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Surface-Knots in 4-Space

An Introduction

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Dedicated to Naoko

Preface

Knot theory is one of the most active research fields in modern mathematics. Knots and links are closed curves (1-dimensional manifolds) in the Euclidean 3-space, and they are related to braids and 3-manifolds. These notions are generalized into higher dimensions. Surface-knots and surface-links are closed surfaces (2-dimensional manifolds) in the Euclidean 4-space, and they are related to 2-dimensional braids and 4-manifolds. Surface-knot theory treats not only closed surfaces but also surfaces with boundaries in 4-manifolds. For example, knot concordance and knot cobordism, that are also important objects in knot theory, are surfaces in the product space of the 3-sphere and the interval.

Although the beginning of the study of surface-knots is due to E. Artin in the 1920s, the crucial research was started by R.H. Fox using the motion picture method, in the 1960s, followed by J. Milnor including researches on knot concordance. Studies using surface diagrams in 3-space were started by D. Roseman in the 1970s and have been extensively done by J.S. Carter and M. Saito since the 1990s. The author has been studying surface-knots using 2-dimensional braids since the 1990s. Theorems on braiding of surface-knots analogous to Alexander and Markov's theorems have been established. Since the late 1990s, invariants of knots and surface-knots using quandles and their (co-)homology theory have been studied. J.S. Carter, D. Jelsovsky, S. Kamada, L. Langford, and M. Saito (CJKLS) constructed invariants called the quandle cocycle invariants, and their invariants are now extended and generalized to various invariants so that they are used to study chirality of knots, hyperbolic volumes and Chern and Simon's invariant, invertibility of surface-knots, triple point numbers of surface-knots, etc.

This book is organized as follows: Chapter 1 is devoted to an introduction and preliminaries. In Chap. 2, we introduce basics of knot theory. In Chap. 3, the motion picture method and a method describing surface-knots by classical diagrams with markers are introduced. How to compute the knot group of a surface-knot from a motion picture is also explained there. Diagrams in 3-space of surface-knots and invariants obtained from diagrams are treated in Chap. 4. We discuss 1-handles attaching to surface-knots in Chap. 5. Spinning constructions of 2-knots and knotted projective planes are introduced in Chap. 6. Knot concordance and knot cobordism are discussed in Chap. 7. Chapter 8 is devoted to the study of quandles and colorings of knots and surface-knots. Fenn and Rourke's notation on quandles and knot quandles are discussed there. In Chap. 9, we introduce the (co-)homology groups of quandles and invariants of knots and surface-knots using them. Presentation of knots using braids and presentation of surface-knots using 2-dimensional braids are introduced in Chap. 10.

This book is written as an introduction to surface-knots, providing basics on surface-knots. I hope that not only the researchers in this field but also graduate students and researchers who are not familiar with this field will enjoy this book.

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Osaka, Japan Seiichi Kamada
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