# Inerter and Its Application in Vibration Control Systems

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# Preface

The topic of this book is a comprehensive introduction of inerter, a new two-terminal mechanical element proposed by Prof. Malcolm Smith from Cambridge University in 2002, and its recent advances in vibration control systems. One of the principal motivations for introducing inerter is due to a snag in the correspondence between passive mechanical and electrical networks, as the capacitor does not have a real corresponding mechanical element. Such a snag has been removed because of the invention of inerter, and as a result a spring–damper–inerter mechanical network can be directly transformed into an inductor–resistor–capacitor electrical network. In this way, the electrical network synthesis theory can be directly applied to the mechanical network synthesis. Based on this, much attention has been drawn to the inerter-based mechanical network synthesis, where the problem of how to realize a positive-real transfer function as specific mechanical networks while considering the simplicity, cost, and other realization requirements is of particular interest. Apart from the research interest on inerter-based mechanical network synthesis, inerter has been applied to a variety of mechanical systems, such as vehicle suspensions, train suspensions, buildings, motorcycle steering compensators, landing gears, wind turbines, etc. A common practice of applying inerters to a specific system is that inerter-based networks, usually more complex than the traditional networks without inerters, are given and then optimize the parameters of the inerter-based networks to achieve a better performance than the traditional ones. Such a procedure is effective to demonstrate the benefits of using inerters, but the drawback is that the basic function of an inerter for general vibration systems is concealed. Since all the application scenarios of inerter are a subset of general vibration systems, it demands a comprehensive investigation of inerters from a vibration system point of view, which is the main motivation of this book.

This book is intended to provide a comprehensive summary of recent results on inerter, and then introduce the recent advances by the author on the application of inerter in vibration systems. In Chap. 1, the concept of inerter, its physical realizations, and state-of-art applications are introduced. In Chap. 2, the influence of inerter on the natural frequencies of vibration systems is discussed. Then, in

Chaps. 3 and 4, the inerter-based vibration isolators and dynamic vibration absorbers are introduced, and the parameters' optimization methods are proposed. In Chap. 5, the inerter concept is extended to semi-active control by introducing the semi-active inerter concept, where physical embodiments of semi-active inerter are also discussed.

The book is intended as a text for graduate students and researchers. A certain basic level of knowledge of mechanical vibration, control theory, and optimization theory is a necessary prerequisite in order to follow the material presented here.

The authors are indebted to all those who have contributed to material presented in this book and whose identities can be deduced from our joint publications that appear in the bibliography.

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# Contents


**1 Introduction** . . . . . 1
1.1 Inerter . . . . . 1
1.2 Network Synthesis . . . . . 2
1.3 The Physical Embodiments of Inerter . . . . . 4
1.4 Inerter-Based Vibration Control Systems . . . . . 8
1.4.1 Passive Vibration Control with Inerters . . . . . 8
1.4.2 Semi-active and Active Vibration Control with Inerters . . . 12
1.5 Conclusions . . . . . 13
References . . . . . 14

**2 Analysis for Inerter-Based Vibration System** . . . . . 19
2.1 Introduction . . . . . 19
2.2 Preliminary . . . . . 20
2.3 Single-Degree-of-Freedom System . . . . . 22
2.4 Two-Degree-of-Freedom System . . . . . 22
2.5 Multi-degree-of-Freedom System . . . . . 24
2.6 Influence of the Inerter Position on the Natural Frequencies . . . . . 29
2.7 Design Procedure and Numerical Example . . . . . 35
2.8 Conclusions . . . . . 38
References . . . . . 38

**3 Inerter-Based Isolation System** . . . . . 41
3.1 Introduction . . . . . 41
3.2 Preliminary . . . . . 42
3.3 Vibration Analysis for Two Simple Inerter-Based Isolators . . . . . 44
3.4 $H_\infty$ Optimization for Inerter-Based Isolators . . . . . 50
3.5 $H_2$ Optimization for Inerter-Based Isolators . . . . . 57
3.6 Conclusions . . . . . 63
Appendix . . . . . 64
References . . . . . 70

**4 Inerter-Based Dynamic Vibration Absorption System** . . . . . 73
4.1 Introduction . . . . . 73
4.2 Preliminary . . . . . 75
4.3 Inerter-Based Dynamic Vibration Absorbers . . . . . 76
4.4 $H_\infty$ Optimization for the IDVAs . . . . . 79
4.4.1 Minmax Optimization Problem Formulation . . . . . 79
4.4.2 Comparison Between the TDVA and IDVAs . . . . . 80
4.5 $H_2$ Optimization for the IDVAs . . . . . 86
4.5.1 $H_2$ Performance Measure and Its Analytical Solution . . . . . 86
4.5.2 Comparison Between the TDVA and IDVAs . . . . . 87
4.6 Conclusions . . . . . 92
Appendix . . . . . 93
References . . . . . 100

**5 Semi-active Inerter and Adaptive Tuned Vibration Absorber** . . . . . 103
5.1 Introduction . . . . . 103
5.2 Preliminary . . . . . 104
5.3 Semi-active Inerter . . . . . 106
5.3.1 The Existing Inerters . . . . . 106
5.3.2 The Controllable-Inertia Flywheel (CIF) . . . . . 106
5.3.3 The CIF-Based Semi-active Inerter . . . . . 108
5.3.4 Modeling of the Proposed Semi-active Inerter . . . . . 108
5.4 Semi-active-Inerter-Based Adaptive Tuned Vibration Absorber . . . 109
5.4.1 Problem Formulation . . . . . 109
5.4.2 Frequency-Tracker-Based (FT) Control . . . . . 110
5.4.3 Phase-Detector-Based (PD) Control . . . . . 110
5.5 Experimental Evaluation . . . . . 111
5.5.1 Experimental Platform Description . . . . . 111
5.5.2 Test Cases . . . . . 114
5.5.3 The Influence of the Inherent Damping of the Semi-active Inerter . . . . . 117
5.6 Conclusions . . . . . 118
References . . . . . 118

**6 Conclusions** . . . . . 121