Part I

Background
The nature of this part is reflected by its title. Our aim here is to quickly introduce the notions and concepts required in the sequel, to prove basic statements about them, and to discuss their interrelation in the light of real interpolation.

In Chapter 1, we describe the classical procedure invented by Calderón and Zygmund, which splits a function in two parts with good properties. We discuss some applications of this procedure and relate it to real interpolation; also, we prove our first stability result for near-minimizers (in a rather elementary setting). Chapter 2 is a brief introduction to the theory of singular integral operators (for us, they serve as “raw material” for stability theorems). In Chapter 3 we prove classical covering theorems due to Besicovitch, Wiener, and Whitney, describe a smooth version of the Calderón–Zygmund decomposition, and present our first serious stability statement. In Chapter 4 we discuss some basic facts about homogeneous spaces of smooth functions and prove that, typically, singular integral operators are bounded on them. Chapter 5 is a brief summary of the real interpolation theory in its relationship with near-minimizers.

In two final Chapters 6 and 7, we dwell on two topics related to the theory under study but staying somewhat apart. In Chapter 6 we give a brief account of the use of near-minimizers for $L$-functionals in ill-posed problems, particularly in those arising in image processing. In Chapter 7, we discuss a different method of producing near-minimizers. This method is not related to decompositions of Calderón–Zygmund type. It is applicable only in the framework of certain spaces of analytic functions of one variable, but leads to very powerful results. The reader interested only in nonclassical Calderón–Zygmund procedures may pass directly to Chapter 8 after Chapter 5.