
Domain Conditions and Social Rationality

Satish Kumar Jain

Domain Conditions and Social Rationality

Satish Kumar Jain
Formerly Professor
Jawaharlal Nehru University
New Delhi, India

ISBN 978-981-13-9671-7 ISBN 978-981-13-9672-4 (eBook)
<https://doi.org/10.1007/978-981-13-9672-4>

© Springer Nature Singapore Pte Ltd. 2019

This work is subject to copyright. All rights are reserved by the Publisher, whether the whole or part of the material is concerned, specifically the rights of translation, reprinting, reuse of illustrations, recitation, broadcasting, reproduction on microfilms or in any other physical way, and transmission or information storage and retrieval, electronic adaptation, computer software, or by similar or dissimilar methodology now known or hereafter developed.

The use of general descriptive names, registered names, trademarks, service marks, etc. in this publication does not imply, even in the absence of a specific statement, that such names are exempt from the relevant protective laws and regulations and therefore free for general use.

The publisher, the authors and the editors are safe to assume that the advice and information in this book are believed to be true and accurate at the date of publication. Neither the publisher nor the authors or the editors give a warranty, expressed or implied, with respect to the material contained herein or for any errors or omissions that may have been made. The publisher remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

This Springer imprint is published by the registered company Springer Nature Singapore Pte Ltd. The registered company address is: 152 Beach Road, #21-01/04 Gateway East, Singapore 189721, Singapore

For

*Abha, Avinash, Mayank,
Rajendra and Subrata*

Preface

One implication of the impossibility theorems of social choice theory, Arrow impossibility theorem being the most important of them, is that all ‘democratic’ methods of arriving at social decisions by combining individual preferences which satisfy Arrow’s independence of irrelevant alternatives, a requirement quite crucial for the unambiguity of social choices, fail to generate rational social preferences for some configurations of individual preferences. The problem is exemplified by the famous voting paradox associated with the majority rule. Under majority rule, it is possible to have alternative x defeating alternative y in a majority vote, alternative y defeating alternative z in a majority vote and alternative z defeating alternative x in a majority vote, thereby making it impossible to choose rationally from among these three alternatives. Thus, in the context of any rule that is to be used for arriving at social decisions on the basis of individual preferences, it is important to know the configurations of individual preferences under which it would be possible to choose rationally. This monograph is almost exclusively concerned with the derivation of conditions for various rules and classes of rules under which the social preferences would be rational. To make the monograph essentially self-contained, the basic social choice theoretic concepts, definitions, propositions and theorems needed for the subject matter of the monograph have been given in Chap. 2. Each of the Chaps. 3–10 deals with a particular rule or a class of rules and is essentially independent of other chapters. Chapter 11 depends, in addition to Chap. 2, on Chaps. 3, 6, 9 and 10. The treatment throughout is rigorous. Unlike most of the literature on domain conditions, care is taken in this monograph with respect to the number of individuals in the ‘necessity’ proofs.

This monograph was written while I was Indian Council of Social Science Research (ICSSR) National Fellow; thanks are due to ICSSR for awarding me the National Fellowship. I wish to thank the Management Development Institute for providing me with affiliation and for providing facilities. I also wish to thank the Economics faculty of the Management Development Institute, particularly Prof. Sunil Ashra and Prof. Rohit Prasad.

Some of the material included in this book I had published as papers in journals and in an edited volume; I thank the publishers of these journals and the edited volume for permission to include material from them in the book. The editorial help from Springer is gratefully acknowledged; I particularly wish to thank Ms. Nupoor Singh. Thanks are due to Kaushal Kishore and Amit Kumar for their help in proofreading.

New Delhi, India

Satish Kumar Jain

Contents

1	Introduction	1
	References	6
2	The Preliminaries	9
2.1	Binary Relations	11
2.2	Social Decision Rules	16
2.3	Latin Squares	24
	References	25
3	The Method of Majority Decision	27
3.1	Characterization of the Method of Majority Decision	29
3.2	Restrictions on Preferences	31
3.3	Conditions for Transitivity	32
3.4	Conditions for Quasi-transitivity	35
3.5	Relationships Among Conditions on Preferences	39
3.5.1	Single-Peakedness	39
3.5.2	Single-Cavedness	39
3.5.3	Separability into Two Groups	40
3.5.4	Value Restriction	41
3.5.5	Dichotomous Preferences	43
3.5.6	Echoic Preferences	43
3.5.7	Antagonistic Preferences	44
3.5.8	Extremal Restriction	44
3.5.9	Taboo Preferences	45
3.5.10	Weak Latin Square Partial Agreement	46
3.5.11	Limited Agreement	48
3.5.12	Latin Square Partial Agreement	48
3.6	Notes on Literature	50
	References	52
4	The Strict Majority Rule	53
4.1	Notation and Definitions	55
4.2	Characterization of the Strict Majority Rule	55
4.3	Restrictions on Preferences	57

4.4	Transitivity Under the Strict Majority Rule	57
4.5	Quasi-transitivity Under the Strict Majority Rule	59
4.6	Interrelationships Among Restrictions on Preferences	61
4.7	Notes on Literature	66
	References	66
5	The Class of Semi-strict Majority Rules	69
5.1	Restrictions on Preferences	71
5.2	Conditions for Transitivity	72
5.3	Conditions for Quasi-transitivity	85
	Reference	92
6	Special Majority Rules	93
6.1	Placement Restriction	94
6.2	Transitivity Under Special Majority Rules	96
6.3	Quasi-transitivity Under Special Majority Rules	100
6.4	Two-Thirds Majority Rule	104
	Reference	108
7	The Class of Strict Majority Rules	109
7.1	Characterization of the Class of Strict Majority Rules	110
7.2	Transitivity Under the Strict Majority Rules	111
7.3	Quasi-transitivity Under the Strict Majority Rules	114
	References	115
8	The Class of Pareto-Inclusive Strict Majority Rules	117
8.1	Definitions of Pareto-Rule and the Class of Pareto-Inclusive Strict Majority Rules	118
8.2	Transitivity Under the Pareto-Inclusive Strict Majority Rules	119
8.3	Quasi-transitivity Under the Pareto-Inclusive Strict Majority Rules	124
9	Social Decision Rules Which Are Simple Games	129
9.1	Characterization of Social Decision Rules Which Are Simple Games	131
9.2	Transitivity Under Simple Games	133
9.3	Quasi-transitivity Under Simple Games	136
	References	138
10	Neutral and Monotonic Binary Social Decision Rules	139
10.1	Characterization Theorems for Neutral and Monotonic Binary Social Decision Rules	141
10.1.1	Characterization of Transitivity	146
10.1.2	Characterization of Quasi-transitivity	147
10.1.3	Characterization of Acyclicity	148

10.2	Conditions for Transitivity	149
10.3	Conditions for Quasi-transitivity	151
10.4	Conditions for Transitivity and Quasi-transitivity When Individual Orderings Are Linear	153
10.5	Notes on Literature	156
	References	158
11	Quasi-transitive Individual Preferences	159
11.1	Some Definitions Involving Quasi-transitive Binary Relations	161
11.2	The Method of Majority Decision	162
11.3	The Special Majority Rules	168
11.4	Social Decision Rules Which Are Simple Games	173
11.5	Neutral and Monotonic Social Decision Rules	175
11.5.1	Characterization of Neutral and Monotonic Binary Social Decision Rules	175
11.5.2	Characterization of Transitivity	179
11.5.3	Alternative Characterization of the Null Social Decision Rule	179
11.5.4	Characterization of Quasi-transitivity	180
11.5.5	Characterization of Acyclicity	182
11.5.6	Condition for Quasi-transitivity	183
11.6	Notes on Literature	184
11.7	Domain Conditions for Acyclicity	185
	References	187
12	Summary and Concluding Remarks	189

About the Author

Satish Kumar Jain is an economist who holds a Master's in Economics from Delhi School of Economics and a Doctorate in Economics from the University of Rochester. He was on the faculty of the Centre for Economic Studies and Planning at Jawaharlal Nehru University for three and a half decades. He was Reserve Bank of India Chair Professor from 2011 to 2013, and was an Indian Council of Social Science Research (ICSSR) National Fellow from 2016 to 2018. He has taught at Shri Ram College of Commerce, Delhi; the Indian Statistical Institute, Delhi; and the Indian Institute of Information Technology, Hyderabad. He has authored *Economic Analysis of Liability Rules* (Springer, 2015); edited *Law and Economics* (Oxford University Press, 2010); and co-edited *Economic Growth, Efficiency, and Inequality* (with Anjan Mukherji, Routledge, 2015). His teaching and research interests include social choice theory, and law and economics.

Abbreviations

A	Anonymity
AP	Antagonistic preferences
AUEV	Absence of unique extremal value
CI	Cyclical indifference
CP	Conflictive preferences
DP	Dichotomous preferences
EP	Echoic preferences
ER	Extremal restriction
EVR	Extremal value restriction
I	Independence of irrelevant alternatives
LA	Limited agreement
LSEVR	Latin Square extremal value restriction
LSIRR-Q	Latin Square intransitive relation restriction-Q
LSLOR	Latin Square linear ordering restriction
LSLOR-Q	Latin Square linear ordering restriction-Q
LSPA	Latin Square partial agreement
LSPA-Q	Latin Square partial agreement-Q
LSUVR	Latin Square unique value restriction
\bar{M}	Strict monotonicity
M	Monotonicity
MMD	Method of majority decision
N	Neutrality
\bar{P}	Pareto-criterion
P	Weak Pareto-criterion
PA	Partial agreement
PI	Pareto-indifference
PP	Pareto-preference
PQT	Pareto quasi-transitivity
PR	Placement restriction
SAP(1)	Strongly antagonistic preferences (1)
SAP(2)	Strongly antagonistic preferences (2)
SC	Single-Cavedness
SDR	Social decision rule

SG	Separability into two groups
SP	Single-Peakedness
SPR	Strict placement restriction
SWF	Social welfare function
TP	Taboo preferences
UVR	Unique value restriction
VR(1)	Value restriction (1)
VR(2)	Value restriction (2)
WCP	Weak conflictive preferences
WER	Weak extremal restriction
WER-Q	Weak extremal restriction-Q
WLSEVR	Weak Latin Square extremal value restriction
WLSPA	Weak Latin Square partial agreement
WM	Weak monotonicity
WPQT	Weak Pareto quasi-transitivity
WPR	Weak Pareto rule

Glossary of Symbols

\sim	Not (negation)
\wedge	And (conjunction)
\vee	Or (disjunction)
\rightarrow	If-then (conditional)
\leftrightarrow	If and only if (biconditional)
iff	If and only if
\forall	For all (universal quantifier)
\exists	There exists (existential quantifier)
\in	Is an element of
\notin	Is not an element of
$B - A$	Complement of set A in set B
$A \cap B$	Intersection of sets A and $B = \{x x \in A \text{ and } x \in B\}$
$A \cup B$	Union of sets A and $B = \{x x \in A \text{ or } x \in B\}$
$A \subseteq B$	A is a subset of B , i.e., $(\forall x)(x \in A \rightarrow x \in B)$
$A = B$	Sets A and B are equal, i.e., $(A \subseteq B \text{ and } B \subseteq A)$
$A \neq B$	Sets A and B are not equal, i.e., $\sim(A = B)$
$A \subset B$	A is a proper subset of B , i.e., $(A \subseteq B \text{ and } A \neq B)$
$A \supseteq B$	A is a superset of B , i.e., $(B \text{ is a subset of } A)$
$A \supset B$	A is a proper superset of B , i.e., $(B \text{ is a proper subset of } A)$
$A \times B$	Cartesian product of sets A and $B = \{(x, y) x \in A \text{ and } y \in B\}$
S	The set of social alternatives
s	Number of social alternatives
$\#X$	Number of alternatives in the set X
N	The set of individuals
n	Number of individuals
$n(), N()$	Number of individuals holding the preferences specified in the parentheses
Φ	The set of permutations of alternatives in S
ϕ	A permutation of alternatives in S
Θ	The set of permutations of individuals in N
θ	A permutation of individuals in N
R	Social binary weak preference relation, socially at least as good as
$P(R), P$	Asymmetric part of R , socially better than

$I(R), I$	Symmetric part of R , socially indifferent to
$C(S, R)$	The set of best elements in S according to R
R_i	Binary weak preference relation of individual i , for individual i at least as good as
$P(R_i), P_i$	Asymmetric part of R_i , for individual i better than
$I(R_i), I_i$	Symmetric part of R_i , for individual i indifferent to
\mathbb{N}	The set of positive integers
\mathcal{B}	The set of all binary relations on the set of social alternatives S
\mathcal{C}	The set of all reflexive and connected binary relations on the set of social alternatives S
\mathcal{A}	The set of all reflexive, connected and acyclic binary relations on the set of social alternatives S
\mathcal{Q}	The set of all reflexive, connected and quasi-transitive binary relations on the set of social alternatives S
\mathcal{T}	The set of all reflexive, connected and transitive binary relations (orderings) on the set of social alternatives S
\mathcal{L}	The set of all reflexive, connected, transitive and anti-symmetric binary relations (linear orderings) on the set of social alternatives S
\mathcal{D}	A set of binary relations on the set of social alternatives S
\mathcal{T}^n	n -fold Cartesian product of \mathcal{T} with itself
\mathcal{Q}^n	n -fold Cartesian product of \mathcal{Q} with itself
\mathcal{D}^n	n -fold Cartesian product of \mathcal{D} with itself
(R_1, \dots, R_n)	A profile of individual orderings, an element of \mathcal{T}^n ; or a profile of reflexive, connected and quasi-transitive individual weak preference relations, an element of \mathcal{Q}^n
\mathbb{D}	A set of profiles of individual binary weak preference relations, a subset of \mathcal{T}^n or a subset of \mathcal{Q}^n
$R A$	Restriction of R to $A \subseteq S$, i.e., $R A = R \cap (A \times A)$
$\mathcal{D} A$	Restriction of \mathcal{D} to $A \subseteq S$, i.e., $\mathcal{D} A = \{R A \mid R \in \mathcal{D}\}$, $\mathcal{D} \subseteq \mathcal{B}$
$D(x, y)$	Almost decisive for the ordered pair (x, y)
$\overline{D}(x, y)$	Decisive for the ordered pair (x, y)
$S(x, y)$	Almost semidecisive for the ordered pair (x, y)
$\overline{S}(x, y)$	Semidecisive for the ordered pair (x, y)
$D_{N-A}(x, y)$	Almost $(N - A)$ —decisive for the ordered pair (x, y)
$\overline{D}_{N-A}(x, y)$	$(N - A)$ —decisive for the ordered pair (x, y)
$S_{N-A}(x, y)$	Almost $(N - A)$ —semidecisive for the ordered pair (x, y)
$\overline{S}_{N-A}(x, y)$	$(N - A)$ —semidecisive for the ordered pair (x, y)
W	The set of winning coalitions
W_m	The set of minimal winning coalitions
B	The set of blocking coalitions
B_s	The set of strictly blocking coalitions
$WLS(xyzx)$	Weak Latin Square with the cyclical arrangement $xyzx$
$LS(xyzx)$	Latin Square with the cyclical arrangement $xyzx$

$T[WLS(xyzx)]$	The set of all orderings of the triple $\{x, y, z\}$ that can be part of $WLS(xyzx)$
$T[LS(xyzx)]$	The set of all orderings of the triple $\{x, y, z\}$ that can be part of $LS(xyzx)$
$Q[WLS(xyzx)]$	The set of all reflexive, connected and quasi-transitive binary weak preference relations of the triple $\{x, y, z\}$ that can be part of $WLS(xyzx)$
$Q[LS(xyzx)]$	The set of all reflexive, connected and quasi-transitive binary weak preference relations of the triple $\{x, y, z\}$ that can be part of $LS(xyzx)$
$B[WLS(xyzx)]$	The set of all binary weak preference relations of the triple $\{x, y, z\}$ that can be part of $WLS(xyzx)$
$B[LS(xyzx)]$	The set of all binary weak preference relations of the triple $\{x, y, z\}$ that can be part of $LS(xyzx)$
$\lceil t \rceil$	Smallest integer greater than or equal to t
$\lfloor t \rfloor$	Largest integer smaller than or equal to t
xUy	$(\forall i \in N) (xR_i y)$
$x\bar{U}y$	$(\forall i \in N) (xP_i y)$
\mathcal{F}_1^T	$\{\mathcal{D} \subseteq \mathcal{F} \mid \text{every } (R_1, \dots, R_n) \in \mathcal{D}^n \text{ yields transitive } R\}$
\mathcal{F}_2^T	$\{\mathcal{D} \subseteq \mathcal{F} \mid \text{some } (R_1, \dots, R_n) \in \mathcal{D}^n \text{ yields intransitive } R\}$
\mathcal{F}_1^Q	$\{\mathcal{D} \subseteq \mathcal{F} \mid \text{every } (R_1, \dots, R_n) \in \mathcal{D}^n \text{ yields quasi-transitive } R\}$
\mathcal{F}_2^Q	$\{\mathcal{D} \subseteq \mathcal{F} \mid \text{some } (R_1, \dots, R_n) \in \mathcal{D}^n \text{ yields non quasi-transitive } R\}$
\mathcal{F}_1^A	$\{\mathcal{D} \subseteq \mathcal{F} \mid \text{every } (R_1, \dots, R_n) \in \mathcal{D}^n \text{ yields acyclic } R\}$
\mathcal{F}_2^A	$\{\mathcal{D} \subseteq \mathcal{F} \mid \text{some } (R_1, \dots, R_n) \in \mathcal{D}^n \text{ yields non-acyclic } R\}$