This part consists of three reviews of various notions of complexity related to statistical learning theory. Chapter 6 by Vladimir V’yugin reviews the fundamental notion of VC dimension and newer measures of complexity widely used in machine learning, including covering numbers, fat-shattering dimension, and Rademacher complexity. Further information about these measures can be found in Part IV; in particular, Chap. 15 in Part IV reviews on-line measures of complexity similar to those discussed in Chap. 6.

Alexander Shen’s Chap. 7 is a review of Kolmogorov complexity and related notions, such as Solomonoff’s \textit{a priori} probability, algorithmic randomness and information, and Hausdorff dimension. It describes numerous applications of these notions to other areas of mathematics and computer science, first of all computational complexity. There are several book-size reviews of Kolmogorov complexity, whereas Alexander Shen’s review is relatively brief while still stating and proving many key technical results.

Chapter 8 by Yuri Kalnishkan is devoted to predictive complexity, which is a natural generalization of Kolmogorov complexity. The Kolmogorov complexity of a binary sequence can be interpreted as the loss of a universal forecaster predicting the elements of the sequence in the on-line mode in the case where the loss is measured using the fundamental logarithmic loss function. There are, however, other important loss functions, such as the square loss in regression and Brier loss in probabilistic prediction. Predictive complexity generalizes Kolmogorov complexity (more precisely, the minus log of Solomonoff’s \textit{a priori} probability) to other loss functions.

Complexity is a huge subject, and this book can only cover a tiny subset of it. Other notions of complexity that are somewhat related to statistical learning are computational and communication complexity. There is a large number of good reviews of these fields of various lengths, including excellent textbooks, such as Sipser’s [3] (computational complexity), Arora and Barak’s [1] (computational complexity and an introduction to communication complexity), and Kushilevitz and Nisan’s [2] (communication complexity).
References