The Potential of Fields in Einstein's Theory of Gravitation

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Preface

The theory of relativity has developed in two phases—special theory of relativity and general theory of relativity. Special theory of relativity adapted the concept of inertial frame to the basic law of constancy of the velocity of light dispensing with the concept of absolute space and time of Galilean–Newtonian mechanics, while the general theory of relativity came into existence as an extension of the special theory of relativity.

With the tools of Riemannian geometry, Einstein was able to formulate a theory that predicts the behaviour of objects in the presence of gravitational, electromagnetic and other forces. Through his general theory of relativity, Einstein redefined gravity. From the classical point of view, gravity is the attractive force between massive objects in three-dimensional space. In general relativity, gravity manifests as a curvature of four-dimensional spacetime. Conversely, curved space and time generates effects that are equivalent to gravitational effects. J. A. Wheelar has described the results as ‘matter tells spacetime how to bend and spacetime returns the complement by telling matter how to move’. On other hand, cosmology is the science of the universe as a whole, and the study of cosmology requires several kinds of physics. Since the dominant force on the cosmic scale is gravitation, this is the basic ingredient that is taken care of by Einstein’s general theory of relativity. The matter distribution of fluids, gases, fields, etc., in the spacetime is given by the Einstein field equations. A cosmological model is a model of our universe that predicts the observed properties of the universe and explains the phenomena of the early universe. In a more restricted sense, cosmological models are the exact solutions of the Einstein field equations for a perfect fluid.

The global geometry of the spacetime is determined by the Riemann curvature tensor, which can be decomposed in terms of the Weyl conformal tensor, Ricci tensor and metric tensor. This decomposition involves certain irreducible tensors. In empty spacetime, the pure gravitational radiation field is described by the Weyl conformal tensor. However, when gravitational waves propagate through matter, the Weyl conformal tensor is still pertinent. In 1962, C. Lanczos thought that the Weyl conformal tensor can also be derived from a simpler tensor field. Moreover, it is known that an electromagnetic field is generated through the covariant differentiation of a vector field. The question now is: Whether it is possible or not to generate the gravitational field through a potential? The answer is yes; one can generate the gravitational field through the covariant differentiation of a tensor field. This tensor field that can act as a potential to the gravitational field is now known as the Lanczos potential. Further, certain physical problems in general relativity are often conveniently described using a tetrad formalism adapted to the geometry of the particular situation.

The present book deals with a detailed study of the Lanczos potential in general relativity and comprises eight chapters. Chapters 1–3 deal with a detailed study of tetrad formalism and its important examples—Newman–Penrose and Geroch–Held–Penrose formalisms. These discussions will then be used in the study of the Lanczos potential. Chapter 4 defines the Lanczos potential, and the equation by means of which the gravitational field is created has been derived. Such equation is called Weyl–Lanczos equation, and this equation and other related results are expressed in terms of Newman–Penrose and Geroch–Held–Penrose formalisms. Chapter 5 gives a general prescription on how to generate a gravitational field of algebraically special fields, which is supported by a number of examples, while Chap. 6 deals with yet another method to obtain the Lanczos potential for a perfect fluid spacetime, and these results are then used to generate the gravitational field of some cosmological models. Chapter 7 defines the Lanczos potentials for some well-known solutions of Einstein field equations, which have been obtained using tetrad formalisms. Apart from tetrad formalism, there are also some other methods to obtain the Lanczos potential. Such methods have been discussed in this chapter and applied to find the Lanczos potential for Gödel cosmological model. Chapter 8, contains some more applications of the Newman–Penrose formalism. A method for finding the solution of Einstein–Maxwell equations, using NP formalism, has been discussed in detail. The interaction between a Petrov type N gravitational field and null electromagnetic field has been considered, and a metric describing this situation has been obtained. A systematic and detailed study of symmetries of the spacetime (which are also known as collineations) has also been made in this chapter. Each chapter of this book ends with a list of references which by no means is a complete bibliography of the Lanczos potential and tetrad formalism; only the work referred to in this book has been included in the list.

Some portions of this book were completed at Universiti Sains Islam Malaysia (USIM), Nilai, Malaysia, during my stay as Visiting Professor. I am highly thankful to Prof. Musa Ahmad, Vice Chancellor of the university, and Dr. Nurul Sima, Head, Department of Mathematics, for their excellent support and encouragement. Thanks are also due to the learned referee for his valuable suggestions and comments. I am also grateful to my publisher for having faith in me and specially to Mr. Shamim Ahmad for introducing me to the world of Springer and his guidance during the preparation of the manuscript.

Aligarh, India Zafar Ahsan
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Contents

About the Author

Zafar Ahsan is a former Professor at the Department of Mathematics, Aligarh Muslim University, India, where he completed his Ph.D. in Mathematics in 1979. He has previously served as a Visiting Associate at the Inter-University Centre of Astronomy and Astrophysics (IUCAA), Pune (1996–2005); a UGC Visiting Professor at Sardar Patel University, Vallabh Vidyanagar, Anand (2004–2005); and as a Visiting Professor at Universiti Sains Islam Malaysia (October–December, 2016). At present, Prof. Ahsan is a Research Fellow at Institut Sains Islam, Malaysia.

Professor Ahsan is a life member of several learned societies including the Indian Association of General Relativity and Gravitation, the Astronomical Society of India, International Electronic Engineering Mathematical Society, Indian Mathematical Society and the Tensor Society of India. Further, he has served as Editor-in-Chief of the *Aligarh Bulletin of Mathematics* (2012–2015), Editor-in-Chief of the *Journal of Tensor Society of India* (2010–2012) and Managing Editor of the *Aligarh Bulletin of Mathematics* (1998–2012). He is currently Editor, *Palestine Journal of Mathematics* and *Journal of Interpolation & Approximation in Scientific Computing*; and a member of the editorial boards of several journals, including the *Global Journal of Advanced Research on Classical and Modern Geometries*, the *Islamic University of Gaza Journal of Natural and Engineering Studies*, and the *Bulletin of the Calcutta Mathematical Society* and the *Journal of the Calcutta Mathematical Society*. With over 40 years of teaching experience, he has published five books and over 100 research papers in several international journals of repute. His research interests are in gravitational waves, symmetries of space–time, exact solutions of Einstein equations, tetrad formalisms and differential geometric structures in general relativity. His current research interest, apart from General Relativity and Gravitation, is in the Qur'an, Science and Modern Cosmology.

Professor Ahsan is the President of Anjuman Faroogh-e-Science (Association for Promotion of Science), Aligarh branch; and has been a Council Member of the Indian Association for General Relativity and Gravitation (2002–2006). He is the recipient of the Nishan-e-Azad award (2017) for the promotion of science in Urdu; the International Einstein Award for Scientific Achievement (2011); the Rashtriya

Gaurav Award (2007); the Shiksha Rattan Puraskar (2007); and the Best Citizens of India award (2006, 2007, 2008). He is a Member of the Research Board of Advisers, American Biography Institute (USA) and the Life Time President of the Tensor Society of India.