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Many-Electron Approaches in Physics, Chemistry and Mathematics

A Multidisciplinary View
Preface

The advent of computers in physics has undoubtedly boosted the development of theoretical approaches to study the structure of matter. The study of many-electron systems followed the technological advancement and led to a breakthrough in understanding important processes in a broad class of subjects in natural science. This success, in turn, provided the confidence to start the process of designing “in silico” new materials or modify “on demand” chemical and physical processes. Nowadays, we face a demand for theoretical predictions with extremely high accuracy in order to create “de novo” or modify to our convenience even the subtlest processes of Nature. In the field of many-electron approaches the question came to an important crossroad: current approaches are either computationally too demanding or conceptually or mathematically not satisfactory, thus where shall we go next?

The most common strategy to overcome this point, in the short term, is that of including “brute force” and/or empirical modifications to the theories in order to get satisfactory answers for, at least, few specific (though relevant) systems. It is becoming clear, however, that there is an alternative strategy, working on a long term but certainly worth to explore, which is based on the interdisciplinary synergy among different disciplines involved in this field. Mathematicians, physicists, chemists, and computer scientists started to converge in common meetings, on common projects, and on common ideas. The points of view are still different, but their complementarity has become clear, and an effort to overcome long-standing prejudicial barriers among the different disciplines is made.

Among these efforts is a workshop on “New Approaches in Many-Electron Theory” (NAMET) which we organized in 2010 to bring people from different fields and disciplines together to exchange results and receive feedback across the scientific fields. The response we obtained was highly positive. It became clear that the exchange of ideas, data, and perspectives was invigorating the background of all the attendants. From several lively discussions it became equally apparent, however, that the way to an optimal merging of interests and goals requires a continuous effort over the next years.

This book represents a natural continuation of NAMET and of our will of feeding this process of exchange and integration further. We have given space to young researchers, with their new ideas which are more naturally open to interdisciplinarity, and complement their contributions with established senior experts,
in an attempt to optimally merge the various views. The resulting product is a general overview of the field, where past perspectives meet current possibilities, and past results become the basis of the search for future innovations. On purpose we have promoted the use of an accessible language for researchers and students of each discipline so that the book is accessible essentially to all mathematicians, physicists, and chemists.

Volker Bach
Luigi Delle Site
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