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# Fuzzy Hypergraphs and Related Extensions

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*We dedicate this book to the memory of Prof. Lotfi A. Zadeh!*

# Foreword

It was stated by G. J. Klir that among the various paradigmatic changes in science and mathematics in the twentieth century, one such change concerned the concept of uncertainty. In science, this change has been manifested by a gradual transition from the traditional view, which states that uncertainty is undesirable in science and should be avoided by all possible means, to an alternative view, which is tolerant of uncertainty and insists that science cannot avoid it. Uncertainty is essential to science and has great utility. An important point in the evolution of the modern concept of uncertainty is the publication of a seminal paper by Lotfi Zadeh.

Fuzzy set theory provides a methodology for carrying out approximate reasoning processes when available information is uncertain, incomplete, imprecise, or vague. This is especially true when observations are expressed in linguistic terms. The success of this methodology has been demonstrated in a variety of fields such as control systems where mathematical models are difficult to specify and in expert systems where rules express knowledge and facts are linguistic in nature. The capability of fuzzy sets to express gradual transitions from membership to non-membership and vice-versa has a broad utility. It provides us not only with meaningful and powerful representations of measurement of uncertainties, but also with a meaningful representation of vague concepts expressed in natural language. Concerning the future of fuzzy logic, it might lie in the new ideas arising from computing with words and perceptions.

Based on Zadeh's fuzzy relations, the first definition of fuzzy graphs was given in 1973 by Arnold Kauffman, the French engineer and professor of applied mechanics and operations research, in the world's first textbook on fuzzy sets and systems. An English version of this book, titled *Introduction to the Theory of Fuzzy Subsets,* was published in 1975. This was the same year Azriel Rosenfeld also provided a concept of a fuzzy graph. He introduced fuzzy analogs of several basic graph-theoretic concepts. Rosenfeld's paper presented the concepts of subgraphs, paths, connectedness, cliques, bridges, trees, and forests and established some of their properties. It appeared in the proceedings of the US-Japan Seminar on Fuzzy Sets and Their Applications. At this seminar, an alternative analysis of fuzzy graphs was also presented by Raymond T. Yeh and S. Y. Bang. Their definition of a fuzzy graph was suitable for cluster analysis and also for database theory. Rosenfeld's paper opened the door for the development of an entire new area in graph theory. Fuzzy graph theory is now a discipline in itself. Hundreds of papers and numerous books have been published in this area. Lee-Kwang and Lee extended crisp hypergraphs by introducing the notion of fuzzy hypergraphs.

This book, *Fuzzy Hypergraphs and Related Extensions*, by Prof. Akram and Dr. Luqman is another strong contribution to fuzzy graph theory. Professor Akram has been a leader in this field for some time. He has published several papers on a wide variety of fuzzy graph-theoretic structures. The results in this book should be very useful in modeling various applications. Crisp hypergraphs have found applications in chemistry, psychology, genetics, human activities, optimization, cellular networks, parallel computing, clustering, information system architecture, social networks, traffic control, engineering, and image processing. The potential for applications of fuzzy hypergraphs can be seen by merely pondering the definition of a fuzzy hypergraph.

It is my hope that researchers will apply the concepts of fuzzy hypergraphs to examine the existential problem of climate change. This may be the most serious problem facing the world at this time. All member states of the United Nations adopted the Agenda 2030 and the Sustainable Development Goals (SDGs). The 17 SDGs describe a universal agenda that applies to and must be implemented by all countries. Among these SDGs is SDG 13 Climate Change. Adverse effects of climate change result in a substantial increase in cruel crimes of human trafficking and modern slavery. These crimes should be brought to a halt. Human trafficking is a prime candidate to be studied using techniques from fuzzy logic. Accurate data concerning trafficking in persons is impossible to obtain. The goal of the trafficker is to be undetected. The size of the problem also makes it very difficult to obtain accurate data. There are also many other reasons for the scarcity of data. I thank Prof. Akram and Dr. Luqman for their timely publication.

Omaha, NE, USA John N. Mordeson

# Preface

Hypergraphs are one of the most successful tools for modeling practical problems in different fields, inclusive of computer science, systems modeling, web information system architecture, service-oriented architecture, and social networks. However, crisp hypergraphs are not sufficient to describe all existing relations between objects. Motivated by this concern, Lee-Kwang and Lee redefined and extended crisp hypergraphs by means of the notion of fuzzy hypergraphs, whose inception had been earlier discussed by Kaufmann. Professors Mordeson and Nair made a real contribution by compiling their comprehensive monograph *Fuzzy Graphs and Fuzzy Hypergraphs*, which motivated us to work in this direction.

Fuzzy set theory was introduced by Lotfi Zadeh in 1965 as a generalization of classical set theory that allows us to represent imprecise and vague phenomena. Since then, fuzzy sets and fuzzy logic have been applied in many real situations that implemented uncertainty. The traditional fuzzy set uses one real value from the unit interval [0,1] in order to represent the grade of membership of objects to a fuzzy set defined on the concerned universe. In some applications such as expert systems, belief systems, and information fusion, not only should we consider the truth-membership supported by evidence but also the falsity-membership against such evidence. Similarly, in most real problems, information consistently comes from more than one agent or from various sources. Due to the limitation of human's knowledge to understand the complex problems, one cannot aspire to apply a single type of uncertain methodology to deal with all such situations. It is therefore necessary to develop generalized mathematical models rather than being satisfied with narrow structures of uncertainty. Researchers have put forward several generalized models of fuzzy sets, including intuitionistic fuzzy sets, bipolar fuzzy sets, $m$-polar fuzzy sets, Pythagorean fuzzy sets, $q$-rung orthopair fuzzy sets, and single-valued neutrosophic sets. We have applied these generalized models to hypergraphs.

This monograph deals with fuzzy hypergraphs, their related extensions, and applications. It originates from our papers published in various scientific journals. This book may be useful for researchers in mathematics, computer scientists, and social scientists alike. In Chap. 1, we present fundamental and technical concepts like fuzzy hypergraphs, fuzzy column hypergraphs, fuzzy row hypergraphs, fuzzy competition hypergraphs, fuzzy $k$-competition hypergraphs and fuzzy neighborhood hypergraphs, and $\mathcal{N}$-hypergraphs, complex fuzzy hypergraphs, $\mu e^{i\theta}$-level hypergraphs, and $C_f$-tempered complex fuzzy hypergraphs. We describe applications of fuzzy competition hypergraphs in decision support systems, including predator–prey relations in ecological niches, social networks, and business marketing.

Chapter 2 defines intuitionistic fuzzy hypergraphs, dual intuitionistic fuzzy hypergraphs, intuitionistic fuzzy line graphs, and 2-section of an intuitionistic fuzzy hypergraph. It also includes applications of intuitionistic fuzzy hypergraphs in planet surface networks, selection of authors of intersecting communities in a social network, and grouping of incompatible chemical substances. We have designed certain algorithms to construct dual intuitionistic fuzzy hypergraphs, intuitionistic fuzzy line graphs, and the selection of objects in decision-making problems. Further, we define complex intuitionistic fuzzy hypergraphs, 2-section, and line graphs of complex intuitionistic fuzzy hypergraphs.

In Chap. 3, we present the notion of $A = [\mu^-, \mu^+]$-tempered interval-valued fuzzy hypergraphs and some of their properties. Moreover, we discuss the notions of vague hypergraphs, dual vague hypergraphs, and $A$-tempered vague hypergraphs. Finally, we describe interval-valued intuitionistic fuzzy hypergraphs and interval-valued intuitionistic fuzzy transversals of $H$.

Chapter 4 discusses the concept of bipolar fuzzy directed hypergraph. We describe certain operations on bipolar fuzzy directed hypergraphs, which include addition, multiplication, vertex-wise multiplication, and structural subtraction. We discuss the concept of $B = (m^+, m^-)$-tempered bipolar fuzzy directed hypergraphs and investigate some of their basic properties. We present an algorithm to compute the minimum arc length of a bipolar fuzzy directed hyperpath.

Chapter 5 includes the notions of regular $m$-polar fuzzy hypergraphs and totally regular $m$-polar fuzzy hypergraphs. We discuss the applications of $m$-polar fuzzy hypergraphs in decision-making problems. Furthermore, the notion of $m$-polar fuzzy directed hypergraph is discussed along with the depiction of certain operations on them. We also describe an application of $m$-polar fuzzy directed hypergraphs in business strategy.

Chapter 6 presents the concepts including $q$-rung orthopair fuzzy hypergraphs, $(\alpha, \beta)$-level hypergraphs, and transversals and minimal transversals of $q$-rung orthopair fuzzy hypergraphs. We implement some interesting notions of $q$-rung orthopair fuzzy hypergraphs into decision-making. We describe additional concepts like $q$-rung orthopair fuzzy directed hypergraphs, dual directed hypergraphs, line graphs, and coloring of $q$-rung orthopair fuzzy directed hypergraphs. We also apply other interesting notions of $q$-rung orthopair fuzzy directed hypergraphs to real-life problems. Further, we study complex q-rung orthopair fuzzy hypergraphs with application.

In Chap. 7, we present $q$-rung picture fuzzy hypergraphs and illustrate the formation of granular structures using $q$-rung picture fuzzy hypergraphs and level hypergraphs. Moreover, we define $q$-rung picture fuzzy equivalence relations and its associated $q$-rung picture fuzzy hierarchical quotient space structures. We also present an arithmetic example in order to demonstrate the benefits and validity of this model.

In Chap. 8, we illustrate the formation of granular structures using $m$-polar fuzzy hypergraphs and level hypergraphs. Further, we define $m$-polar fuzzy hierarchical quotient space structures. The mappings between the $m$-polar fuzzy hypergraphs depict the relationships among granules occurred in different levels. The consequences reveal that the representation of partition of universal set is more efficient through $m$-polar fuzzy hypergraphs as compared to crisp hypergraphs. We also present some examples and a real world problem to signify the validity of our proposed model.

Chapter 9 discusses the concepts including single-valued neutrosophic hypergraphs, dual single-valued neutrosophic hypergraphs, and transversal single-valued neutrosophic hypergraphs. Additionally, we discuss the notions of intuitionistic single-valued neutrosophic hypergraphs, and dual intuitionistic single-valued neutrosophic hypergraphs. We describe an application of intuitionistic single-valued neutrosophic hypergraphs in a clustering problem. Then, we present other related concepts like single-valued neutrosophic directed hypergraphs, single-valued neutrosophic line directed graphs, and dual single-valued neutrosophic directed hypergraphs. Finally, in this section, we describe the applications of single-valued neutrosophic directed hypergraphs. Additionally, we define complex neutrosophic hypergraphs and $T$-related complex neutrosophic hypergraphs with applications.

In Chap. 10, we present bipolar neutrosophic hypergraphs and $B$-tempered bipolar neutrosophic hypergraphs. We describe the concepts of transversals, minimal transversals, and locally minimal transversals of bipolar neutrosophic hypergraphs. Furthermore, we put forward some applications of bipolar neutrosophic hypergraphs in marketing and biology. We also introduce bipolar neutrosophic directed hypergraphs, regular bipolar neutrosophic directed hypergraphs, homomorphism, and isomorphism on bipolar neutrosophic directed hypergraphs. To conclude, we describe an efficient algorithm to solve decision-making problems.

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