

Mathematics Education – An Asian Perspective

Series editors

Berinderjeet Kaur, National Institute of Education, Singapore, Singapore
Catherine Vistro-Yu, Ateneo de Manila University, Manila, Philippines

Aims and Scope

Mathematics Education – An Asian Perspective facilitates high quality publications on rigorous aspects of mathematics education in Asia. This will be achieved by producing thematic books that capture knowledge and practices on mathematics education in Asia from both the insider and outsider perspectives. The series helps to establish a much needed Asian perspective to mathematics education research in the international landscape.

Over the last decade or so, several international comparative studies have shed light on systems of schooling that were otherwise not very much sought after. Several educational systems in Asia, in particular East Asia have consistently produced stellar outcomes for mathematics in both TIMSS and PISA despite the fact that both studies measure achievement in mathematics in distinct ways that are very much orthogonal to each other, while other Asian systems have not been able to replicate the same level of success. Though one may occasionally chance upon a publication on some aspect of mathematics education in Asia, there appears to be in general a dearth of publications on mathematics education in Asian countries from the perspectives of scholars from Asia. Hence it is apparent that there is a gap in the availability of knowledge on mathematics education from the region in the international space.

This series has a wide scope with emphasis on relevancy and timeliness. It encompasses the general trends in educational research such as theory, practice and policy. Books in the series are thematic and focus both on macro and micro topics. An example of a themed book on a macro topic could be one on “School mathematic curricula – An Asian perspective” while a themed book on a micro topic could be one on “The pedagogy of ‘simultaneous equations’ in Asian classrooms”.

More information about this series at <http://www.springer.com/series/11679>

Tin Lam Toh · Berinderjeet Kaur ·
Eng Guan Tay
Editors

Mathematics Education in Singapore

 Springer

Editors

Tin Lam Toh
National Institute of Education
Nanyang Technological University
Singapore, Singapore

Berinderjeet Kaur
National Institute of Education
Nanyang Technological University
Singapore, Singapore

Eng Guan Tay
National Institute of Education
Nanyang Technological University
Singapore, Singapore

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Foreword

Since Confucius and Socrates, educators have recognised the double purpose of education: to impart the meaning and significance of the past, and to prepare young people for the challenges of the future.

When we could still assume that what we learn in school will last for a lifetime, teaching routine cognitive skills was rightly at the centre of education. These days, the dilemma for educators is that the skills that are easiest to teach and easiest to test have become the skills that are easiest to digitise and automate. Today, when we can access content via search engines, and when routine cognitive tasks are being digitised and outsourced, the focus must shift to enabling people to become lifelong learners. Lifelong learning is about constantly learning, unlearning and relearning when the contexts change. It entails continuous processes of reflection, anticipation and action. Reflective practice is needed to take a critical stance when deciding, choosing and acting, by stepping back from what is known or assumed and by taking different perspectives. Anticipation mobilises cognitive skills, such as analytical or critical thinking, to foresee what may be needed in future or how actions taken today might have consequences for the future. Both reflective practice and anticipation contribute to the willingness to take responsible actions, in the belief that it is within the power of all of us to shape and change the course of events. This is how agency is built.

Modern schools need to help students constantly evolve and grow, and to find and adjust their right place in a changing world. They need to prepare students for rapid change, to learn for jobs that have not been created, to tackle societal challenges that we cannot yet imagine and to use technologies that have not yet been invented. And they need to prepare students for an interconnected world in which students understand and appreciate different perspectives and world views, interact successfully and respectfully with others, and take responsible action towards sustainability and collective well-being.

No country I know of is constantly reimagining the education of tomorrow's students as systematically and rigorously as Singapore does. This book is about the why, the what and the how of mathematics education in Singapore. But beyond that, it provides a unique window into the ways in which Singapore designs its

instructional system and builds the capacity to deliver it consistently in every classroom. Having gone through this book, readers will understand that the stellar performance of Singapore in the global PISA mathematics and science tests is not an artefact of context and culture, but the result of carefully designed policy and practice.

This book begins with providing Singapore's answer to what students should learn in mathematics and why, and how this answer evolved as the world kept changing. It presents a mathematics curriculum that is characterised by rigour (building what is being taught on a high level of cognitive demand), by focus (aiming at conceptual understanding by prioritising depth over breadth of content) and by coherence (sequencing instruction based on a scientific understanding of learning progressions and human development). While great attention is paid to remain true to the mathematics discipline, much thought is also given to interdisciplinary learning and building students' capacity to see problems through multiple lenses. Singapore's curriculum also carefully balances knowledge of mathematics content with knowledge about the underlying nature and principles of the discipline; to help students address unknown future problems, it gives priority to knowledge, skills and attitudes that can be learned in one context and applied to others. A great strength of this part is that it goes beyond laying out curricular intentions and also provides a critical analysis of the challenges that lie in translating those intentions into classroom practice.

Building on this, the second part shows how a demanding curriculum can be consistently realised in diverse classrooms. Bringing teachers along with the ideas of a twenty-first-century curriculum is the heart of Singapore's success. This is not simply about teachers knowing mathematics, teachers knowing how students learn mathematics and teachers knowing their students, but it is about enabling teachers to design innovative pedagogical practice, with due attention to the needs of all learners. It is about framing learning in relevant and realistic contexts, and using approaches that are problem-based, project-based and centred around co-creation with their colleagues and their students. Professionalising teaching in this way has ensured that teachers have a deep understanding not only of the curriculum as a product, but of the process of designing pedagogies that will best communicate the ideas behind the curriculum.

As readers will see, Singapore does whatever it takes to develop ownership of professional practice by the teaching profession. In turn, professional discretion accorded to teachers allows them greater latitude in developing student creativity and critical thinking skills that are central to success in the twenty-first century and that are much harder to develop in highly prescriptive learning environments. And when teachers feel a sense of ownership over their classrooms, when students feel a sense of ownership over their learning, that is when productive teaching takes place. Singapore's answer to this has been to strengthen trust, transparency, professional autonomy and the collaborative culture of the profession all at the same time. This is how Singapore has created an open-source community of teachers and unlocked teachers' creativity simply by tapping into the desire of people to contribute, collaborate and be recognised for their contributions.

What makes this book a fascinating read is to see how all the pieces fit together, the why, the what and the how. It underlines how Singapore's success in education is a story about leadership and alignment between policy and practice; about setting ambitious standards; about focusing on building teacher and leadership capacity to develop vision and strategy at the school level; and about a culture of continuous improvement that benchmarks practice against the best in the world.

Paris, France

Andreas Schleicher
Director for the Directorate of Education and Skills
The Organisation for Economic Co-operation
and Development (OECD)

Series Editors' Introduction

The third volume of the book series *Mathematics Education: An Asian Perspective*, entitled “*Mathematics Education in Singapore*” and edited by Tin Lam Toh, Berinderjeet Kaur and Eng Guan Tay offers a one-stop resource on the why, the what and the how of the current state of mathematics education in Singapore. As noted by Andreas Schleicher in the foreword, it provides a unique window into the ways in which Singapore designs its instructional system and builds the capacity to deliver it consistently in every classroom.

The chapters in this book provide a rich source of information and analyses from a scholarly insider's view. Myths about the exotic East continue to circulate. Do Singapore teachers drill to kill? Do Singapore schools resemble a factory production line? Are Singapore students and people unimaginative and dour? This book, like the country, will surprise the reader with its energy, openness, humility and pragmatism. This volume is thus a much needed and worthy contribution to the mathematics education literature.

There is no doubt that this book contributes towards reducing the dearth in the availability of knowledge about mathematics education in Asia for the international audience. We hope researchers will find it a valuable resource and, for all, an enjoyable read.

Singapore
Manila, Philippines

Berinderjeet Kaur
Catherine Vistro-Yu

Contents

1	Surprising Singapore	1
	Eng Guan Tay, Tin Lam Toh and Berinderjeet Kaur	
Part I The Singapore School Mathematics Curriculum		
2	Overview of Singapore’s Education System and Milestones in the Development of the System and School Mathematics Curriculum	13
	Berinderjeet Kaur	
3	The Intended School Mathematics Curriculum	35
	Ngan Hoe Lee, Wee Leng Ng and Li Gek Pearlyn Lim	
4	The Enacted School Mathematics Curriculum	55
	Yew Hoong Leong and Berinderjeet Kaur	
5	Beyond School Mathematics	67
	Weng Kin Ho, Pee Choon Toh, Kok Ming Teo, Dongsheng Zhao and Kim Hoo Hang	
6	Singapore’s Participation in International Benchmark Studies—TIMSS, PISA and TEDS-M	101
	Berinderjeet Kaur, Ying Zhu and Wai Kwong Cheang	
Part II Teaching and Learning Practices in Singapore Mathematics Classrooms		
7	Problem Solving in the Singapore School Mathematics Curriculum	141
	Tin Lam Toh, Chun Ming Eric Chan, Eng Guan Tay, Yew Hoong Leong, Khiok Seng Quek, Pee Choon Toh, Weng Kin Ho, Jaguthsing Dindyal, Foo Him Ho and Fengming Dong	

8	Innovative Pedagogical Practices	165
	Joseph B. W. Yeo, Ban Heng Choy, Li Gek Pearlyn Lim and Lai Fong Wong	
9	Problems in Real-World Context and Mathematical Modelling	195
	Chun Ming Eric Chan, Kit Ee Dawn Ng, Ngan Hoe Lee and Jaguthsing Dindyal	
10	Patterns Across the Years—Singapore Learners’ Epistemology	217
	Swee Fong Ng and Boon Liang Chua	
11	Metacognition in the Teaching and Learning of Mathematics	241
	Ngan Hoe Lee, Kit Ee Dawn Ng and Joseph B. W. Yeo	
12	Students’ Perspectives of Good Mathematics Lessons, Homework and How Their Teachers Facilitate Learning of Mathematics	269
	Berinderjeet Kaur and Wei Yeng Karen Toh	
13	Low Attainers and Learning of Mathematics	287
	Tin Lam Toh and Berinderjeet Kaur	
14	Use of Technology in Mathematics Education	313
	Wee Leng Ng, Beng Chong Teo, Joseph B. W. Yeo, Weng Kin Ho and Kok Ming Teo	
Part III Teacher Education and Professional Development		
15	The National Institute of Education and Mathematics Teacher Education: Evolution of Pre-service and Graduate Mathematics Teacher Education	351
	Eng Guan Tay, Weng Kin Ho, Lu Pien Cheng and Paul M. E. Shutler	
16	Exemplary Practices of Mathematics Teachers	385
	Yew Hoong Leong, Berinderjeet Kaur, Ngan Hoe Lee and Tin Lam Toh	
17	Continuing from Pre-service: Towards a Professional Development Framework for Mathematics Teachers in the Twenty-First Century	405
	Kit Ee Dawn Ng, Joseph Kai Kow Yeo, Boon Liang Chua and Swee Fong Ng	
18	Models of Teacher Professional Development	429
	Berinderjeet Kaur, Lu Pien Cheng, Lai Fong Wong and Cynthia Seto	

19 Teaching Simultaneous Linear Equations: A Case of Realistic Ambitious Pedagogy 451
Yew Hoong Leong, Eng Guan Tay, Khiok Seng Quek
and Sook Fwe Yap

20 Productive Teacher Noticing: Implications for Improving Teaching 469
Ban Heng Choy and Jaguthsing Dindyal

Part IV Conclusion

21 Reviewing the Past, Striving in the Present and Moving Towards a Future-Ready Mathematics Education 491
Tin Lam Toh, Berinderjeet Kaur and Eng Guan Tay

Editors and Contributors

About the Editors

Tin Lam Toh is an Associate Professor and currently the Deputy Head of the Mathematics and Mathematics Education Academic Group in the National Institute of Education, Nanyang Technological University of Singapore. He obtained his Ph. D. from the National University of Singapore in 2001. He continues to do research in mathematics as well as mathematics education. He has published papers in international scientific journals in both areas.

Berinderjeet Kaur is a Professor of Mathematics Education at the National Institute of Education in Singapore. She holds a Ph.D. in Mathematics Education from Monash University in Australia. She has been with the institute for the last 30 years and is one of the leading figures of mathematics education in Singapore. In 2010, she became the first full professor of mathematics education in Singapore. She has been involved in numerous international studies of mathematics education and was the Mathematics Consultant to TIMSS 2011. She was also a core member of Mathematics Expert Group (MEG) for PISA 2015. She is passionate about the development of mathematics teachers and in turn the learning of mathematics by children in schools. Her accolades at the national level include the public administration medal in 2006 by the President of Singapore, the long public service with distinction medal in 2016 by the President of Singapore and in 2015, in celebration of 50 years of Singapore's nation building, recognition as an outstanding educator by the Sikh community in Singapore for contributions towards nation building.

Eng Guan Tay is an Associate Professor and the Head in the Mathematics and Mathematics Education Academic Group of the National Institute of Education at Nanyang Technological University, Singapore. He obtained his Ph.D. in the area of Graph Theory from the National University of Singapore. He has continued his research in graph theory and mathematics education and has had papers published

in international scientific journals in both areas. He is the co-author of the books *Counting, Graph Theory: Undergraduate Mathematics* and *Making Mathematics Practical*. He has taught in Singapore junior colleges and also served a stint in the Ministry of Education.

Contributors

Chun Ming Eric Chan National Institute of Education, Singapore, Singapore

Wai Kwong Cheang National Institute of Education, Singapore, Singapore

Lu Pien Cheng National Institute of Education, Singapore, Singapore

Ban Heng Choy National Institute of Education, Singapore, Singapore

Boon Liang Chua National Institute of Education, Singapore, Singapore

Jaguthsing Dindyal National Institute of Education, Singapore, Singapore

Fengming Dong National Institute of Education, Singapore, Singapore

Kim Hoo Hang Jurong Junior College, Singapore, Singapore

Foo Him Ho Ministry of Education, Singapore, Singapore

Weng Kin Ho National Institute of Education, Singapore, Singapore

Berinderjeet Kaur National Institute of Education, Singapore, Singapore

Ngan Hoe Lee National Institute of Education, Singapore, Singapore

Yew Hoong Leong National Institute of Education, Singapore, Singapore

Li Gek Pearlyn Lim National Institute of Education, Singapore, Singapore

Kit Ee Dawn Ng National Institute of Education, Singapore, Singapore

Swee Fong Ng National Institute of Education, Singapore, Singapore

Wee Leng Ng National Institute of Education, Singapore, Singapore

Khiok Seng Quek National Institute of Education, Singapore, Singapore

Cynthia Seto Academy of Singapore Teachers, Singapore, Singapore

Paul M. E. Shutler National Institute of Education, Singapore, Singapore

Eng Guan Tay National Institute of Education, Singapore, Singapore

Beng Chong Teo National Institute of Education, Singapore, Singapore

Kok Ming Teo National Institute of Education, Singapore, Singapore

Pee Choon Toh National Institute of Education, Singapore, Singapore

Tin Lam Toh National Institute of Education, Singapore, Singapore

Wei Yeng Karen Toh National Institute of Education, Singapore, Singapore

Lai Fong Wong National Institute of Education, Singapore, Singapore

Sook Fwe Yap National Institute of Education, Singapore, Singapore

Joseph B. W. Yeo National Institute of Education, Singapore, Singapore

Joseph Kai Kow Yeo National Institute of Education, Singapore, Singapore

Dongsheng Zhao National Institute of Education, Singapore, Singapore

Ying Zhu National Institute of Education, Singapore, Singapore

Acronyms

AME	Association of Mathematics Educators
AS	Academic subject
AST	Academy of Singapore Teachers
AU	Academic unit
BA (Ed)	Bachelor of Arts with Education
BSc (Ed)	Bachelor of Science with Education
CAI	Computer-assisted instruction
CAS	Computer algebra system
21CC	21st Century Competencies
CCA	Co-curricular activity
CCT	Critical and creative thinking
CDIS	Curriculum Development Institute of Singapore
CER	Creative Reflection
Cert. Ed	Certificate in Education
CIR	Critical Reflection
CK	Content knowledge
CLT	Cognitive load theory
CPA	Computer application
C-P-A	Concrete–pictorial–abstract
CS	Curriculum studies
C-V	Concrete-Virtual
DI	Differentiated instruction
Dip Ed	Diploma in Education
DOE	Desired Outcomes of Education
EmC ²	Change reflection model
EMR	Emotive Reflection
FM	Further Mathematics
GC	Graphing calculator
GCE—N(A) Level	General Certificate of Education (Normal [Academic]) Level

GCE—N(T) Level	General Certificate of Education (Normal [Technical]) Level
GCE—O Level	General Certificate of Education (Ordinary) Level
GDP	Gross domestic product
gRAT	Group readiness assurance test
GSP	Geometer's Sketchpad
ICAN	Improving Confidence And Numeracy
ICT	Information and communication technology
ICTMA	International Community of Teachers of Mathematical Modelling and Applications
IE	Institute of Education
IEA	International Association for the Evaluation of Educational Achievement
IHL	Institutes of higher learning
IP	Integrated Programme
IPW	Interdisciplinary Project Work
iRAT	Individual readiness assurance test
ITE	Institute of Technical Education
JC	Junior college
LAPM	Low Attainers in Primary Mathematics
LC	Laboratory class
LOVE Mathematics framework	Linking Opportunities in a Variety of Experiences to the learning of Mathematics
LPS	Learner's Perspective Study
LS	Lesson study
LT	Lead teacher
MAM	Mathematical Applications and Modelling
MAS	Mathematical Sciences
MATHLET	Mathematical Learning Experience Toolkit
MC	Module credit
MCK	Mathematics Content Knowledge
MEd (Math)	Master of Education (Mathematics)
MEd	Master of Education
MME	Mathematics and Mathematics Education Academic Group
MMO	Mathematical Modelling Outreach
MOE	Ministry of Education, Singapore
mp 1	ICT Masterplan 1
mp 2	ICT Masterplan 2
mp 3	ICT Masterplan 3
mp 4	ICT Masterplan 4
MPCK	Mathematics pedagogical content knowledge
MProSE	Mathematical Problem Solving for Everyone
MPS	Mathematical Problem Solving

MSc (MfE)	Master of Science (Mathematics for Educators)
MTC	Mathematics Teachers Conference
MTT	Master Teacher
N(A)	Normal (Academic)
N(T)	Normal (Technical)
NE	National Education
NES	New education system
NHSMS	NUS High School of Mathematics and Science
NIE	National Institute of Education, Singapore
NLCs	Networked Learning Communities
NTU	Nanyang Technological University
NUS	National University of Singapore
OECD	Organisation for Economic Co-operation and Development
OPAL	One Portal All Learners
PAP	People's Action Party
PCK	Pedagogical content knowledge
PD	Professional development
PDCM	Professional Development Continuum Model
PGDE	Postgraduate Diploma in Education
PIRLS	Progress in International Reading Literacy Study
PISA	Programme for International Student Assessment
PLCs	Professional learning communities
PMT	Pre-class Milestone Tasks
PRWC	Problems in real-world context
PSLE	Primary School Leaving Examination
R&D	Research and development
RAP	Realistic ambitious pedagogy
RAT	Readiness assurance test
SBDP	School-based development programme
SK	Subject knowledge
SDT	Self-determination theory
SMAPP	Singapore Mathematics Assessment and Pedagogy Project
SMCF	School mathematics curriculum framework
SMMT	School Mathematics Mastery Test
SPMS	School of Physical and Mathematical Sciences
ST	Senior teacher
STEM	Science, technology, engineering and mathematics
SUSS	Singapore University of Social Science
TA	Teaching assistantship
TBL	Team-based learning

TE 21	Teacher Education in the 21st Century
TEDS-M	Teacher Education and Development Study in Mathematics
TELT	Teacher Education and Learning to Teach
TfU	Teaching for understanding
TGM	Teacher Growth Model
TIMSS	Trends in International Mathematics and Science Study
TLLM	Teach Less, Learn More
TRAI SI	Training Administration System on Internet
TRU	Teaching for Robust Understanding
TSLN	Thinking Schools, Learning Nation
TSP	NTU-NIE Teaching Scholars Programme
TTC	Teachers' Training College
UCLES	University of Cambridge Local Examinations Syndicate
V-P-A	Virtual–pictorial–abstract