Part VI

BRAID GROUP ACTION

In the classical theory of semisimple Lie algebras, the Weyl group plays an important role. Now the Weyl group is not quite a subgroup of the corresponding simply connected Lie group; only a finite covering of it is. As Tits has shown [9], one can choose such a covering which is naturally a quotient of the braid group. In particular, there is a small obstruction to making the Weyl group act on a simple integrable module for the Lie algebra; what acts naturally is a quotient of the braid group, which is a finite covering of the Weyl group. Since in this case, the obstruction involves only signs, it is almost invisible. In the quantum case, the obstruction becomes quite serious, and in this case, not even a finite covering of the Weyl group can be made to act; the braid group still acts, but in general not through a finite quotient.

In Part VI we explain how the braid group acts on integrable U-modules and on U itself. (In fact there are several braid group actions, but they are related to each other in a simple way.)

The symmetries $T'_{i,e}, T''_{i,e}$ of an integrable U-module have already been introduced in Chapter 5. In Chapter 39 it is shown that these symmetries satisfy the braid group relations, hence they define braid group actions. These symmetries are studied simultaneously with the analogous symmetries of U (see Chapter 37) which also satisfy the braid group relations.

In Chapters 38 and 40 we study the connection between the symmetries of U and the inner product $(,)$ on f. In Chapter 41 we define a braid group action on $R\hat{U}$ and on its integrable modules for any $R$.

In Chapter 42 we assume that the Cartan datum is simply laced and of finite type and we use the braid group actions to give a purely combinatorial parametrization of the canonical basis $B$ in terms of reduced expressions for the longest element of $W$. 