Magnetohydrodynamics:
Waves and Shock Waves in Curved Space-Time

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Magnetohydrodynamics: Waves and Shock Waves in Curved Space-Time

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PREFACE

For seventy years, we have known that Einstein's theory is essentially a theory of propagation of waves for the gravitational field. Confusion enters, however, through the fact that the word ***wave,*** in physics, implies sometimes repetition and sometimes not. This confusion is often increased by he use of Fourier transforms, by which a disturbanse which appears to be without repetition is resolved into periodic wave-trains with all frequencies. But, in a general curved space-time, we have nothing corresponding to Fourier transforms.

Here, we consider systematically waves corresponding to the propagation of discontinuities of physical quantities describing either fields (essentially electromagnetic fields and gravitational field), or the motion of a fluid, or together, in magnetohydrodynamics, the changes in time of a field and of a fluid.

The main equations, for the different studied phenomena, constitute a ***hyperbolic system*** and the study of a formal Cauchy problem is possible. We call ***ordinary waves*** the case in which the derivative of superior order appearing in the system are discontinuous at the traverse of a hypersurface, the wave front ; we call shock waves the case where the derivatives of an order inferior by one are discontinuous at the traverse of a wave front.

From 1950, many well-known scientits (Taub, Synge, Choquet-Bruhat, etc.) have studied the corresponding equations for different physical phenomena : systems associated to the electromagnetic and gravitational fields, to hydrodynamics and to magnetohydrodynamics.

The purpose of this book -which is a book of mathematical physics- is to give a synthesis of the results and to precise some new results, using a unique method, the method of ***the tensor-distributions.*** In particular, this method is a powerful tool for the study of the gravitational shock waves and for the magnetohydrodynamical shock waves, where a complete theory may be given. Such a theory is specially adapted to astrophysics, in particular for the treatment of the pulsars.

The same method may be used for the quantization of some fields in a curved space-time. Therefore, we have included in the book a Note concerning an approach of the quantization of fields in this context.

We hope that this book is clear and contains sufficiently detailed mathematical proofs, with the suitable mathematical rigor. One of its purposes is to give an easy access to the different works on this subject during half-century and also to most recent results.

A. Lichnerowicz
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