

BRST Symmetry and de Rham Cohomology

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Preface

Since Dirac proposed quantization for systems with constraints, there have been considerable progresses in Hamiltonian quantization method associated with Becchi-Rouet-Stora-Tyutin (BRST) charge. Especially in topological solitons such as $O(3)$ nonlinear sigma model, $CP(N)$ model, Skymion model and chiral bag model, there appear geometrical constraints which can be rigorously treated in the Hamiltonian quantization scheme by exploiting Stückelberg fields in extended phase spaces. In this book, by including ghost and antighost fields in these extended phase spaces, we construct the BRST invariant effective Lagrangian for these solitons and some other models described below.

Moreover, exploiting the Hamiltonian quantization method, there have been attempts to quantize the geometrically constrained systems such as free particles on a sphere and on a torus to investigate the BRST symmetries involved in the systems. Both symplectic embedding and Hamilton-Jacobi quantization schemes have been also developed to analyze Proca model, self-dual master Lagrangian and nonholonomic system. The BRST symmetries in $SU(3)$ linear sigma model, fractional spin statistics of $CP(1)$ model with Hopf term and gauge symmetry enhancement in enlarged $CP(N)$ model coupled with $U(2)$ Chern-Simons term have been studied in the Hamiltonian quantization method.

Phenomenologically, flavor symmetry breaking effect on $SU(3)$ Skymion has been investigated to yield relevant mass spectra including Weyl ordering corrections associated with the constraints. Strangeness in $SU(3)$ chiral bag model, which is a hybrid of the Skymion and MIT bag model, has been evaluated in terms of baryon octet and decuplet magnetic moments to predict data of SAMPLE and HAPPEX experiments on proton strange form factor. Most of physical systems are supposed to possess constraints and thus significance of the Hamiltonian quantization for these systems is being emphasized increasingly.

Finally, in this book, BRST charge, de Rham cohomology and closed algebra of quantum field operators have been investigated in 't Hooft-Polyakov monopole, which is classified as second class system in the Dirac quantization formalism. To this end, the first class Hamiltonian of the monopole has been constructed to define a monopole charge in $U(1)$ subgroup of $U(2)$ gauge group in the first

class configuration and to investigate Bogomol'nyi bound on extended internal two-sphere. The explicit form of the BRST invariant Hamiltonian has been studied to discuss geometric aspects of the corresponding de Rham cohomology.

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