Information Processing</td>
<td>585</td>
</tr>
<tr>
<td>14.7.2</td>
<td>Neural Networks and Reservoir Computing</td>
<td>587</td>
</tr>
<tr>
<td>14.7.3</td>
<td>Reservoir Computing and Implementation</td>
<td>589</td>
</tr>
<tr>
<td>14.7.4</td>
<td>Application of Reservoir Computing to Time Series Prediction</td>
<td>591</td>
</tr>
<tr>
<td>References</td>
<td></td>
<td>594</td>
</tr>
</tbody>
</table>

**15 Physical Random Number Generations and Photonic Integrated Circuits for Chaotic Generators**

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.1</td>
<td>Introduction to Physical Random Number Generations</td>
<td>597</td>
</tr>
<tr>
<td>15.2</td>
<td>Signatures in Optical Feedback Induced Chaos</td>
<td>599</td>
</tr>
<tr>
<td>15.3</td>
<td>Probability Distributions of Chaotic Signals</td>
<td>604</td>
</tr>
<tr>
<td>15.4</td>
<td>Chaotic Physical Random Number Generations</td>
<td>606</td>
</tr>
<tr>
<td>15.4.1</td>
<td>Principles of Chaotic Physical Random Number Generations</td>
<td>606</td>
</tr>
<tr>
<td>15.4.2</td>
<td>Fast Chaotic Random Number Generators</td>
<td>611</td>
</tr>
<tr>
<td>15.4.3</td>
<td>All Optical Chaotic Random Number Generators</td>
<td>615</td>
</tr>
<tr>
<td>15.5</td>
<td>Photonic Integrated Circuit for Chaotic Generators</td>
<td>618</td>
</tr>
<tr>
<td>15.6</td>
<td>Application for Random Number Key Distribution with Chaotic Semiconductor Lasers</td>
<td>622</td>
</tr>
<tr>
<td>References</td>
<td></td>
<td>624</td>
</tr>
</tbody>
</table>

**Appendix: Chaos**

**Index**
About the Author

Junji Ohtsubo received the B.S. degree in Electronics from the Kyushu Institute of Technology in 1973, and the M.S. and Ph.D. degrees in Electronics from Hokkaido University in 1975 and 1978, respectively. In 1978, he joined the Mechanical Engineering Laboratory, MITI, Japan. During 1981–1982, he was a Research Associate at the Institute of Optics, University of Rochester. He joined Shizuoka University, Shizuoka, Japan, as Associate Professor in 1985 and was Professor at the Department of Systems Engineering from 1993. He is now Professor Emeritus of Shizuoka University. His current research interests are nonlinear dynamics in optics, chaos in semiconductor lasers, optical information processing and computing, optical security systems, statistical optics, speckle, and optical metrology.

Professor Ohtsubo is a Fellow of the Optical Society of America and a member of IEEE, SPIE, ASP, the Japanese Society of Applied Physics, the Optical Society of Japan, and the Laser Society of Japan.