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Muhammad Akram

Single-Valued Neutrosophic Graphs

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Muhammad Akram
Department of Mathematics
University of the Punjab
Lahore, Pakistan

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To my lovely parents and worthy teachers!

Foreword

The Königsberg bridge problem originated in the city of Königsberg, located on the river Pregel. The city had seven bridges, which connected two islands with the mainland. People staying there always wondered whether there was any way to walk over all the bridges once and only once and return to the same place where they started the walk. In 1736, Euler came out with the solution in terms of graph theory. He proved that it was not possible to walk through the seven bridges exactly one time. In coming to this conclusion, Euler formulated the problem in terms of graph theory. Each landmark was represented as a point (node) and every bridge as an edge. This led to the formation of graph theory. Graph theory is a beautiful part of mathematics. Not only computer science is heavily based on graph theory, but there are a lot of applications of graph theory in operational research, combinatorial optimization and bioinformatics.

Neutrosophy was introduced by Smarandache in 1995, as a new branch of philosophy, which is a generalization of dialectics. Neutrosophy is the base of neutrosophic set, neutrosophic logic, neutrosophic probability and statistics, and neutrosophic calculus that have many real applications. A single-valued neutrosophic set is a special neutrosophic set and can be used expediently to deal with the real-world problems, especially in decision support.

This book presents readers with fundamental concepts, including single-valued neutrosophic, neutrosophic graph structures, bipolar neutrosophic graphs, domination in bipolar neutrosophic graphs, bipolar neutrosophic planar graphs, interval-valued neutrosophic graphs, interval-valued neutrosophic graph structures, rough neutrosophic digraphs, neutrosophic rough digraphs, neutrosophic soft graphs and intuitionistic neutrosophic soft graphs. This book also presents practical applications of the concepts in real world. Therefore, the book presents a valuable contribution for students and researchers in neutrosophic graphs and their applications.

The author, Muhammad Akram, is a well-known international researcher in the field of neutrosophic graphs and he manifests a great enthusiasm and strong potential in developingthe neutrosophic environment and applying it to practical problems.

Gallup, USA

Florentin Smarandache
University of New Mexico

Preface

The concept of fuzzy sets was introduced by Zadeh in 1965. Since then, fuzzy sets and fuzzy logic have been applied in many real applications to handle uncertainty. The traditional fuzzy set uses one real value from the unit interval [0, 1] to represent the grade of membership of fuzzy set defined on the universe. In some applications, including an expert system, belief system and information fusion, we should consider not only the truth-membership supported by the evident but also the falsity-membership against by the evident. That is beyond the scope of fuzzy sets. In 1983, Atanassov introduced the intuitionistic fuzzy sets which are a generalization of fuzzy sets. The intuitionistic fuzzy sets consider both truth-membership ($T_A(x)$) and falsity-membership ($F_A(x)$) with $T_A(x)$, $F_A(x) \in [0, 1]$ and $T_A(x) + F_A(x) \leq 1$. Intuitionistic fuzzy sets can only handle incomplete information and not the indeterminate information and inconsistent information which exist commonly in the belief system. In intuitionistic fuzzy sets, hesitancy is $1 - T_A(x) - F_A(x)$ by default. In a neutrosophic set [163], indeterminacy is quantified explicitly and truth-membership, indeterminacy-membership and falsity-membership are independent. This assumption is very important in a lot of situations such as information fusion when we try to combine the data from different sensors. Neutrosophy was introduced by Smarandache in 1995. "It is a branch of philosophy which studies the origin, nature and scope of neutralities, as well as their interactions with different ideational spectra". Neutrosophy is the base of neutrosophic set, neutrosophic logic, neutrosophic probability and statistics, and neutrosophic calculus. A single-valued neutrosophic set is a special neutrosophic set and can be used expediently to deal with the real-world problems, especially in decision support. Thus, a single-valued neutrosophic set is a powerful general formal framework which generalizes the concept of fuzzy set and intuitionistic fuzzy set. The work presented here intends to overcome the lack of a mathematical approach towards indeterminate information and inconsistent information. This monograph deals with single-valued neutrosophic graphs and their applications. It is based on a number of papers by the author, which have been published in various scientific journals. This book may be useful for researchers in mathematics, computer scientists and social scientists alike.

In Chap. 1, a concise review of the single-valued neutrosophic sets is presented. Certain types of single-valued neutrosophic (neutrosophic, for short) graphs are discussed. Applications of neutrosophic graphs are described. Moreover, the energy of neutrosophic graphs with applications is presented.

In Chap. 2, certain concepts of neutrosophic graph structures and some of their properties are presented. Moreover, some interesting applications of neutrosophic graph structures are discussed.

In Chap. 3, certain bipolar neutrosophic graphs are studied. Domination in bipolar neutrosophic graphs is presented. Bipolar neutrosophic planar graphs and bipolar neutrosophic line graphs are discussed. Further, some applications of bipolar neutrosophic graphs are described.

In Chap. 4, the concept of interval-valued neutrosophic graphs is presented. Certain types including k-competition interval-valued neutrosophic graphs, p-competition interval-valued neutrosophic graphs and m-step interval-valued neutrosophic competition graphs are discussed.

In Chap. 5, certain notions of interval-valued neutrosophic graph structures are presented. The concepts of interval-valued neutrosophic graph structures with examples are elaborated. Moreover, the concept of φ-complement of an interval-valued neutrosophic graph structure is discussed. Finally, some related properties, including φ-complement, totally self-complementary and totally strong self-complementary, of interval-valued neutrosophic graph structures are described.

In Chap. 6, the concepts of rough neutrosophic digraphs and neutrosophic rough digraphs are presented. Further, applications of rough neutrosophic digraphs and neutrosophic rough digraphs in decision-making problems are described. Moreover, comparative analysis of rough neutrosophic digraphs and neutrosophic rough digraphs is given.

In Chap. 7, the notions of neutrosophic soft graphs and intuitionistic neutrosophic soft graphs are presented. Further, applications of neutrosophic soft graphs and intuitionistic neutrosophic soft graphs are discussed. Moreover, the notion of neutrosophic soft rough graphs is described. Finally, in Chap. 8, applications of neutrosophic soft rough graphs are considered.

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Lahore, Pakistan Muhammad Akram

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About the Author

Muhammad Akram is Professor at the Department of Mathematics, University of the Punjab, Pakistan. He earned his Ph.D. in fuzzy mathematics from the Government College University, Pakistan. His research interests include numerical solutions of parabolic PDEs, fuzzy graphs, fuzzy algebras and new trends in fuzzy set theory. He has published six monographs and over 265 research articles in international peer-reviewed journals. He has been an editorial board member of 10 international academic journals and a reviewer/referee for 120 international journals, including *Mathematical Reviews* (USA) and *Zentralblatt MATH* (Germany). Seven students have successfully completed their Ph.D. research work under his supervision. Currently, he is supervising six Ph.D. students.

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