PART C

APPLICATIONS OF
DISCRETE $D^m$-SPLINES
INTRODUCTION

The problems studied in this part of the book mainly arise in oil research and involve disciplines such as Geophysics and Geology. In every case, the question is to construct, from a set of data of the corresponding function, a regular (explicit or possibly parametric) surface, i.e. a surface of class \(C^1\) or \(C^2\). This set may be made up, either of a large number \(N\) of point values of the function and, eventually, of its first derivatives, or of an infinity of such values at points continuously distributed on curves or open subsets of \(\mathbb{R}^2\). Sometimes these surfaces present discontinuities, e.g. the faulted surfaces in Geophysics.

From the numerical point of view, one cannot use interpolation methods, even if any exist, because they are too expensive. In the case of a finite set of \(N\) data, for example, interpolation methods involve processes, such as construction of triangulations or solution of linear systems, that are of order greater than \(N\), generally, and \(N\) is supposed to be large. For the same reason, one cannot consider interpolating or smoothing \((m, s)\)-splines, which both lead to linear systems of order \(N + m(m + 1)/2\).

The method of discrete \(D^m\)-splines is worth using on two accounts: flexibility of modelling and cost-in-use. As will be seen, the theory of Chapter VI fits easily to the situation of Chapters VIII–XI. It also fits to the case of parametric surfaces, treated in Chapter XII. On the other hand, the nature of discrete smoothing \(D^m\)-spline allows us to disconnect the number \(M\) of degrees of freedom of the approximant from the number \(N\) of data, which enables us to choose \(M\) much smaller than \(N\).