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Neha Yadav · Anupam Yadav
Manoj Kumar

An Introduction to Neural Network Methods for Differential Equations

Springer

Neha Yadav
Department of Applied Sciences
ITM University
Gurgaon, Haryana
India

Anupam Yadav
Department of Sciences and Humanities
National Institute of Technology
Uttarakhand
Srinagar, Uttarakhand
India

Manoj Kumar
Department of Mathematics
Motilal Nehru National Institute of
Technology
Allahabad
India

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Preface

Artificial neural networks, or neural networks, represent a technology that is rooted in many disciplines like mathematics, physics, statistics, computer science and engineering. Neural networks have various applications in the area of mathematical modelling, pattern recognition, signal processing and time-series analysis, etc. It is an emerging field for researchers and scientists in the industry and academics to work on. Also, many problems in science and engineering can be modelled with the use of differential equations such as problems in physics, chemistry, biology and mathematics. Due to the importance of differential equations, many methods have been developed in the literature for solving them, but they have their own shortcomings.

This book introduces a variety of neural network methods for solving differential equations arising in science and engineering. Emphasis is placed on the deep understanding of the neural network techniques, which have been presented in a mostly heuristic and intuitive manner. This approach will enable the reader to understand the working, efficiency and shortcomings of each neural network technique for solving differential equations.

The objective of this book is to provide the readers with a sound understanding of the foundations of neural network, comprehensive introduction to neural network methods for solving differential equations along with the recent developments in the techniques. The main purpose to write this textbook is stated in its title *An Introduction to Neural Network Methods for Differential Equations*. This book aims to get started with the neural network techniques for solving differential equations easily, quickly and pleasantly to beginners, regardless of their background—physics, chemistry, mathematics or engineering. This book is a comprehensive text on neural network methods for solving differential equations, and the subject matter is presented in an organized and systematic way. The book may serve as a background for readers who do not have in-depth knowledge of differential equations and neural networks together with building a basic skill set that can be used to be master in it. Our presentation in the book is aimed at developing the insights and techniques that are most useful for attacking new problems. To compile this book, we had to borrow

ideas from different sources and the credit goes to all the original developers of these networks; we have presented a list of references for each section.

This book has been compiled in four chapters. The Introduction provides a glimpse of the organization of the book and a general introduction. Chapter 1 consists of a brief overview of differential equations and the physical problems arising in science and engineering. Chapter 2 illustrates the history of neural networks starting from the 1940s beginning to the 1980s renewed enthusiasm. A general introduction to neural networks and learning technologies is presented in Chap. 3. This chapter also includes a description of multilayer perceptron and its learning methods. In Chap. 4, we introduce the different neural network methods for solving differential equations. The recent developments in all the techniques is also presented in this section. The conclusion is also presented at the end of Chap. 4, which concludes the topics presented in the book. An exhaustive list of references is given at the end of the book.

Neha Yadav
Anupam Yadav
Manoj Kumar

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Introduction

A series of problems in many scientific fields can be modelled with the use of differential equations such as problems in physics, chemistry, biology, economics, etc. Although model equations based on established physical laws may be constructed using analytical tools and are frequently inadequate for the purpose of obtaining their closed form solution. Due to the importance of differential equations many methods have been proposed in the existing literature for their solution. Principal numerical methods available for solving differential equations are Finite difference method (FDM), Finite element method (FEM), Finite volume method (FVM), the boundary element method (BEM), etc. These methods generally require discretisation of the domain into a number of finite elements (FEs), which is not a straightforward task.

In contrast, for FE-type approximation, neural networks can be considered as approximation schemes where the input data for a design of network consist of only a set of unstructured discrete data points. Thus an application of neural network for solving differential equations can be regarded as a mesh-free numerical method. The solution via neural network is differentiable, closed analytic form and easily used in any subsequent calculation. Most other techniques offer a discrete solution or a solution of limited differentiability. This book presents the general introduction to neural networks and a brief description of different neural network methods for solving ordinary and partial differential equations.

Neural networks are simplified models of the biological nervous system and therefore have drawn their motivation from the kind of computing performed by a human brain. In general, the neural network is a highly interconnected network of a large number of processing elements called neurons in an architecture inspired by the brain. The neural network learns by examples and thus can be trained to acquire knowledge about the system. Once the training has been performed appropriately, the network can be put to effective use for solving 'unknown' instances of the problem. Neural networks adopt various learning mechanisms among which supervised and unsupervised learning methods have turned out to be very popular.

Neural networks have been successfully applied to problems in the fields of pattern recognition, image processing, forecasting and optimization, etc.

Initially, most of the work in solving differential equations using neural network is restricted to the case of solving the system of algebraic equations which result from the discretisation of the domain. The solution of a linear system of equations is mapped onto the architecture of a neural network and the solution to the system of given equations is then obtained by the minimization of the network's energy function. Another approach to the solution of differential equations is based on the fact that certain types of splines, for instance B_1 splines, can be derived by the superposition of piecewise linear activation functions. The solution of differential equations using B_1 splines as basis functions can be obtained by solving a system of linear or nonlinear equations in order to determine the coefficients of splines. Such a solution is mapped directly on the architecture of a feedforward neural network by replacing each spline with the sum of piecewise linear activation functions that correspond to the hidden units. This method considers local basis function and in general requires many splines in order to yield accurate solution. Furthermore, it is not easy to extend these techniques to multidimensional domains.

In this book we present different neural network methods for solution of differential equations, which provides many attractive features towards the solution: (i) The solution using neural network is differentiable, closed analytic form and easily used in any subsequent calculation; (ii) Method is general and can be applied to solve ordinary as well as partial differential equations with higher order complexities; (iii) Method requires less number of model parameters than any other technique and hence requires less memory space; (iv) Provides a solution with very good generalization properties.

The objective of this book is to provide the reader with a sound understanding of the foundations of neural network and a comprehensive introduction to different neural network methods for solving differential equations. Our presentation is aimed at developing the insights and techniques that are most useful for attacking new problems. However, the matter presented in this book is available in different books and research articles but we summarized the important useful material in an effective manner, which can serve as an introduction to new researchers and be helpful both as a learning tool and as a reference.

The structure of the book is as follows. The book is divided into four chapters.

Chapter 1, entitled "Overview of Differential Equations", introduces fundamentals of differential equation problems with some appropriate examples. This chapter also explains some existing numerical methods with examples for the solution of differential equations.

Chapter 2, entitled "History of Neural Networks", presents the origin of Neural Network in the existing literature.

Chapter 3, entitled "Preliminaries of Neural Networks", introduces the fundamentals of neural networks along with their learning algorithms and major architectures.

Chapter 4, entitled "Neural Network Methods for Solving Differential Equations", contains different neural network methods for solving differential equations of various kinds and complexities. This chapter also contains some worked out numerical examples arising in real-life applications.

MATLAB code for the solution of differential equations based on neural network has been also given in the Appendix section.