For JV, the silliest girl I know.
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About the Author

Magnus Lie Hetland is an experienced Python programmer, having used the language since the late 1990s. He is also an associate professor of algorithms at the Norwegian University of Science and Technology and has taught algorithms for more than a decade. Hetland is the author of *Beginning Python*.
Stefan Turalski is just another coder who is perfectly happy delivering pragmatic, not necessarily software, solutions and climbing the impassable learning curve.

He has more than a decade of experience building solutions in such diverse domains as knowledge management, embedded networking, healthcare, power and gas trading, and, in the last few years, finance.

Focusing on code optimization and systems integration, he has dabbled (or almost drowned) in quite a few programming languages and has abused a number of open source and commercial software frameworks, libraries, servers, and so on.

Stefan is currently working on a highly scalable, low-latency, intraday risk valuation system at a financial institution in London. His latest interests revolve around functional and reactive programming, F#, Clojure, Python, OpenCL, and WebGL.

He still cannot believe that he was trusted enough to help on the second edition of Magnus Lie Hetland’s superb book. Stefan hopes that his (and your) brain cells injured while studying the algorithmic problems covered by the author will recover stronger and wiser!
Acknowledgments

Thanks to everyone who contributed to this book, either directly or indirectly. This certainly includes my algorithm mentors, Arne Halaas and Bjørn Olstad, as well as the entire crew at Apress and my brilliant tech editors, Alex Martelli (for the first edition) and Stefan Turalski. Thanks to all the readers who pointed out errors in the first edition; I hope I have corrected most of them. I’d especially like to thank Gerald Senarclens de Grancy, who supplied an extensive, well-annotated list of errata covering the entire book. Thanks to Nils Grimsmo, Jon Marius Venstad, Ole Edsberg, Rolv Seehuus, and Jorg Rødsjø for useful input; to my girlfriend, Janne Varvára Seem, my parents, Kjersti Lie and Tor M. Hetland, and my sister, Anne Lie-Hetland, for their interest and support; and to my uncle Axel, for checking my French. Finally, a big thank-you to the Python Software Foundation for their permission to reproduce parts of the Python standard library and to Randall Munroe for letting me include some of his wonderful XKCD comics.
This book is a marriage of three of my passions: algorithms, Python programming, and explaining things. To me, all three of these are about aesthetics—finding just the right way of doing something, looking until you uncover a hint of elegance, and then polishing that until it shines (or at least until it is a bit shinier). Of course, when there’s a lot of material to cover, you may not get to polish things quite as much as you want. Luckily, though, most of the content in this book is prepolished because I’m writing about really beautiful algorithms and proofs, as well as one of the cutest programming languages out there. As for the third part, I’ve tried hard to find explanations that will make things seem as obvious as possible. Even so, I’m sure I have failed in many ways, and if you have suggestions for improving the book, I’d be happy to hear from you. Who knows, maybe some of your ideas could make it into a future edition. For now, though, I hope you have fun with what’s here and that you take any newfound insight and run with it. If you can, use it to make the world a more awesome place, in whatever way seems right.