Solid Mechanics and Its Applications

Volume 227

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Aims and Scope of the Series

The fundamental questions arising in mechanics are: Why?, How?, and How much? The aim of this series is to provide lucid accounts written by authoritative researchers giving vision and insight in answering these questions on the subject of mechanics as it relates to solids.

The scope of the series covers the entire spectrum of solid mechanics. Thus it includes the foundation of mechanics; variational formulations; computational mechanics; statics, kinematics and dynamics of rigid and elastic bodies; vibrations of solids and structures; dynamical systems and chaos; the theories of elasticity, plasticity and viscoelasticity; composite materials; rods, beams, shells and membranes; structural control and stability; soils, rocks and geomechanics; fracture; tribology; experimental mechanics; biomechanics and machine design.

The median level of presentation is to the first year graduate student. Some texts are monographs defining the current state of the field; others are accessible to final year undergraduates; but essentially the emphasis is on readability and clarity.

More information about this series at http://www.springer.com/series/6557
Fatigue Crack Growth

Detect—Assess—Avoid
Preface

From time to time in technical practice, damages occur due to mechanical loading. Reasons are often small defects or cracks that are already in the component or initiate during the operation. Under service load—time variable loading—cracks can grow. In general the crack growth is initially stable, i.e., the crack grows a small amount with each cycle. This is known as fatigue crack growth. Depending on the manner of the loading, the geometry of the component and the material, a fatigue crack can grow over thousands of cycles without becoming unstable. If the loading or the crack length reaches a critical limit, unstable crack growth occurs and the component or the whole structure fails.

This book deals with the fatigue crack growth process. Therefore, at first the dimensioning of components and structures in accordance with current approaches of the classical strength of materials is described. After the description of many cases of damage caused by the crack growth as well as the principles of damage analyses and non-destructive testing, the basics of fracture mechanics and fatigue crack growth for mode I, mode II, mode III as well as mixed mode are presented. The experimental determination of fracture-mechanical material parameters, e.g., the fracture toughness, the threshold value, or the fatigue crack growth curve is described afterwards. The previously mentioned concepts and material parameters are valid for cyclic loading with constant amplitude. However, constant amplitude loading is very rare in practice, so that the fatigue crack growth under service loading and its influence on the residual lifetime is discussed in detail in the next chapter. Subsequently, the calculation of the residual lifetime using analytical and numerical simulation tools is in the focus. The book concludes with practical examples such as a leak in a pipeline, an investigation of fatigue crack growth in a high-speed train tire or a simulation of the fatigue crack growth in a press frame.

The authors hope that readers will find here a solid basis for further study on fatigue crack growth and subsequent application of the described concepts and methods. Moreover, they hope these readers will enjoy studying the variety of examples contained in the book. Let the study of this book lead to a smaller number of damage cases caused by fatigue crack growth.
The book is translation from the third edition of the German language book “Ermüdungsrisse – Erkennen, Sicher beurteilen, Vermeiden” published by Springer Vieweg in 2012. In this context we thank all persons who contributed to the German edition, especially Dr.-Ing. Andre Riemer (University of Paderborn) as well as Thomas Zipsner and Imke Zander (Editorial Office Springer Vieweg).

For preparation of the drawings in English, we thank Birgit Felske. Moreover, the authors thank Springer Science Media for undertaking its publication. Our special thanks are expressed to Nathalie Jacobs and Cynthia Feenstra for their cooperation and assistance in the editing and the final preparation for printing.

Paderborn
Rostock
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Hans Albert Richard
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Symbols

\( A \) Area
\( A_{\text{min}} \) Minimum area
\( A_0 - A_3 \) Coefficients in the Newman crack opening function
\( A-D \) Constants of the Richard interpolation formula
\( C \) Reduction rate according to ASTM (American Society for Testing and Material)
\( C_{\text{FM}} \) Material-dependent coefficient of the NASGRO equation
\( C_P \) Retardation factor
\( C_{\text{P}} \) Material-dependent coefficient in the Paris law
\( C_{\text{E}} \) Material-dependent factor in the Erdogan–Ratwani law
\( C_{\text{th}} \) Parameter in the Newman empirical function for describing the R-dependence of the threshold value
\( D \) Diameter
\( E \) Modulus of elasticity, Young’s Modulus
\( F \) Force
\( F_a \) Force amplitude
\( F_m \) Force mean
\( F_{\text{max}}, F_{\text{min}} \) Maximum, minimum force
\( F_0, F_u \) Maximum, minimum force
\( \Delta F \) Cyclic force
\( G \) Weight
\( G \) Energy release rate
\( G_{I}, G_{II}, G_{III} \) Energy release rate for mode I, mode II, mode III
\( G_{IC} \) Critical energy release rate
\( H \) Cumulative frequency
\( I \) Area moment of inertia
\( J \) Value of the J-integral
\( J_{IC} \) Critical value of the J-integral
\( K \) Stress intensity factor
\( K_C \) Critical stress intensity factor
\( K_{I}, K_{II}, K_{III} \) Stress intensity factor for mode I, mode II, mode III
$K_{I,Bl,max}$, $K_{I,Bl,min}$ Maximum, minimum stress intensity factor of the baseline level loading
$K_{I,block}$ Maximum stress intensity factor of the block load
$K_{IC}$, $K_{IIC}$, $K_{III}$ Fracture toughness for mode I, mode II, mode III
$K_{I,max}$, $K_{I,min}$ Stress intensity factors for mode I under maximum, minimum load
$K_{II,max}$, $K_{II,min}$ Stress intensity factors for mode II under maximum, minimum load
$K_{I,max,eff}$, $K_{I,min,eff}$ Effective maximum, minimum stress intensity factor
$K_{I,max,req}$ Virtual stress intensity factor for taking the residual stress into account in the Willenborg model
$K_{I,ol}$ Stress intensity factor of an overload
$K_{I,op}$ Crack opening stress intensity factor
$K_{I,R}$ Residual stress intensity factor
$K_{I,ul}$ Stress intensity factor of an underload
$K_{I,zul}$ Allowable stress intensity factor
$Q$ Critical stress intensity factor
$V$ Equivalent stress intensity factor
$V_{max}$, $V_{min}$ Maximum, minimum equivalent stress intensity factor
$\Delta K$ Cyclic stress intensity factor
$\Delta K_I$, $\Delta K_{II}$, $\Delta K_{III}$ Cyclic stress intensity factor for mode I, mode II, mode III
$\Delta K_{I,0}$ Initial cyclic stress intensity factor
$\Delta K_{I,Bl}$ Cyclic stress intensity factor of the baseline level loading
$\Delta K_{IC}$ Cyclic stress intensity at which unstable crack propagation starts: $\Delta K_{IC} = K_{IC} \cdot (1 - R)$
$\Delta K_{I,eff}$ Effective cyclic stress intensity factor
$\Delta K_{I,eff,th}$ Effective threshold value
$\Delta K_{I,rms}$ Root mean square of the cyclic stress intensity of a load spectrum
$\Delta K_{I,th}$ Threshold value for mode I (threshold value for the cyclic stress intensity factor for mode I)
$\Delta K_{II,th}$, $\Delta K_{III,th}$ Threshold value for mode II, mode III (threshold value for the cyclic stress intensity factor for mode II, mode III)
$\Delta K_{I,zul}$ Allowable cyclic stress intensity factor
$\Delta K_{th}$ Threshold value of the two-criteria concept
$\Delta K_{th,0}$ Threshold value $\Delta K_{th}$ for $R = 0$
$\Delta K_{V}$ Cyclic equivalent stress intensity factor
$M$ Moment
$M_B$ Bending moment
$M_T$ Torque
$L_j$ Beam element length (strip yield model)
Symbols

$N$ Number of load cycle
$N$ Normal force
$N_\text{Bl}$ Residual life
$N_{\text{Bl}}$ Number of load cycles of the baseline level loading
$N_\text{D}$ Number of load cycles at fatigue strength
$N_{\text{D}}, N_{\text{DI}}$ Number of retardation load cycles, corrected number of retardation load cycles
$N_i$ Initiation life
$N_f$ Overall service life
$Q$ Shear force
$R$ Ratio of minimum to maximum stress or of minimum to maximum stress intensity: $R = \frac{\sigma_{\text{min}}}{\sigma_{\text{max}}} = \frac{K_{\text{min}}}{K_{\text{max}}}$
$R_{\text{block}}$ Block loading ratio
$R_{\text{el}}, R_p$ Stress ratio from which $\Delta K_{\text{th}} = \text{const.}$ applies for positive, negative $R$-ratios
$R_e$ Yield strength
$R_{\text{eff}}$ Effective stress ratio $R_{\text{eff}} = \frac{K_{\text{min,eff}}}{K_{\text{max,eff}}}$
$R_{\text{in}}$ Tensile strength
$R_{\text{ol}}$ Overload ratio
$R_{p0.2}$ 0.2 % yield strength
$R_{\text{SO}}$ Shut-off ratio
$R_t$ Surface roughness
$S_B$ Safety against fracture
$S_D$ Safety against fatigue fracture
$S_E$ Safety against fatigue crack growth
$S_F$ Safety against yielding
$S_R$ Safety against unstable crack propagation
$T$ Temperature
$U$ Electrical potential difference
$U$ Elastic energy
$U_0$ Initial electrical potential difference
$\dot{U}$ Elastic energy density
$V$ Volume
$V_j$ Fictitious crack opening displacement (strip yield model)
$W$ Work of external forces
$W$ Section modulus
$W$ Wheeler exponent
$W_B$ Section modulus against bending
$W_{\text{min}}$ Minimum section modulus
$W_p$ Polar area moment of inertia
$W_T$ Section modulus against torsion
$Y_i, Y_{\text{II}}, Y_{\text{III}}$ Geometry factor, standardized stress intensity factor for mode I, mode II, mode III
$a$ Crack depth, crack length
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$a_0$</td>
<td>El Haddad parameter</td>
</tr>
<tr>
<td>$a_A$</td>
<td>Initial crack length</td>
</tr>
<tr>
<td>$a_C$</td>
<td>Critical crack depth, crack length</td>
</tr>
<tr>
<td>$a_{det}$</td>
<td>Detection limit: crack length discoverable using non-destructive testing</td>
</tr>
<tr>
<td>$a_{pl}$</td>
<td>Plastic crack length correction</td>
</tr>
<tr>
<td>$a_{th}$</td>
<td>Crack length at which the threshold value of fatigue crack growth is exceeded</td>
</tr>
<tr>
<td>$\Delta a$</td>
<td>Crack increment</td>
</tr>
<tr>
<td>$b$</td>
<td>Half-axis of an ellipse</td>
</tr>
<tr>
<td>$b_1$</td>
<td>Surface coefficient</td>
</tr>
<tr>
<td>$b_2$</td>
<td>Size coefficient</td>
</tr>
<tr>
<td>$c$</td>
<td>Crack length</td>
</tr>
<tr>
<td>$d$</td>
<td>Diameter, width</td>
</tr>
<tr>
<td>$da/dN$</td>
<td>Crack growth rate</td>
</tr>
<tr>
<td>$(da/dN)_{th}$</td>
<td>Crack growth rate near the threshold value</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency</td>
</tr>
<tr>
<td>$f_{ij}^I, f_{ij}^II, f_{ij}^III$</td>
<td>Dimensionless functions</td>
</tr>
<tr>
<td>$m_p$</td>
<td>Material-dependent exponent in the Paris law</td>
</tr>
<tr>
<td>$m_E$</td>
<td>Material-dependent exponent in the Erdogan–Ratwani law</td>
</tr>
<tr>
<td>$n_{ol}$</td>
<td>Number of interspersed overloads</td>
</tr>
<tr>
<td>$n_{FM}, p, q$</td>
<td>Material-dependent exponent of the NASGRO equation</td>
</tr>
<tr>
<td>$p$</td>
<td>Internal pressure</td>
</tr>
<tr>
<td>$r, \varphi$</td>
<td>Polar coordinates</td>
</tr>
<tr>
<td>$t$</td>
<td>Time</td>
</tr>
<tr>
<td>$t$</td>
<td>Thickness, specimen thickness</td>
</tr>
<tr>
<td>$u, v, w$</td>
<td>Displacements</td>
</tr>
<tr>
<td>$w$</td>
<td>Specimen width</td>
</tr>
<tr>
<td>$x, y, z$</td>
<td>Cartesian coordinates</td>
</tr>
<tr>
<td>$a$</td>
<td>Angle</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Constraint factor</td>
</tr>
<tr>
<td>$\alpha_{th}$</td>
<td>Principal shear stress angle</td>
</tr>
<tr>
<td>$\alpha_K$</td>
<td>Stress concentration factor</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Angle</td>
</tr>
<tr>
<td>$\varepsilon$</td>
<td>Strain</td>
</tr>
<tr>
<td>$\varepsilon_{ij}$</td>
<td>Strain tensor</td>
</tr>
<tr>
<td>$\varepsilon_m$</td>
<td>Mean strain</td>
</tr>
<tr>
<td>$\varepsilon_{max}, \varepsilon_{min}$</td>
<td>Maximum, minimum strain</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Crack opening function: ratio of $K_{I,op}$ to $K_{I,max}$</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>$(3 - \nu)/(1 + \nu)$ for plane stress state (ESZ); $3 - 4\nu$ for plane strain state (EVZ)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson’s ratio</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Notch radius</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density</td>
</tr>
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Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
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</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Normal stress</td>
</tr>
<tr>
<td>$\sigma_1, \sigma_2, \sigma_3$</td>
<td>Principal normal stresses</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>Stress amplitude</td>
</tr>
<tr>
<td>$\sigma_{a,zul}$</td>
<td>Highest stress amplitude of the cumulative frequency distribution</td>
</tr>
<tr>
<td>$\sigma_{a,zul}$</td>
<td>Allowable stress amplitude</td>
</tr>
<tr>
<td>$\sigma_A$</td>
<td>Fatigue strength for a certain $R$-ratio</td>
</tr>
<tr>
<td>$\sigma_C$</td>
<td>Critical stress</td>
</tr>
<tr>
<td>$\sigma_D$</td>
<td>Fatigue strength</td>
</tr>
<tr>
<td>$\sigma_F$</td>
<td>Yield stress</td>
</tr>
<tr>
<td>$\sigma_{ij}$</td>
<td>Stress tensor</td>
</tr>
<tr>
<td>$\sigma_j$</td>
<td>Contact stress in the strip yield model</td>
</tr>
<tr>
<td>$\sigma_m$</td>
<td>Mean stress</td>
</tr>
<tr>
<td>$\sigma_{\text{max}}, \sigma_{\text{min}}$</td>
<td>Maximum, minimum normal stress</td>
</tr>
<tr>
<td>$\sigma_N$</td>
<td>Nominal stress</td>
</tr>
<tr>
<td>$\sigma_{op}$</td>
<td>Crack opening stress</td>
</tr>
<tr>
<td>$\sigma_r, \sigma_\phi, \sigma_z$</td>
<td>Normal stresses in cylindrical coordinates</td>
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<tr>
<td>$\sigma_{\text{Sch}}$</td>
<td>Fatigue strength under fluctuating stress</td>
</tr>
<tr>
<td>$\sigma_V$</td>
<td>Equivalent stress</td>
</tr>
<tr>
<td>$\sigma_{V,a}$</td>
<td>Equivalent stress amplitude</td>
</tr>
<tr>
<td>$\sigma_{V,\text{max}}$</td>
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<tr>
<td>$\sigma_W$</td>
<td>Fatigue strength under alternating stress</td>
</tr>
<tr>
<td>$\sigma_{x, y, z}$</td>
<td>Normal stress in $x, y, z$ direction</td>
</tr>
<tr>
<td>$\sigma_{zul}$</td>
<td>Allowable stress</td>
</tr>
<tr>
<td>$\Delta \sigma$</td>
<td>Cyclic normal stress</td>
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<tr>
<td>$\Delta \sigma_{\text{th}}$</td>
<td>Cyclic Stress at which fatigue crack growth initiates</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Shear stress</td>
</tr>
<tr>
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<td>Shear stress amplitude</td>
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<tr>
<td>$\tau_C$</td>
<td>Critical shear stress</td>
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<tr>
<td>$\tau_H$</td>
<td>Principal shear stress</td>
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<tr>
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<tr>
<td>$\tau_{r\phi}, \tau_{rz}, \tau_{\phi z}$</td>
<td>Shear stresses in cylindrical coordinates</td>
</tr>
<tr>
<td>$\tau_{xy}, \tau_{yz}, \tau_{zx}$</td>
<td>Shear stresses in Cartesian coordinates</td>
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<tr>
<td>$\tau_z$</td>
<td>Non-planar shear stress</td>
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<td>$\Delta \tau$</td>
<td>Cyclic shear stress</td>
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<tr>
<td>$\phi_0$</td>
<td>Kinking angle</td>
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<td>$\psi_0$</td>
<td>Twisting angle</td>
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<tr>
<td>$\omega, \omega_{pl}$</td>
<td>Size of plastic zone</td>
</tr>
<tr>
<td>$\omega_{\text{max}}, \omega_{ol}$</td>
<td>Size of primary plastic zone</td>
</tr>
<tr>
<td>$\omega_{\text{min}}$</td>
<td>Size of reversed-plastic zone</td>
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