Domain Conditions and Social Rationality

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For

Abha, Avinash, Mayank,
Rajendra and Subrata

Preface

One implication of the impossibility theorems of social choice theory, Arrow impossibility theorem being the most important of them, is that all 'democratic' methods of arriving at social decisions by combining individual preferences which satisfy Arrow's independence of irrelevant alternatives, a requirement quite crucial for the unambiguity of social choices, fail to generate rational social preferences for some configurations of individual preferences. The problem is exemplified by the famous voting paradox associated with the majority rule. Under majority rule, it is possible to have alternative x defeating alternative y in a majority vote, alternative y defeating alternative z in a majority vote and alternative z defeating alternative x in a majority vote, thereby making it impossible to choose rationally from among these three alternatives. Thus, in the context of any rule that is to be used for arriving at social decisions on the basis of individual preferences, it is important to know the configurations of individual preferences under which it would be possible to choose rationally. This monograph is almost exclusively concerned with the derivation of conditions for various rules and classes of rules under which the social preferences would be rational. To make the monograph essentially self-contained, the basic social choice theoretic concepts, definitions, propositions and theorems needed for the subject matter of the monograph have been given in Chap. 2. Each of the Chaps. 3–10 deals with a particular rule or a class of rules and is essentially independent of other chapters. Chapter 11 depends, in addition to Chap. 2, on Chaps. 3, 6, 9 and 10. The treatment throughout is rigorous. Unlike most of the literature on domain conditions, care is taken in this monograph with respect to the number of individuals in the 'necessity' proofs.

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New Delhi, India Satish Kumar Jain

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About the Author

Satish Kumar Jain is an economist who holds a Master's in Economics from Delhi School of Economics and a Doctorate in Economics from the University of Rochester. He was on the faculty of the Centre for Economic Studies and Planning at Jawaharlal Nehru University for three and a half decades. He was Reserve Bank of India Chair Professor from 2011 to 2013, and was an Indian Council of Social Science Research (ICSSR) National Fellow from 2016 to 2018. He has taught at Shri Ram College of Commerce, Delhi; the Indian Statistical Institute, Delhi; and the Indian Institute of Information Technology, Hyderabad. He has authored *Economic Analysis of Liability Rules* (Springer, 2015); edited *Law and Economics* (Oxford University Press, 2010); and co-edited *Economic Growth, Efficiency, and Inequality* (with Anjan Mukherji, Routledge, 2015). His teaching and research interests include social choice theory, and law and economics.

Abbreviations

A	Anonymity
AP	Antagonistic preferences
AUEV	Absence of unique extremal value
CI	Cyclical indifference
CP	Conflictive preferences
DP	Dichotomous preferences
EP	Echoic preferences
ER	Extremal restriction
EVR	Extremal value restriction
I	Independence of irrelevant alternatives
LA	Limited agreement
LSEVR	Latin Square extremal value restriction
LSIRR-Q	Latin Square intransitive relation restriction-Q
LSLOR	Latin Square linear ordering restriction
LSLOR-Q	Latin Square linear ordering restriction-Q
LSPA	Latin Square partial agreement
LSPA-Q	Latin Square partial agreement-Q
LSUVR	Latin Square unique value restriction
$\overline{\mathrm{M}}$	Strict monotonicity
M	Monotonicity
MMD	Method of majority decision
N	Neutrality
$\overline{\mathrm{P}}$	Pareto-criterion
P	Weak Pareto-criterion
PA	Partial agreement
PI	Pareto-indifference
PP	Pareto-preference
PQT	Pareto quasi-transitivity
PR	Placement restriction
SAP(1)	Strongly antagonistic preferences (1)
SAP(2)	Strongly antagonistic preferences (2)
SC	Single-Cavedness
SDR	Social decision rule

SG	Separability into two groups
SP	Single-Peakedness
SPR	Strict placement restriction
SWF	Social welfare function
TP	Taboo preferences
UVR	Unique value restriction
VR(1)	Value restriction (1)
VR(2)	Value restriction (2)
WCP	Weak conflictive preferences
WER	Weak extremal restriction
WER-Q	Weak extremal restriction-Q
WLSEVR	Weak Latin Square extremal value restriction
WLSPA	Weak Latin Square partial agreement
WM	Weak monotonicity
WPQT	Weak Pareto quasi-transitivity
WPR	Weak Pareto rule

Glossary of Symbols

$\sim$	Not (negation)
$\wedge$	And (conjunction)
$\vee$	Or (disjunction)
$\rightarrow$	If-then (conditional)
$\leftrightarrow$	If and only if (biconditional)
iff	If and only if
$\forall$	For all (universal quantifier)
$\exists$	There exists (existential quantifier)
$\in$	Is an element of
$\notin$	Is not an element of
$B - A$	Complement of set A in set B
$A \cap B$	Intersection of sets A and $B = \{x \vert x \in A \text{ and } x \in B\}$
$A \cup B$	Union of sets A and $B = \{x \vert x \in A \text{ or } x \in B\}$
$A \subseteq B$	A is a subset of B, i.e., $(\forall x)(x \in A \rightarrow x \in B)$
$A = B$	Sets A and B are equal, i.e., $(A \subseteq B$ and $B \subseteq A)$
$A \neq B$	Sets A and B are not equal, i.e., $\sim(A = B)$
$A \subset B$	A is a proper subset of B, i.e., $(A \subseteq B$ and $A \neq B)$
$A \supseteq B$	A is a superset of B, i.e., (B is a subset of A)
$A \supset B$	A is a proper superset of B, i.e., (B is a proper subset of A)
$A \times B$	Cartesian product of sets A and $B = \{(x, y) \vert x \in A \text{ and } y \in B\}$
S	The set of social alternatives
s	Number of social alternatives
$\#X$	Number of alternatives in the set X
N	The set of individuals
n	Number of individuals
$n(), N()$	Number of individuals holding the preferences specified in the parentheses
Φ	The set of permutations of alternatives in S
ϕ	A permutation of alternatives in S
Θ	The set of permutations of individuals in N
θ	A permutation of individuals in N
R	Social binary weak preference relation, socially at least as good as
$P(R), P$	Asymmetric part of R, socially better than

$I(R), I$	Symmetric part of R, socially indifferent to
$C(S, R)$	The set of best elements in S according to R
R_i	Binary weak preference relation of individual i, for individual i at least as good as
$P(R_i), P_i$	Asymmetric part of R_i, for individual i better than
$I(R_i), I_i$	Symmetric part of R_i, for individual i indifferent to
$\mathbb{N}$	The set of positive integers
$\mathscr{B}$	The set of all binary relations on the set of social alternatives S
$\mathscr{C}$	The set of all reflexive and connected binary relations on the set of social alternatives S
$\mathscr{A}$	The set of all reflexive, connected and acyclic binary relations on the set of social alternatives S
$\mathscr{Q}$	The set of all reflexive, connected and quasi-transitive binary relations on the set of social alternatives S
$\mathscr{T}$	The set of all reflexive, connected and transitive binary relations (orderings) on the set of social alternatives S
$\mathscr{L}$	The set of all reflexive, connected, transitive and anti-symmetric binary relations (linear orderings) on the set of social alternatives S
$\mathscr{D}$	A set of binary relations on the set of social alternatives S
$\mathscr{T}^n$	n-fold Cartesian product of $\mathscr{T}$ with itself
$\mathscr{Q}^n$	n-fold Cartesian product of $\mathscr{Q}$ with itself
$\mathscr{D}^n$	n-fold Cartesian product of $\mathscr{D}$ with itself
$(R_1, \ldots, R_n)$	A profile of individual orderings, an element of $\mathscr{T}^n$; or a profile of reflexive, connected and quasi-transitive individual weak preference relations, an element of $\mathscr{Q}^n$
$\mathbb{D}$	A set of profiles of individual binary weak preference relations, a subset of $\mathscr{T}^n$ or a subset of $\mathscr{Q}^n$
$R\|A$	Restriction of R to $A \subseteq S$, i.e., $R\|A = R \cap (A \times A)$
$\mathscr{D}\|A$	Restriction of $\mathscr{D}$ to $A \subseteq S$, i.e., $\mathscr{D}\|A = \{R\|A \,\|R \in \mathscr{D}\}, \mathscr{D} \subseteq \mathscr{B}$
$D(x, y)$	Almost decisive for the ordered pair (x, y)
$\overline{D}(x, y)$	Decisive for the ordered pair (x, y)
$S(x, y)$	Almost semidecisive for the ordered pair (x, y)
$\overline{S}(x, y)$	Semidecisive for the ordered pair (x, y)
$D_{N-A}(x, y)$	Almost $(N - A)$—decisive for the ordered pair (x, y)
$\overline{D}_{N-A}(x, y)$	$(N - A)$—decisive for the ordered pair (x, y)
$S_{N-A}(x, y)$	Almost $(N - A)$—semidecisive for the ordered pair (x, y)
$\overline{S}_{N-A}(x, y)$	$(N - A)$—semidecisive for the ordered pair (x, y)
W	The set of winning coalitions
W_m	The set of minimal winning coalitions
B	The set of blocking coalitions
B_s	The set of strictly blocking coalitions
$WLS(xyzx)$	Weak Latin Square with the cyclical arrangement *xyzx*
$LS(xyzx)$	Latin Square with the cyclical arrangement *xyzx*

$T[WLS(xyzx)]$	The set of all orderings of the triple $\{x, y, z\}$ that can be part of $WLS(xyzx)$
$T[LS(xyzx)]$	The set of all orderings of the triple $\{x, y, z\}$ that can be part of $LS(xyzx)$
$Q[WLS(xyzx)]$	The set of all reflexive, connected and quasi-transitive binary weak preference relations of the triple $\{x, y, z\}$ that can be part of $WLS(xyzx)$
$Q[LS(xyzx)]$	The set of all reflexive, connected and quasi-transitive binary weak preference relations of the triple $\{x, y, z\}$ that can be part of $LS(xyzx)$
$B[WLS(xyzx)]$	The set of all binary weak preference relations of the triple $\{x, y, z\}$ that can be part of $WLS(xyzx)$
$B[LS(xyzx)]$	The set of all binary weak preference relations of the triple $\{x, y, z\}$ that can be part of $LS(xyzx)$
$\lceil t \rceil$	Smallest integer greater than or equal to t
$\lfloor t \rfloor$	Largest integer smaller than or equal to t
xUy	$(\forall i \in N)$ (xR_iy)
$x\overline{U}y$	$(\forall i \in N)$ (xP_iy)
$\mathscr{T}_1^T$	$\{\mathscr{D} \subseteq \mathscr{T} \mid$ every $(R_1, \ldots, R_n) \in \mathscr{D}^n$ yields transitive $R\}$
$\mathscr{T}_2^T$	$\{\mathscr{D} \subseteq \mathscr{T} \mid$ some $(R_1, \ldots, R_n) \in \mathscr{D}^n$ yields intransitive $R\}$
$\mathscr{T}_1^Q$	$\{\mathscr{D} \subseteq \mathscr{T} \mid$ every $(R_1, \ldots, R_n) \in \mathscr{D}^n$ yields quasi-transitive $R\}$
$\mathscr{T}_2^Q$	$\{\mathscr{D} \subseteq \mathscr{T} \mid$ some $(R_1, \ldots, R_n) \in \mathscr{D}^n$ yields non quasi-transitive $R\}$
$\mathscr{T}_1^A$	$\{\mathscr{D} \subseteq \mathscr{T} \mid$ every $(R_1, \ldots, R_n) \in \mathscr{D}^n$ yields acyclic $R\}$
$\mathscr{T}_2^A$	$\{\mathscr{D} \subseteq \mathscr{T} \mid$ some $(R_1, \ldots, R_n) \in \mathscr{D}^n$ yields non-acyclic $R\}$