

The Zakharov System and its Soliton Solutions

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Preface

V.E. Zakharov is a famous Russian (Soviet) mathematical physicist. He made excellent contributions to plasma physics, turbulence and soliton theory. In 1962, he considered the collapse of Langmuir waves, and in 1972, he proposed a system coupled by the electric field and the perturbation of particles when studying the interaction between plasma and laser. From the viewpoint of mathematics, such system (which was known as Zakharov system) is the nonlinear Schrödinger equation coupled with the wave equation with strong nonlinearity. Zakharov computed the soliton solution for this system, and explained clearly the unresolved phenomenon for a long time in laser target shooting, namely, the density hollow phenomenon near the critical surface. Hence, this system caused great concern among the international physics community. Since then, many research works have been done on Zakharov system both in mathematics and physics, and many innovative, significant achievements which are of great impact on other research fields have been obtained. In mathematics, a lot of researchers, for example, I. Bejenaru, J. Bourgain, J. Colliander, L. Glangetas, B. Guo, Z. Guo, Z. Hani, S. Herr, J. Holmer, C. Kenig, N. Masmoudi, F. Merle, K. Nakanishi, T. Ozawa, G. Ponce, H. Pecher, F. Pusateri, C. Sulem, P.L. Sulem, D. Tataru, Y. Tsutsumi, N. Tzirakis, J. Shatah, and L. Vega made a series of important works on problems about the existence of global solution, uniqueness, blow-up, low regularity and singular limit.

This book introduces the mathematical theories, research methods and results for Zakharov type system, including the existence of global solution in energy space, uniqueness, blow-up, low regularity, large time behavior and the singular limit. As the developments in this literature are quite quick, we are not aiming to cover all the important results for such system. However, through the contents introduced in the book, we hope the readers can quickly trace the main subject of Zakharov system and conduct related research on this system.

Owing to the limited time and knowledge of the authors, there must be some improper errors and omissions in the book. Any suggestions and comments are welcome.

Beijing, China
June 2015

Boling Guo

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