Advances in Industrial Control
Steven X. Ding

Model-Based Fault Diagnosis Techniques

Design Schemes, Algorithms and Tools

Second Edition

Springer
To My Parents and Eve Limin
The series *Advances in Industrial Control* aims to report and encourage technology transfer in control engineering. The rapid development of control technology has an impact on all areas of the control discipline. New theory, new controllers, actuators, sensors, new industrial processes, computer methods, new applications, new philosophies..., new challenges. Much of this development work resides in industrial reports, feasibility study papers and the reports of advanced collaborative projects. The series offers an opportunity for researchers to present an extended exposition of such new work in all aspects of industrial control for wider and rapid dissemination.

When assessing the performance of a control system, it is easy to overlook the fundamental question of whether the actual system configuration and set up has all the features and hardware that will enable the process to be controlled per se. If the system can be represented by a reasonable linear model, then the characteristics of a process that create limitations to achieving various control performance requirements can be identified and listed. Such information can be used to produce guidelines that give a valuable insight as to what a system can or cannot achieve in terms of performance. In control systems analysis textbooks, these important properties are often given under terms such as “input–output controllability” and “dynamic resilience”.

It is interesting to see similar questions arising in the study of fault detection and isolation (FDI) systems. At a fundamental level, the first question is not one of the performance of the fault detection and analysis system, but of whether the underlying process has the structure and properties to allow faults to be detected, isolated and identified. As with the analysis of the control case, if the system can be represented by a linear model then definitions and conditions can be given as to whether the system is generically fault detectable, fault isolatable and fault identifiable. Fault detectability is about whether a system fault would cause changes in the system outputs independently of the type and size of the fault, fault isolatability is a matter of whether the changes in the system output caused by different faults are distinguishable (from for example, system output changes caused by the presence of a disturbance) and finally fault identifiability is about whether the mapping from
the system output to the fault is unique since if this is so then the fault is identifiable. With the fundamental conditions verified, the engineer can proceed to designing the FDI system. All these issues, along with design techniques based on models with demonstrative case study applications can be found in this comprehensive second edition of Professor Steven Ding’s book *Model-Based Fault Diagnosis Technique: Design Schemes, Algorithms and Tools* that has now entered the *Advances in Industrial Control* series of monographs.

The key practical issues that complicate the design of a FDI system come from two sources. Firstly from the process: Many process plants and installations are often subject to unknown disturbances and it is important to be able to distinguish these upsets from genuine faults. Similarly process noise, emanating from the mechanisms within the process and from the measurements sensors themselves, is usually present in real systems so it is important that process measurement noise does not trigger false alarms. The second set of issues arises from FDI design itself where model uncertainty is present. This may exhibit itself as simply imperfect process-operational knowledge with the result that the FDI system is either too sensitive or too insensitive. Alternatively, model uncertainty (model inaccuracy) may well exist and the designer will be advised to use a robust FDI scheme. Professor Ding provides solutions, analysis and discussion of many of these technical FDI issues in his book.

A very valuable feature of the book presentation is the use of five thematic case study examples used to illuminate the substantial matters of theory, algorithms and implementation. The case study systems are:

- speed control of a dc motor;
- an inverted pendulum control system;
- a three-tank system;
- a vehicle lateral dynamical system; and
- a continuous stirred tank heater system.

Further, a useful aspect of these case study systems is that four of them are linked to laboratory-scale experimental rigs, thus presenting the academic and engineering reader with the potential to obtain direct applications experience of the FDI techniques described.

The first edition of this book was a successful enterprise and since its publication in 2008 the model-based FDI field has grown in depth and insight. Professor Ding has taken the opportunity to update the book by adding more recent research findings and including a new case study example from the industrial process area. The new edition is a very welcome addition to the *Advances in Industrial Control* series.

Industrial Control Centre, Glasgow, Scotland, UK

M.J. Grimble

M.A. Johnson
Preface

Model-based fault diagnosis is a vital field in the research and engineering domains. In the past years since the publication of this book, new diagnostic methods and successful applications have been reported. During this time, I have also received many mails with constructive remarks and valuable comments on this book, and enjoyed interesting and helpful discussions with students and colleagues during classes, at conferences and workshops. All these motivated me to work on a new edition.

The second edition retains the original structure of the book. Recent results on the robust residual generation issues and case studies have been added. Chapter 14 has been extended to include additional fault identification schemes. In a new chapter, fault diagnosis in feedback control systems and fault-tolerant control architectures are addressed. Thanks to the received remarks and comments, numerous revisions have been made.

A part of this book serves as a textbook for a Master course on *Fault Diagnosis and Fault Tolerant Systems*, which is offered in the Department of Electrical Engineering and Information Technology at the University of Duisburg-Essen. It is recommended to include Chaps. 1–3, 5, 7 (partly), 9, 10, 12–15 (partly) in this edition for such a Master course. It is worth mentioning that this book is so structured that it can also be used as a self-study book for engineers in the application fields of automatic control.

I would like to thank my Ph.D. students and co-worker for their valuable contributions to the case study. They are Tim Könings (inverted pendulum), Hao Luo (three-tank system and CSTM), Jedsada Saijai and Ali Abdo (vehicle lateral dynamic system), Ping Liu (DC motor) and Jonas Esch (CSTM).

Finally, I would like to express my gratitude to Oliver Jackson from Springer-Verlag and the Series Editor for their valuable support.

Duisburg, Germany

Steven X. Ding
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Notation

∀ For all
∈ Belong to
⊂ Subset
∪ Union
∩ Intersection
≡ Identically equal
≈ Approximately equal
:= Defined as
⇒ Implies
⇔ Equivalent to
≫ (≪) Much greater (less) than
max (min) Maximum (minimum)
sup (inf) Supremum (infimum)

\( \mathbb{R} \) and \( \mathbb{C} \) Field of real and complex numbers
\( \mathbb{C}_+ \) and \( \overline{\mathbb{C}_+} \) Open and closed right-half plane (RHP)
\( \mathbb{C}_- \) and \( \overline{\mathbb{C}_-} \) Open and closed left-half plane (LHP)
\( \mathbb{C}_{j\omega} \) Imaginary axis
\( \mathbb{C}_1 \) and \( \overline{\mathbb{C}_1} \) Open and closed plane outside of the unit circle
\( \mathbb{R}^n \) Space of real \( n \)-dimensional vectors
\( \mathbb{R}^{n \times m} \) Space of \( n \) by \( m \) matrices
\( \mathcal{RH}_\infty, \mathcal{RH}_\infty^{n \times m} \) Denote the set of \( n \) by \( m \) stable transfer matrices, see [198] for definition
\( \mathcal{RH}_2, \mathcal{RH}_2^{n \times m} \) Denote the set of \( n \) by \( m \) stable, strictly proper transfer matrices, see [198] for definition
\( \mathcal{LH}_\infty, \mathcal{LH}_\infty^{n \times m} \) Denote the set of \( n \) by \( m \) transfer matrices, see [198] for definition
$X^T$  
Transpose of $X$

$X^\perp$  
Orthogonal complement of $X$

$X^{-1}$  
Inverse of $X$

$X^-$  
Pseudo-inverse of $X$ (including left or right inverse)

$\text{rank}(X)$  
Rank of $X$

$\text{trace}(X)$  
Trace of $X$

$\text{det}(X)$  
Determinant of $X$

$\lambda(X)$  
Eigenvalue of $X$

$\bar{\sigma}(X)$ ($\sigma_{\text{max}}(X)$)  
Largest (maximum) singular value of $X$

$\sigma(X)$ ($\sigma_{\text{min}}(X)$)  
Least (minimum) singular value of $X$

$\sigma_i(X)$  
The $i$th singular value of $X$

$\text{Im}(X)$  
Image space of $X$

$\text{Ker}(X)$  
Null space of $X$

$\text{diag}(X_1, \ldots, X_n)$  
Block diagonal matrix formed with $X_1, \ldots, X_n$

$\text{prob}(a < b)$  
Probability that $a < b$

$\mathcal{N}(a, \Sigma)$  
Gaussian distribution with mean vector $a$ and covariance matrix $\Sigma$

$x \sim \mathcal{N}(a, \Sigma)$  
$x$ is distributed as $\mathcal{N}(a, \Sigma)$

$\mathcal{E}(x)$  
Mean of $x$

$\text{var}(x)$  
Variance of $x$

$G(p)$  
Transfer matrix, $p$ is either $s$ for a continuous-time system or $z$ for a discrete-time system

$G^*(j\omega) = G^T(-j\omega)$  
Conjugate of $G(j\omega)$

$(A, B, C, D)$  
Shorthand for the state space representation

$\text{rank}(G(s))$  
Normal rank of $G(s)$, see [105] for definition