OPTIMAL DESIGN AND RELATED AREAS
IN OPTIMIZATION AND STATISTICS
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Aims and Scope
Optimization has been expanding in all directions at an astonishing rate during the last few decades. New algorithmic and theoretical techniques have been developed, the diffusion into other disciplines has proceeded at a rapid pace, and our knowledge of all aspects of the field has grown even more profound. At the same time, one of the most striking trends in optimization is the constantly increasing emphasis on the interdisciplinary nature of the field. Optimization has been a basic tool in all areas of applied mathematics, engineering, medicine, economics and other sciences.

The Springer Series in Optimization and Its Applications publishes undergraduate and graduate textbooks, monographs and state-of-the-art expository works that focus on algorithms for solving optimization problems and also study applications involving such problems. Some of the topics covered include nonlinear optimization (convex and nonconvex), network flow problems, stochastic optimization, optimal control, discrete optimization, multi-objective programming, description of software packages, approximation techniques and heuristic approaches.
OPTIMAL DESIGN AND RELATED AREAS
IN OPTIMIZATION AND STATISTICS

Edited By

LUC PRONZATO
CNRS/Université de Nice Sophia Antipolis, France

ANATOLY ZHIGLJAVSKY
Cardiff University, UK

Springer
Henry Wynn has been involved in some of the major advances in experimental design since 1970, stemming from the work in optimal experimental design of Jack Kiefer and collaborators. His first major result concerned the algorithm for constructing optimal designs, which is now often referred to as the Fedorov–Wynn Algorithm.\textsuperscript{1} In joint work with Kiefer he was one of the first to study optimal design for correlated observations\textsuperscript{2} and helped to edit

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Kiefer’s collected works. He was also a member of the team headed by Jerome Sacks, which wrote the now highly cited paper on computer experiments\(^3\) and continues to be active in the area. An interest in the application of information theory, which started with the introduction of Maximum Entropy Sampling, led to joint work with Sebastiani.\(^4\)

An early paper on multiple comparisons,\(^5\) written with Bloomfield while they were both Ph.D. students, and further work with late Robert Bohrer led to a long-standing collaboration with Daniel Naiman, through which they introduced the concept of a discrete tube.\(^6\) Notable was their use of ideas from computational geometry and topology, which paralleled work on continuous tube theory by Naiman and by Robert Adler and co-workers. Tube theory gives tight inclusion–exclusion (Boole-Bouferroni-Fréchet) bounds for the probability of unions of events. The observation that the close connection between discrete tubes, unions of orthants, monomial ideals and Hilbert functions could be used to get tight systems reliability bounds led to papers.\(^7,8\) This work is continuing with Eduardo Sáenz-de-Cabezón (presented at MEGA2007) using the latter’s new algorithm for minimal free resolutions.

Henry Wynn was intimately involved with the introduction of robust engineering methodologies into Europe following his regular participation in a series of National Science Foundation funded workshops in the late 1980s led by Jerome Sacks. This work has led to publications, particularly in the engineering literature, and extensive collaboration with industrial partners, notably Fiat CRF, well funded by both UK and EU grants. Much of this work is joint with Bates.\(^9\)

Initial joint work with Anatoly Zhigljavsky led to a three-way long-term collaboration with Zhigljavsky and Luc Pronzato into the applications of dynamical systems to search and optimization. The novel idea is that a careful renormalization can convert certain search and optimization algorithms into dynamical systems, and rates of convergence can then be translated into rates


of expansion of such systems as measured by Kolmogorov-Shannon entropy. The first tranche of work is summarized in the monograph\textsuperscript{10} and includes an improvement over the celebrated Golden Section (Fibonacci) line search algorithm, which they have called the GS4 algorithm. Other joint work, begun there, is a detailed study of the attractors of algorithms of steepest descent type.\textsuperscript{11} An important insight has been the link with certain classes of optimal experimental design algorithm, which stems from the fact that both classes of algorithms can be interpreted as updating of measures, spectral measure in the case of renormalized steepest descent.

Following an initial paper on the application of Gröbner bases to experimental design,\textsuperscript{12} Wynn and Pistone collaborated with Riccomagno in a monograph,\textsuperscript{13} which was an early contribution to the field of “Algebraic Statistics”. With others, particularly the Genoa group led by Lorenzo Robbiano, they staged the successful series of GROSTAT workshops every year from 1998 to 2003. The field is growing rapidly with major contributions in the USA by Persi Diaconis, Berndt Sturmfels, Stephen Fienberg and a strong cadre of young researchers.

A long-standing collaboration with Giovagnoli began with work on group invariant orderings (majorization and its generalizations)\textsuperscript{14} and widened to include the introduction of D-ordering.\textsuperscript{15} Their new work includes the study of measures of agreement and a duality theory for generalized Lorenz ordering and integral stochastic orderings.

Henry Wynn is a professor of statistics at the London School of Economics and leads a research group. From 2001 to 2005 he was also a scientific co-director of EURANDOM, the international stochastics institute based at Eindhoven Technical University (TUE). He has a B.A. in honours mathematics from the University of Oxford and a Ph.D. in mathematical statistics from Imperial College, London. Following a period as a lecturer and then as a reader at Imperial College he became a professor of mathematical statistics at City University, London, in 1985 and Dean of Mathematics from 1987 to 1995. At City University he co-founded the Engineering Design Centre, for which he was a co-director. He moved, in 1995, to the University of Warwick

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as founding director of the Risk Initiative and Statistical Consultancy Unit (RISCU), which he helped build as a leading centre of its kind, well supported by a range of research grants. He holds a number of honours, including the Guy Medal in Silver from the Royal Statistical Society. He claims not to have wholly discarded a radical enthusiasm that led him to San Francisco in 1966, to Paris in support of the students in 1968 and to stand against the council candidate to become the President of the Royal Statistical Society at the age of 32.
Preface

The present volume is a collective monograph devoted to applications of the optimal design theory in optimization and statistics. The chapters reflect the topics discussed at the workshop “W-Optimum Design and Related Statistical Issues” that took place in Juan-les-Pins, France, in May 2005. The title of the workshop was chosen as a light-hearted celebration of the work of Henry Wynn. It was supported by the Laboratoire I3S (CNRS/Université de Nice, Sophia Antipolis), to which Henry is a frequent visitor.

The topics covered partly reflect the wide spectrum of Henry’s research interests. Algorithms for constructing optimal designs are discussed in Chap. 1, where Henry’s contribution to the field is acknowledged. Steepest-ascent algorithms used to construct optimal designs are very much related to general gradient algorithms for convex optimization. In the last ten years, a significant part of Henry’s research was devoted to the study of the asymptotic properties of such algorithms. This topic is covered by Chaps. 2 and 3. The work by Alessandra Giovagnoli concentrates on the use of majorization and stochastic ordering, and Chap. 4 is a hopeful renewal of their collaboration. One of Henry’s major recent interests is what is now called algebraic statistics, the application of computational commutative algebra to statistics, and he was partly responsible for introducing the experimental design sub-area, reviewed in Chap. 5. One other sub-area is the application to Bayesian networks and Chap. 6 covers this, with Chap. 7 being strongly related. Chapters 8 and 9 focus on nonlinear regression, a topic with strong links to both design and optimization.

We hope that the volume will be of interest to both the specialist in the areas covered and also tempt the non-expert. Although several papers are in the nature of a review they all contain substantial new material and are essentially the beginning of new fields needing a continuing research effort.

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Sophia Antipolis, France
Cardiff, UK

Luc Pronzato
Anatoly Zhigljavsky
List of Contributors

Peter E. Caines
Department of Electrical and Computer Engineering
McGill University
Montreal, Canada
peterc@cim.mcgill.ca

Rob Deardon
Department of Mathematics and Statistics, University of Guelph
Guelph, ON, Canada
rdeardon@uoguelph.ca

Alessandra Giovagnoli
Department of Statistical Sciences
University of Bologna
Via Belle Arti 41
Bologna 40126, Italy
alessandra.giovagnoli@unibo.it

Rebecca Haycroft
School of Mathematics
Cardiff University
Senghennydd Road
Cardiff CF24 4AG, UK
haycroftRJ@cf.ac.uk

Alexandr Ivanov
Kyiv Polytechnic Institute
National Technical University
37 Peremogy Avenue
03056 Kyiv, Ukraine
Ivanov@paligora.kiev.ua

Nikolai Leonenko
School of Mathematics
Cardiff University
Senghennydd Road
Cardiff CF24 4AG, UK
LeonenkoN@cardiff.ac.uk

Johnny Marzialetti
Department of Statistical Sciences
University of Bologna
Via Belle Arti 41, Bologna 40126
Italy
marzialetti@stat.unibo.it

Andrej Pázman
Department of Applied Mathematics and Statistics
Faculty of Mathematics, Physics and Informatics
Comenius University
84248 Bratislava, Slovakia
pazman@center.fmph.uniba.sk

Giovanni Pistone
Department of Mathematics
Politecnico di Torino
Corso Duca degli Abruzzi, 24
10129 Torino
Italy
giovanni.pistone@polito.it
Luc Pronzato  
Laboratoire I3S, CNRS – UNSA  
Les Algorithmes – Bât. Euclide B  
2000 route des Lucioles  
B.P. 121  
06903 Sophia Antipolis  
France  
pronzato@i3s.unice.fr

Eva Riccomagno  
Dipartimento di Matematica  
Università degli Studi di  
Genova  
Via Dodecaneso 35  
16149 Genova  
Italia  
riccomag@dima.unige.it

Maria Piera Rogantin  
Dipartimento di Matematica  
Università degli Studi di  
Genova  
Via Dodecaneso 35  
16149 Genova  
Italia  
rogantin@dima.unige.it

Jim Q. Smith  
Department of Statistics  
The University of Warwick  
Gibbet Hill Road  
Coventry CV4 7AL, UK  
J.Q.Smith@warwick.ac.uk

Ben Torsney  
Department of Statistics  
University of Glasgow  
15 University Gardens  
Glasgow G12 8QW UK  
bent@stats.gla.ac.uk

Henry P. Wynn  
London School of Economics  
and Political Science  
Houghton Street  
London WC2A 2AE UK  
h.wynn@lse.ac.uk

Anatoly Zhigljavsky  
Cardiff University  
School of Mathematics  
Senghennydd Road  
Cardiff CF24 4AG UK  
zhiglavlavskyAA@cf.ac.uk
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