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Recent Developments in Anisotropic Heterogeneous Shell Theory

General Theory and Applications of Classical Theory - Volume 1

Springer

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I have no satisfaction in
formulas unless I feel their
numerical magnitude.

William Thomson–Lord Kelvin

In memoriam of
Professor L. Librescu

Preface

The theory of shells is an independent and highly developed science, logically based on the theory of elasticity. Constructions consisting of thin-walled elements have found widespread applications in mechanical engineering, civil and industrial construction, ships, planes, and rockets building, as well as transport systems. The development of different shell models requires the application of hypotheses based on elasticity theory leading to a reduction in terms of two-dimensional equations that describe the deformation of the shell's middle surface. The solution of shell problems requires use of various numerical methods and involves great difficulties of computational nature. The authors present discrete–continuum approaches which they developed for solving problems of elasticity theory and which allow to reduce the initial problem to systems of ordinary differential equations. These are then solved by the stable numerical method of discrete orthogonalization and will be presented in this book. On the basis of these approaches, a solution for a wide class of problems of stationary deformation of anisotropic heterogeneous shells is obtained.

Many structural elements of present-day engineering, manufactured in the form of shells of various shape and complex structure with different kinds of fixation, are under the action of distributed and local loads. The wide use of shell-like elements can be attributed to the wish for satisfying the requirements associated with complex operating conditions of machines, flying vehicles, different structures, and other aggregates [7, 10–13, 15]. The complication of construction and of the structural forms of shell-like members necessitates developing a corresponding theory and methods for solving static and dynamic problems of shells made of anisotropic inhomogeneous materials.

Allowing for the aforesaid, this monograph considers approaches to solving different classes of static problems in linear formulations for anisotropic inhomogeneous shells. In this context, classic theory and different refined models are used that take specific features of shells made of modern composite materials into account. In a number of cases, the problems are solved in spatial formulation. This gives us the possibility to analyze thick-walled inhomogeneous shells and allows us to estimate the applicability of the applied theories.

In order to solve important classes of problems from the applied point of view and by taking into account the necessity of their effective realization, it seems to be expedient to use different shell models based on some simplifying assumptions. In this context, the wide use of classic theory based on the hypothesis of the invariability of the normal should be noted. It can be explained by a rather simple mathematical formulation of initial relations and by the fact that a great deal of the used shell elements possess parameters for which use of this hypothesis can be assumed to be feasible. Such circumstances make it necessary to develop methods for solving different complicated classes of problems within the framework of this hypothesis. In the case of shells made of modern composite materials, for which anisotropy and inhomogeneity in mechanical properties are typical, as well as of thick-walled shells, and shells subjected to action of local loads, it becomes necessary to take the effect of transverse strains and stresses into account, which are neglected by the classic theory. The efficiency of the following realization should be taken into account when constructing a refined shell theory. It is directly connected with a mechanical interpretation of the adopted assumptions, with the simplicity of mathematical formulation of initial relations, and with the order of the resolving equations.

A special feature during the development of plate and shell theories consists of connecting a mathematical model for a specific class of problems and developing the corresponding solution method. This fact is supported by the Kirchhoff–Love theory of thin shells [8, 9], where the objective is pursued by describing adequately the behavior of plates and shells and by retaining the simplicity of these models to such an extent that it is possible to solve problems with existing computing facilities. This interconnection becomes all the more evident in present times, when computers are widely used for solving shell theory problems.

> The necessity for obtaining numerical answers to numerically posed questions is a central stimulus for origin of new theories during all the path mathematics was developed. One may say with confidence that the most part of the science, which is named today as classical applied mathematics, is originated due to the desire to get rid of laborious calculations. Now, when the possibilities of computers are increased significantly, the center of gravity has shifted: the emphasis is on the search of the most efficient and convenient methods for their performing instead of total relieving of having to do arithmetic calculations (Casti and Kalaba [2]).

> In order to ensure that the powerful computational potentialities, which we have at our disposal, would be exploited, it is necessary to carry out bulky preliminary analysis of problem statements and analytical methods making some sense in application. If previously researchers were striving to simplify the problems so the linear functional equations would be obtained, at the present time an objective is to reduce computation problems to the Cauchy problems for ordinary differential equations (linear or nonlinear). As soon as a physical, economical, technical, or biological problem is reduced to the Cauchy problem for ordinary differential equations, its total solution may be thought of as approaching to completion (Bellman and Calaba [6]).

Along with universal approaches to solving problems of mechanics and mathematical physics, based on using finite elements [3, 4, 14, 16, 17], boundary element [1, 5], and other discrete methods, nowadays approaches that allow to reduce a problem to ordinary differential equations, based on approximation by other variables, using analytical tools, have found their wide application to solving certain classes of problems.

The approaches proposed are realized in computational complexes using stable numerical methods. It allows us solving problems with high accuracy for shells of different shape and structure in varying parameters in a wide range. Possibilities of the approaches proposed are supported by the examples of the number of complicated problems including analysis of specific structural elements which are widely used in different engineering applications.

In this context, the present monograph represents some approaches to numerical–analytical methods for solving the problems of mechanics of shells with various structure and shape based on the classical, refined, and spatial models.

The monograph consists of two books, each of which consists of three chapters. A summary of the chapters is as follows.

Book 1

Chapter 1: Elastic bodies in the form of thin- and thick-walled anisotropic shells are considered. The shells may be made of both homogeneous and inhomogeneous materials with discrete (multilayer) structure or of continuously inhomogeneous materials (functionally gradient materials). The stationary deformation of shells of the above class is analyzed by using various mechanical models. The basic relations of the theory of elasticity, which include the equations of equilibrium, geometrical, and physical relations, are presented. By using classical and refined shell theories, the original three-dimensional problem is reduced to a two-dimensional one. The fundamental equations of the classical (Kirchhoff–Love) shell theory, which are based on the hypothesis of undeformed normals, are presented. It is assumed that all of the shell layers are stiffly joined and operate mutually without sliding and separation. It is assumed that geometrical and mechanical parameters of the shells and mechanical loads applied to them are such that when considering the shell as a unit stack, the hypothesis of undeformed normals is valid. In case of laminated shells made of new composite materials with low shearing stiffness, essential anisotropy, and inhomogeneity of mechanical properties, whose characteristics of layers are highly dissimilar, the refined model based on the straight-line hypothesis is used. The basic equations of the model are presented. The various boundary physically consistent conditions at the bounding surfaces of the shells are specified.

Chapter 2: The numerical–analytical methods for solving boundary-value and boundary-value eigenvalue problems for the systems of ordinary differential equations and partial differential equations with variable coefficients are presented.

In order to solve one-dimensional problems, the discrete-orthogonalization method is proposed. Such an approach is based on reducing the boundary-value problem to a number of Cauchy problems and on their orthogonalization at some points of the integration interval which provides stability of calculations. In case of boundary-value eigenvalue problems, such an approach is employed in combination with an incremental search method. In order to solve two-dimensional problems, an approach based on reducing the original partial system to systems of ordinary differential equations by making use of spline approximation, solved by the discrete-orthogonalization method, is proposed. Employing spline-functions has the following advantages: Stability with respect to local disturbances, i.e., spline behavior in the vicinity of a point does not influence the spline behavior as a whole, as it does, for example, in polynomial approximation; more satisfactory convergence in contrast to the case of polynomials being applied as approximation functions; simplicity and convenience in calculation and implementation of spline-functions with the help of modern computers. Besides that a nontraditional approach to solving problems of the above class is proposed. The approach employs discrete Fourier series, i.e., Fourier series for functions specified on the discrete set of points. The two-dimensional boundary-value problem is solved by reducing it to a one-dimensional one as a result of introducing auxiliary functions and by separation of variables when using discrete Fourier series. Taking into account the calculation possibilities of modern computers, which make it possible to calculate a large number of series terms, the problem can be solved with high accuracy.

Chapter 3: The results of studying stationary deformation of anisotropic inhomogeneous shells of various classes by using the classical Kirchhoff–Love theory and numerical approaches outlined in the Chap. 2 of the present book are presented. The stress–strain problems for shallow, noncircular cylindrical shells and shells of revolution are solved. The various types of boundary conditions and loadings are considered. Distributions of stress and displacement fields in the shells of the above classes are analyzed for various geometrical and mechanical parameters. The practically important stress problem for a high-pressure glass-reinforced balloon is solved. The dynamical characteristics of an inhomogeneous orthotropic plate under various boundary conditions are studied. The problem of free vibrations of a circumferential inhomogeneous truncated conical shell is solved. The effect of variation in thickness, mechanical parameters, and boundary conditions on the behavior of natural frequencies and vibration modes of a plate and cone is analyzed. Much attention is given to validation of the reliability of the results obtained by numerical calculations.

Book 2

Chapter 1: The solutions of stress–strain problems for a wide class of anisotropic inhomogeneous shells obtained by the refined model are presented. Studying these problems results in the calculations of severe difficulties due to partial differential equations with variable coefficients. For solving the problem, spline-collocation and discrete-orthogonalization methods are used. The influence of geometrical and mechanical parameters, of the boundary conditions, and of the loading character on the distributions of stress and displacement fields in shallow, spherical, conical, and noncircular cylindrical shells is analyzed. The dependence of the stress–strain pattern on shell thickness variation is studied. The problem was solved also in the case of the thickness varying in two directions. It is studied how the rule of variation in the thickness of the shells influences their stress–strain state. Noncircular cylindrical shells with elliptical and corrugated sections are considered.

The results obtained in numerous calculations support the efficiency of the discrete-orthogonalization approach proposed in the monograph for solving static problems for anisotropic inhomogeneous shells when using the refined model.

Chapter 2: A wide class of problems of natural vibrations of anisotropic inhomogeneous shells is solved by using a refined model. Shells with constructional (variable thickness) and structural inhomogeneity (made of functionally gradient materials) are considered. The initial boundary-value eigenvalue partial derivative problems with variable coefficients are solved by spline-collocation, discrete-orthogonalization, and incremental search methods. In case of hinged shells, the results obtained by making use of analytical and proposed numerical methods are compared and analyzed. It is studied how the geometrical and mechanical parameters as well as the type of boundary conditions influence the distribution of dynamical characteristics of the shells under consideration. The frequencies and modes of natural vibrations of an orthotropic shallow shell of double curvature with variable thickness and various values of curvature radius are determined. For the example of cylindrical shells made of a functionally gradient material, the dynamical characteristics have been calculated with the thickness being differently varied in circumferential direction. The values of natural frequencies obtained for this class of shells under some boundary conditions are compared with the data calculated by the three-dimensional theory of elasticity.

Chapter 3: The model of the three-dimensional theory of elasticity is employed in order to study stationary deformation of hollow anisotropic inhomogeneous cylinders of finite length. Solutions of problems of the stress–strain state and natural vibrations of hollow inhomogeneous finite-length cylinders are presented, which were obtained by making use of spline-collocation and discrete-orthogonalization methods. The influence of geometrical and mechanical parameters, of boundary conditions, and of the loading character on distributions of stress and displacement fields, as well as of dynamical characteristics in the above cylinders is analyzed. For some cases, the results obtained by three-dimensional and shell theories are compared. When solving dynamical problems for orthotropic hollow cylinders with different boundary conditions at the ends, the method of straight-line methods in combination with the discrete-orthogonalization method was also applied. Computations for solid anisotropic finite-length cylinders with different end conditions were carried out by using the semi-analytical finite element method. In case of free ends, the results of calculations the natural frequencies were compared with those determined experimentally. The results of calculations of mechanical behavior of anisotropic inhomogeneous circular cylinders demonstrate the efficiency of the discrete–continual approaches proposed in the monograph for solving shell problems using the three-dimensional model of the theory of elasticity.

References

1. Banerjee PK (1994) The boundary element methods in engineering. Mc Graw-Hill College
2. Bellman R, Kalaba R (1965) Quasilinearization and nonlinear boundary-value problems. Elsevier, Amsterdam
3. Bernadou M (1996) Finite element methods for thin shell problems. Wiley, New York
4. Bischoff M, Wall WA, Bletzinger KU, Ramm E (2004) Models and finite elements for thin-walled structures. In: Stein E, De Borst R, Hughes TJR (eds) Encyclopedia of computational mechanics. Volume 2: solids and structures. Wiley, Chichester, pp 59–137
5. Brebbia CA, Walker S (1984) Boundary element technique in engineering. Buttterworth
6. Casti J, Kalaba R (1973). Imbedding methods in applied mathematics. Addison-Wesley, Reading, MA
7. Farshad M (1992) Design and analysis of shell structures farshad of Plate and Shell Structures. Springer, Berlin
8. Love AEH (1952) Mathematical theory of elasticity. Cambridge University Press, Cambridge
9. Love AEH (1888) On the small free vibrations and deformations of thin elastic shells. Philos Trans R Soc Lond Ser A 179:491–546
10. Maan HJ (2003) Design of plate and shell structures. Wiley, New York
11. Mungan I, Abelj F (2011) Fifty years of progress for shell and spatial structures. Multi Science Publishing Co Ltd
12. Pietraszkiewicz W (2014) Shell-structures: theory and application. CRC Press, Boca Raton
13. Ramm E, Wall WA (2005) Computational Methods for Shells. Special Issue of Comput Methods Appl Mech Eng 194:2285–2707
14. Reddy JN (2005) An introduction to the finite element method. McGraw-Hill Education, New York
15. Reddy JN (2007) Theory and analysis of elastic plates and shells. CRC Press, Taylor and Francis, Boca Raton
16. Zienkiewicz OC, Taylor RL (1989) .The finite element method. McGraw-Hill, New York
17. Zienkiewicz OC, Taylor RL, Too JM (1971). Reduced integration technique in general analysis of plates and shells. Int J Numer Methods Eng 3:275–290

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