Part VIII
Biomathematics
Medical and biological sciences are nowadays spearheading important research and development efforts around the world. In this, they both benefit from and are benefactors of progress in applied mathematics, with many fundamental contributions arising in the field of harmonic analysis. Prominent examples of interactions between the mathematical and biomedical sciences include the role of the Kaczmarz algorithm in computed tomography and of Radon transforms in magnetic resonance imaging—both rewarded with Nobel Prizes in physiology and medicine. There is no doubt that this trend is going to continue. This is supported by the growing importance of fields such as systems biology, which reflects the fact that biological and medical models become increasingly more complex and involved, or bioinformatics, which addresses the issue of rapid growth in available medical data. With this in mind, we present some contributions describing state-of-the-art applications of harmonic analysis to current problems in the medical and biological sciences.

Alex Chen, Andrea L. Bertozzi, Paul D. Ashby, Pascal Getreuer, and Yifei Lou introduce us to the field of atomic force microscopy (AFM)—a powerful tool to study biological, chemical, and physical processes at the atomic level. The authors detail the role of mathematical advancements in this novel imaging modality. This includes the discussion of the role of sampling methodologies in AFM image reconstruction, as well as a review of a number of interpolation and inpainting approaches that are useful in AFM applications.

Gregory S. Chirikjian analyzes the role of representation theory and numerical harmonic analysis in the mechanics of double-helical DNA molecules. Through modeling of DNA as an elastic filament capable of bending and twisting, the author introduces the representation theory on unimodular Lie groups, with special emphasis placed on the three-dimensional group of rigid-body motions. The associated unitary irreducible representations are then used to provide explicit solutions of the diffusion equations describing the DNA structure. The result is a simplified model for a distribution of DNA poses.

Martin Ehler, Julia A. Dobrosotskaya, Emily J. King, and Robert F. Bonner present state-of-the-art mathematical applications in ophthalmology, the branch of medicine that deals with the human eye. It is not at all unexpected that the image analysis of our visual system poses a number of captivating problems of fundamental importance to our health. As an example the authors consider the early detection of age-related retinal diseases. Among such diseases is AMD (age-related macular degeneration), the most common cause of blindness among the elderly populations in the developed world. A number of image analysis tools is employed to understand its mechanics, including computational models for rhodopsin bleaching kinetics, variational inpainting techniques, and multispectral analysis.

The problems in magnetic resonance (MR) are analyzed by Evren Özarslan, Cheng G. Koay, and Peter J. Basser. Nuclear MR is a technique that allows us to obtain information about the imaged domain via the analysis of diffusion processes in that domain. The chapter focuses on MR performed in the $q$-space (i.e., after the mapping by the Fourier transform). This technique allows the researchers to analyze microscopic tissue structures, which otherwise are inaccessible to conventional MR
imaging. Hermite functions are one of their mathematical tools. Applications to reconstruction of certain two- and three-dimensional signals from one-dimensional measurements are also provided.

A long history of the use of Fourier analysis in the study of structured materials is revisited by Richard O. Prum and Rodolfo H. Torres. Their goal is to analyze the nature of nonpigmentary coloration in the tissues of living organisms. Their groundbreaking work mathematically establishes the fact that coherent light scattering can also be achieved as a result of reflection from quasi-ordered collagen fibers. This result manifests itself in our perception of certain nano-structures as colors.

Paul Hernandez-Herrera, David Jiménez, Ioannis A. Kakadiaris, Andreas Koutsogiannis, Demetrio Labate, Fernanda Laezza, and Manos Papadakis give us a harmonic analysis view on neuroscience imaging. The chapter begins with an extensive, historical, accessible overview of modern neuron imaging techniques. This is followed by a detailed study of the approximation errors due to the action of a group of orthogonal transformations on Euclidean space. These results depend on efficient directional representations, with examples including such novel representation systems as shearlets and curvelets. All this fascinating work culminates in an algorithm for computation of realistic models for naturally occurring neuronal dendrites.