Part II

GEOMETRIC REALIZATION OF f

The algebra $f$ has a canonical basis $B$ with very remarkable properties. This gives an extremely rigid structure for $f$ and also (in the $Y$-regular case) for each $\Lambda_{\lambda}$. Part II will introduce the canonical basis of $f$. At the same time, $f$ will be constructed in a purely geometric way, in terms of perverse sheaves on the moduli space of representations of a quiver.

Chapter 8 contains a review of the theory of perverse sheaves over an algebraic variety in positive characteristic. As far as definitions are concerned, it would have been possible to stay in characteristic zero and use $\mathcal{D}$-modules instead of perverse sheaves. This would certainly have been more elementary, but would have deprived us of the possibility of using the Weil conjecture and its consequences which are available in the framework of perverse sheaves on varieties in positive characteristic.

In Chapter 9 we introduce a class of perverse sheaves attached to a quiver and the operations of induction and restriction for perverse sheaves in this class. In Chapter 10, we study the Fourier-Deligne transform of perverse sheaves in our class. This is necessary for understanding the effect of changing the orientation of the quiver. In Chapter 11 we study linear categories with a given periodic functor (a functor which has some power equal to identity). These are needed to handle the case where the Cartan datum is not symmetric. (The geometry associated to a non-symmetric Cartan datum is very closely related to that associated to a symmetric Cartan datum, together with an action of a finite cyclic group.)

In Chapter 12 we study quivers with a cyclic group action. The geometric construction of $f$ and of its canonical basis (up to signs) is given in Chapter 13.

In Chapter 14, we discuss various properties of the canonical basis. For example, the property expressed in Theorem 14.3.2 is responsible for the existence of a canonical basis in the simple integrable $U$-modules (see Theorem 14.4.11). Perhaps the deepest property of the canonical basis is
expressed by the positivity theorem 14.4.13, which states (for symmetric Cartan data) that the structure constants of \( f \) are given by polynomials with positive integer coefficients.

Theorem 14.4.9 gives a natural bijection between the canonical basis of \( f \) for a non-symmetric Cartan datum and the fixed point set of a cyclic group action on the canonical basis of the analogous algebra corresponding to a symmetric Cartan datum.