Part IV

Discretization: Concepts and Analysis
Introduction to Part IV

Discretization: Concepts and Analysis

This part summarizes recent trends and addresses future research directions in the field of tailored discrete concepts for PDE constrained optimization with elliptic and parabolic PDEs in the presence of pointwise constraints. It covers the range from the treatment of mesh grading over relaxation techniques, and adaptive a posteriori finite element concepts, to the modern treatment of optimal control problems with parabolic PDEs.

Thomas Apel and Dieter Sirch deal, in *A Priori Mesh Grading for Distributed Optimal Control Problems*, with $L^2$-error estimates for finite element approximations of control constrained distributed optimal control problems governed by linear partial differential equations whose solutions develop singularities. In order to avoid a reduced convergence order, graded finite element meshes are used.

Michael Hinze and Arnd Rösch review, with *Discretization of Optimal Control Problems*, the state of the art in designing discrete concepts for optimization problems with PDE constraints with emphasis on structure conservation of solutions on the discrete level, and on error analysis for the discrete variables involved.

Kristina Kohls, Arnd Rösch and Kunibert G. Siebert contribute, with *A Posteriori Error Estimators for Control Constrained Optimal Control Problems*, a framework for the a posteriori error analysis of control constrained optimal control problems with linear PDE constraints, which is solely based on reliable and efficient error estimators for the corresponding linear state and adjoint equations.

Ira Neitzel and Fredi Tröltzsch review, in *Numerical Analysis of State-constrained Optimal Control Problems for PDEs*, Lavrentiev-type regularization of both distributed and boundary control problems, as well as a priori error estimates for elliptic control problems with finite dimensional control space and finitely many state-constraints.


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