

Advanced Structured Materials

Volume 67

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Variational Continuum Multiphase Poroelasticity

Theory and Applications

 Springer

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ISSN 1869-8433

Advanced Structured Materials

ISBN 978-981-10-3451-0

DOI 10.1007/978-981-10-3452-7

ISSN 1869-8441 (electronic)

ISBN 978-981-10-3452-7 (eBook)

Library of Congress Control Number: 2016960776

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The registered company is Springer Nature Singapore Pte Ltd.

The registered company address is: 152 Beach Road, #22-06/08 Gateway East, Singapore 189721, Singapore

*To our beloved wives Ilaria and Pamela,
to our children Lorenza, Alessandro and Dylan,
and to our parents*

Foreword

It is a great pleasure to present this nice book in a subject which occupied my studies for a long while. The authors have enthusiastically started a cultural and scientific endeavour which surely deserves a great investment in intelligence and study. Indeed to base the continuum mechanics of porous media on variational principles presents some relevant difficulties of mathematical nature as well as very important issues related to applicability to real-world problems. It has been a pleasure to see how the younger (compared with me) authors have approached the increasingly difficult problems which they met in pursuing their scientific objective and I hope that the discussions which we had were of some use. Actually the mathematical description of the flow of a compressible fluid in a deformable medium presents relevant difficulties and we cannot expect that these are solved quickly and naturally without an important change of the conceptual paradigm used to confront them. This monograph presents some ideas of the authors which are framed nicely in the logic started by the papers by Biot and, more recently, by Coussy and it seems to me that many of the presented methods are well grounded. The state of the art is examined from the point of view of the authors and seems rather complete, while the formulation of the mathematical models follows the standards commonly accepted in continuum mechanics. One can expect therefore that this work will have some beneficial effects in the scientific community interested in such a kind of problems. Indeed: (1) mathematicians will find a wealth of interesting problems to be studied and formalised, (2) engineers may find interesting methods for forecasting the behaviour of applicable mechanical systems, (3) theoretical mechanicians may find a further evidence about the importance of Lagrangian methods.

The enthusiasm of the authors may have led them to underestimate the relevance of some of their simplifying assumptions: however, this is the needed approach for attacking problems which resisted too much to the efforts of scientists. It is very good that they did not want to follow my invitation to prudence and to circumspection and they finally wanted to dare to formulate models in which the microstructure of the deformable matrix was explicitly taken into account: maybe it

could have been done in a better way. However, it is better to start an investigation instead of postponing it, while waiting for the moment in which the logical tools are ripened. Indeed the logical tools will ripen under the push of the conjectures which are presented, for instance, in this monograph. I wish to the authors a long scientific career, which seems to me has started under the best auspices.

Rome, Italy
October 2016

Francesco dell'Isola

Preface

The main objective of this monograph is to provide a comprehensive picture of the Variational Macroscopic Theory of Porous Media (VMTPM), a general two-phase variational continuum theory with microstructure which we have been developing since 2013, based on a previous theory originally proposed in 2011. Therefore, this book contains a detailed derivation of VMTPM based on canonical arguments of variational continuum mechanics, followed by the presentation of several applications to consolidation problems we believe to be of relevance in both geomechanics and biomechanics. The intent is to show the variational consistency of this theory and to exemplify its capability to describe a large class of linear and non-linear mechanical behaviors observed in two-phase saturated materials.

During these years, VMTPM was consolidated in the theoretical fundamentals and corroborated with studies showing its capability of predicting established experimental evidences as well as of encompassing paradigms of widespread use in multiphase poroelasticity applied to geomechanics and biomechanics, such as Terzaghi's stress partitioning principle and Biot's equations. Most of the results produced by this research have been published on specialized journals and presented at international meetings in the field. Nevertheless, we believe that the monograph format provides the ideal ground to report a revisited exposition of this variational theory keeping uniformity of treatment and of notation.

In this contribution, we strove to provide a theoretical approach capable of attaining a *medium-independent* framework, presenting to the poroelasticity community a set of equations which *any other continuum theory of poroelasticity should be downward compatible to*. This is indeed rather an ambitious plan, since it requires a general enough statement of the variational model, as well as a due discussion of a number of limit cases which should be consistently embraced by any candidate general medium-independent theory of this alleged kind. Accordingly, to achieve generality, the variational theory is developed in this work proceeding from a finite kinematic description. Just to mention a few of the limit cases specifically addressed here, it is shown that VMTPM is downward compatible to single-continuum elasticity when porosity achieves zero or unity limit conditions; special care was also taken in showing that the kinematics and the mechanics

of VMTPM consistently include the description of fluid flow outside of a porous body, and consistently address the presence of free solid-fluid surfaces. A discussion is also included on the extent to which the equations of this theory apply, beyond the purely mechanical context, to media with inelastic dissipative behavior, such as in elastoplasticity. Hence, the monograph format provided a wider editorial template suitable to accommodate this more extended treatment.

This work was written for an intended audience including investigators in the fields of continuum mechanics, geomechanics and biomechanics, as they will find in this contribution not only a thorough presentation of VMTPM as a theoretical framework for porous media, but also several of its applications of relevance for their research.

The authors would like to acknowledge Prof. Luciano Rosati from University of Naples Federico II, Dr. Alessandro della Corte from University of Rome La Sapienza, and Dr. Shihab Asfour from University of Miami for their scientific contribution to the material presented in Chaps. 1 and 5.

Finally, the authors wish to thank Prof. Francesco dell'Isola for the encouragement to undertake the task of writing this monographic work, for his support, and for the many insightful discussions on the roots of continuum mechanics.

Benevento, Italy
Miami, USA

Roberto Serpieri
Francesco Travascio

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