

**TRIANGULAR NORM-BASED MEASURES  
AND GAMES WITH FUZZY COALITIONS**

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# TRIANGULAR NORM-BASED MEASURES AND GAMES WITH FUZZY COALITIONS

*by*

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*To Hadassah and Sieglinde*

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*Dan Butnariu  
Erich Peter Klement*