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Mathematical Programming and Game Theory

Springer

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Preface

Mathematical programming and game theory models are applied frequently in management, business, and social studies. This volume deals with certain topics of fundamental importance in mathematical programming, game theory, and other related sciences that are presented in the form of 12 chapters. It is a peer-reviewed volume under Indian Statistical Institute Series with a primary focus on recent topics that discuss new challenges from theory and practice. Some pioneers in the field and some prominent young researchers have contributed chapters to this volume. This volume presents an integration of mathematical programming and game theory models that use different methodologies to improve the decision making associated with the new challenges of the present and future problems.

The linear complementarity problem (LCP) is normally identified as a problem of mathematical programming, and it provides a unifying framework for several optimization problems like linear programming, linear fractional programming, convex quadratic programming, and bimatrix game problem. More specifically, LCP models the optimality conditions of these problems. Chapter 1 by D. Dubey and S. K. Neogy starts with the presentation of various mathematical programming problems and bimatrix game problem as the linear complementarity problem. Rest of the chapter is devoted to a study of the properties of some matrix classes in the linear complementarity theory and its usefulness for solving LCP by Lemke's algorithm. Under what conditions a linear complementarity problem can be solved as a linear programming problem is also discussed. Finally, various generalizations that appear in various applications in engineering, management science, and game theory are also discussed.

Chapters 2–4 deal with mathematical programming problems that arise in graph theory. Chapter 2 by R. B. Bapat considers two problems, namely the problems of maximizing the spectral radius and the number of spanning trees in a class of bipartite graphs with certain degree constraints, and the optimal graph for both the problems is conjectured to be a Ferrers graph. Several necessary and sufficient conditions under which the removal of an edge in a graph does not affect the resistance distance between the end-vertices of another edge are presented in this chapter. A brief survey of the problem and references to the literature containing

results and open problems are also given. A new proof of the formula for the number of spanning trees in a Ferrers graph is presented, which is different from the proof of Ehrenborg and van Willigenburg that uses electrical networks and resistances. Chapter 3 by Masahiro Hachimori considers optimization problem on orientations of a given graph, where the values of the objective functions are determined by the out-degrees of the resulting directed graph and the constraints contain acyclicity of the orientations. A survey of the applications of such optimization problems in polytope theory, shellability of simplicial complexes, and acyclic partitions are also discussed. Another interesting problem is to look for a nontrivial class of graphs for which optimization problems that are presented in this chapter can be solved in a polynomial time. Chapter 4 deals with the Max-Flow-Min-Cut property and total dual integrality. A matrix inequality $A\mathbf{x} \geq \mathbf{b}$ (resp. to $A\mathbf{x} \leq \mathbf{b}$) is called *totally dual integral* if the linear program $\min\{\langle \mathbf{w}, \mathbf{x}\rangle | A\mathbf{x} \geq \mathbf{b}\}$ (resp. to $\max\{\langle \mathbf{w}, \mathbf{x}\rangle | A\mathbf{x} \leq \mathbf{b}\}$) has an integral optimal dual solution $\mathbf{y}$ for every integral cost vector $\mathbf{w}$ for which the above linear program has a finite optimum. Motivated by the pluperfect and (weak) perfect graph theorems for the set covering problem by Fulkerson and Lovász, Seymour introduced the concept of the so-called Max-Flow-Min-Cut property (the MFMC property) of clutters, which is the packing counterpart of the totally dual integrality built in the perfection. A clutter $\mathcal{C}$ has the MFMC property if, for its clutter matrix $M(\mathcal{C})$, the linear system $M(\mathcal{C})\mathbf{x} \geqq \mathbf{1}, \mathbf{x} \geqq \mathbf{0}$ is totally dual integral. Conforti and Cornuéjols conjectured that a clutter has the packing property if and only if it has the MFMC property (Conjecture 1). Cornuéjols, Guenin, and Margot conjectured that the blocking number of every ideal minimally nonpacking clutter is 2. Furthermore, they proved that Conjecture 1 implies Conjecture 2. In this chapter, K. Kashiwabara and T. Sakuma provide a framework to attack Conjecture 2.

Chapter 5 deals with an important combinatorial optimization problem, namely travelling salesman problem (TSP). The objective of TSP is to find an optimal tour that visits every node in a finite set of nodes and returns to the origin node on a graph, given the matrix of distances between any two nodes. In this chapter, Tiru Arthanari and Kun Qian study TSP, followed by some preliminaries in graph theory. The authors then compare the Dantzig, Fulkerson, and Johnson (DFJ) formulation, Carr's cycle-shrink relaxation (an LP formulation), and multi-stage insertion (MI) formulation given by Arthanari. Various advantages of the MI formulation are discussed. With the same LP relaxation values as the classic DFJ formulation, the MI formulation has only n^3 variables and n^2 constraints, compared to DFJ with $n(n-1)$ variables and $2^{n-1}+n-1$ constraints. Using CPLEX, a commercial LP solver, the MI formulation has been shown to be competitive compared to other formulations of TSP. An interpretation of the MI formulation as a hypergraph minimum cost flow problem and some theoretical computational complexity results on the algorithms involved in solving the hypergraph minimum cost flow problem, namely the flow and potential algorithm, are also presented.

Chapter 6 by D. Aussel, J. Dutta, and T. Pandit discusses the links between equilibrium problems and variational inequalities. Under the most natural assumption, the equilibrium problem is shown to be equivalent to an associated variational

inequality and the existence results for equilibrium problems can be obtained from the existence results for variational inequality problems and vice versa. The authors also study a problem of existence of Nash equilibrium in an oligopolistic market and show that it is equivalent to a variational inequality under the most natural economic assumption. Further, the relation between the quasi-equilibrium problem and quasi-variational inequality is also studied.

Chapter 7 by Y. Kimura presents approximation techniques as the solution to convex minimization problems by using iterative sequences with resolvent operators and proposes an iterative scheme for an approximation of the solution to a common minimization problem for a finite family of convex functions.

Chapter 8 by Sushmita Gupta, Sanjukta Roy, Saket Saurabh, and Meirav Zehavi deals with an emerging area of research within algorithmic game theory: multivariate analysis of games. This chapter presents a survey of the landscape of work on various stable marriage problems and the use of parametrized complexity as a toolbox to study computationally hard variants of these problems. The entire survey is divided into three broad topics, namely strategic manipulation, maximum(minimum) sized matching in the presence of ties, and notions of fair or equitable stable matchings.

Chapter 9 by M. Kaneko deals with quasi-linear utility functions that are widely used in economics and game theory as convenient tools. The author makes an explicit connection between approximate quasi-linearity and expected utility theory and presents two applications of their results to the theories of cooperative games with side payments and of Lindahl-ratio equilibrium for a public goods economy with quasi-linearity.

Chapter 10 by L. Mallozzi and A. Sacco presents a cooperative game theoretical model for a multi-commodity network flow problem. In this game, each player receives a return for shipping his commodity and considers the possibility to have uncertainty on the costs. A cooperative game under interval uncertainty is presented for the model, and the existence of core solutions is also investigated.

Chapter 11 by Andrey Garnaev and Wade Trappe discusses an interesting topic on pricing competition between cell phone carriers in a growing market of customers. A game theoretical model for the competition between service providers, such as cell phone carriers, in a market of customers that is growing, was investigated. Solving this game helps to show how the loyalty factor associated with the carriers might impact the prices and relative market share between the carriers.

Chapter 12 by Reinoud Joosten and Robin Meijboom presents and analyzes a stochastic game in which transition probabilities between states are not fixed as in standard stochastic games, but depend on the history of the play, i.e., the players' past action choices. For the limiting average reward criterion, the authors determine the set of jointly convergent pure-strategy rewards which can be supported by equilibria involving threats. Further, for expository purposes, a stylized fishery game is analyzed. In each period, two agents choose between catching with restraint and catching without restraint. The resource is in either of two states, *high* or *low*. Restraint is harmless to the fish, but it is a dominated action at each stage. The lesser the restraint shown during the play, the higher the probabilities that the system

moves to or stays in *low*. The latter state may even become 'absorbing temporarily'; i.e., transition probabilities to *high* temporarily become zero, while transition probabilities to *low* remain nonzero. Future research should combine various modifications and extensions of the original Small Fish Wars with the innovation presented here.

It is hoped that the results presented in this research monograph will inspire young researchers for further contributions to the fields of mathematical programming, game theory, and graph theory, especially in the form of novel applications and development of computational techniques.

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S. K. Neogy is Professor at Indian Statistical Institute, New Delhi. He obtained his Ph.D. from the same institute, and his primary areas of research are mathematical programming and game theory. He is the co-editor of the following books: *Modeling, Computation and Optimization and Mathematical Programming and Game Theory for Decision Making* (both from World Scientific). He has also been a co-editor of the special issue of several journals: *Annals of Operations Research, entitled Optimization Models with Economic and Game Theoretic Applications* (2016), *International Game Theory Review, Entitled Operations Research and Game Theory* (2001), and *Applied Optimization and Game-Theoretic Models*, Parts I and II (2015). He has published widely in several international journals of repute like *Mathematical Programming, Linear Algebra and its Applications,* OR *Spektrum, SIAM Journal on Matrix Analysis and Applications, SIAM Journal on Optimization, International Journal of Game Theory, Dynamic Games and Applications, Annals of Operations Research,* and *Mathematical Analysis and Applications*. He is a reviewer of zbMATH and Mathematical Reviews.

Ravindra B. Bapat obtained his Ph.D. from the University of Illinois at Chicago and is Professor at the Stat-Math Unit, Indian Statistical Institute, New Delhi. He was earlier associated with Northern Illinois University in DeKalb, Illinois, and the University of Mumbai, India, before joining Indian Statistical Institute, New Delhi, in 1983. He held visiting positions at various universities in the USA and visited several institutes in countries including France, Holland, Canada, China, and Taiwan for collaborative research and seminars. His main areas of research are nonnegative matrices, matrix inequalities, matrices in graph theory, and generalized inverses. He has published over 140 research papers in these areas in journals of repute and guided several Ph.D. students. He is the author of several books on linear algebra including *Linear Algebra and Linear Models* and *Graphs and Matrices* (both published by Springer). He also wrote a book on Mathematics for

the General Reader, in Marathi, which won the State Government Award for 2004 for the Best Literature in Science. In 2009, he was awarded the J.C. Bose Fellowship. He has been on the editorial boards of several journals: *Linear and Multilinear Algebra, Electronic Journal of Linear Algebra, Indian Journal of Pure and Applied Mathematics*, and *Kerala Mathematical Association Bulletin*. He has been elected Fellow of the Indian Academy of Sciences, Bangalore, and the Indian National Science Academy, New Delhi. He has served as President of the Indian Mathematical Society during its centennial year 2007–2008. For the past several years, he has been actively involved with the Mathematics Olympiad Program in India as the national coordinator for the program. He has also served as Head, Indian Statistical Institute, New Delhi, during 2007–2011.

Dipti Dubey is Postdoctoral Fellow at Indian Statistical Institute, New Delhi. A Ph.D. from the Indian Institute of Technology Delhi, her primary area of research is mathematical programming and game theory.

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