

TOPICS IN PHILOSOPHICAL LOGIC

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TOPICS IN  
PHILOSOPHICAL LOGIC



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I take pleasure in dedicating this book to my students who have over the years been also my collaborators in logical research, and in particular to:

**BAS VAN FRAASSEN**

**NEIL A. GALLAGHER**

**JAMES W. GARSON**

**BARBARA ANNE HUNT**

**RICHARD K. MARTIN**

**ROBERT K. MEYER**

**ANNE CROSS PELON**

**BRIAN SKYRMS**

**ERNEST SOSA**

## PREFACE

The aim of the book is to introduce the reader to some new areas of logic which have yet to find their way into the bulk of modern logic books written from the more orthodox direction of the mainstream of developments. Such a work seems to me much needed, both because of the intrinsic value and increasing prominence of the nonstandard sector of logic, and because this particular sector is of the greatest interest from the standpoint of philosophical implications and applications.

This book unites a series of studies in philosophical logic, drawing for the most part on material which I have contributed to the journal literature of the subject over the past ten years. Despite the fact that some of these essays have been published in various journals at different times, they possess a high degree of thematic and methodological unity. All of these studies deal with material of substantial current interest in philosophical logic and embody a fusion of the modern techniques of logical and linguistic-philosophical analysis for the exploration of areas of logic that are of substantial philosophical relevance.

Chap. VII on 'Venn Diagrams for Plurative Syllogisms' was written in collaboration with my student Mr. Neil A. Gallagher, and Chap. XIII on 'Topological Logic' was written in collaboration with my student Mr. James W. Garson. I am grateful to these gentlemen for agreeing to the inclusion of this material here. I express my appreciative thanks to Miss Dorothy Henle and Miss Judy Bazy for their help in preparing this material for the printer and in seeing it through the press. I should also like to thank Mr. Alasdair Urquhart for his help with the proofreading.

*Pittsburgh, August, 1967*

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