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Lattice Quantum Chromodynamics

Practical Essentials

 Springer

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To our wives and children

Preface

More than 40 years after its invention, lattice field theory has developed into an interdisciplinary research area, with close connections between physics, mathematics and informatics. Monte Carlo simulations of lattice Quantum Chromodynamics (QCD) describe the physics of hadrons from their constituents: quarks and gluons. These studies are computationally challenging. In this book we provide an overview of the techniques central to lattice QCD, including modern developments. The book has four chapters. The first chapter explains the formulation of quarks and gluons on a Euclidean lattice. The second chapter introduces Monte Carlo methods and details the numerical algorithms to simulate lattice gauge fields. The third chapter explains the mathematical and numerical techniques needed to study quark fields and the computation of quark propagators. The fourth chapter is devoted to the physical observables constructed from lattice fields and explains how to measure them in simulations. The book is aimed at enabling graduate students who are new to the field to carry out explicitly the first steps and prepare them for research in lattice QCD.

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1. U. Wolff, *Quantum Field Theory* (Lecture note, HU Berlin, Unpublished, 2010)

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