Part X

Applications and Algorithms in the Physical Sciences
One of the many ways in which harmonic analysis distinguishes itself among other areas of modern mathematics is through the emphasis placed on algorithm development and the connections that it builds with applied sciences. This phenomenon goes back to Joseph Fourier, whose main motivation for introducing what we know as the Fourier transform, was his work on heat flow and thermal conduction. Other prominent examples of these interactions include the role of Radon transform in Magnetic Resonance Imaging, the impact of Kaczmarz algorithm on Computed Tomography, and the role played by the Phase Problem in X-ray crystallography—all rewarded with Nobel Prizes. The continuation of these trends is certain, as is illustrated by the selection of four excellent chapters devoted to state-of-the-art applications of recent developments in harmonic analysis.

An example of such a fundamental connection to applied sciences is the concept of Laplace transform, which is treated in the first chapter. NAIL A. GUMEROV and RAMANI DURAIWAMI present a detailed analysis of the spherical harmonic rotation coefficients, together with new, fast, and stable recursive algorithms for their computation. Spherical harmonics form an orthonormal basis for the space of square integrable functions on the unit sphere. As such, they have many practical applications, ranging from computation of electron configurations in quantum mechanics, providing solutions for many fundamental equations in mathematical physics, to applications in geostatistics and astrophysics. Detailed description of the involved algorithms is provided and illustrated with numerical examples.

BRIAN O’DONNELL, ALEXANDER MAURER, and ANTONIA PAPANDREOU-SUPPAPPOLA give an excellent overview of the role of modern time-frequency signal processing techniques in molecular biology. Highly localized waveform analysis and parameter estimation are the main tools used to detect and analyze variations in the profiles of antibodies to discriminate between pathogens. When combined with the recent developments in measuring expression levels for large numbers of genes, proteins, or peptides, these methods become a powerful tool with such possible applications, as diagnosis of infectious diseases before they become symptomatic.

Harmonic analysis inspired representations of 3D objects are treated in the third chapter of this part. Efficient visualization of complex 3D phenomena plays an important role in such diverse fields as computer graphics, X-ray crystallography, or magnetic resonance imaging. DAVID A. SCHUG, GLENN R. EASLEY, and DIANNE P. O’LEARY analyze novel geometric multiscale representation systems called shearlets, and demonstrate the advantages arising from including directional information in the multiresolution analysis, over classical wavelet techniques. Resulting 3D edge detection algorithms are carefully studied and compared with traditional 2D methods.

In the final chapter, SHERRY E. SCOTT introduces the readers to the rich field of fluid dynamics and its many interactions with wavelet theory. This gives us a better understanding of the role played by novel mathematical models in analysis and monitoring of Earth’s climate and weather. The key notion in this presentation is the concept of ergodicity defect—a value that captures the deviation of a system from ergodicity, and which can serve as a diagnostic tool in a variety of geoscience applications.