Lightlike Submanifolds of Semi-Riemannian Manifolds and Applications

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Lightlike Submanifolds of Semi-Riemannian Manifolds and Applications

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Preface

This book is about the lightlike (degenerate) geometry of submanifolds needed to fill a gap in the general theory of submanifolds. The growing importance of lightlike hypersurfaces in mathematical physics, in particular their extensive use in relativity, and very limited information available on the general theory of lightlike submanifolds, motivated the present authors, in 1990, to do collaborative research on the subject matter of this book.

Based on a series of author's papers (Bejancu [3], Bejancu-Duggal [1,3], Duggal [13], Duggal-Bejancu [1,2,3]) and several other researchers, this volume was conceived and developed during the Fall '91 and Fall '94 visits of Bejancu to the University of Windsor, Canada.

The primary difference between the lightlike submanifold and that of its non-degenerate counterpart arises due to the fact that in the first case, the normal vector bundle intersects with the tangent bundle of the submanifold. Thus, one fails to use, in the usual way, the theory of non-degenerate submanifolds (cf. Chen [1]) to define the induced geometric objects (such as linear connection, second fundamental form, Gauss and Weingarten equations) on the lightlike submanifold. Some work is known on null hypersurfaces and degenerate submanifolds (see an up-to-date list of references on pages 138 and 140 respectively).

Our approach, in this book, has the following outstanding features: (a) It is the first-ever attempt of an up-to-date information on null curves, lightlike hypersurfaces and submanifolds, consistent with the theory of non-degenerate submanifolds. (b) Our geometric technique is most general, and, has potential for further research on this new topic in differential geometry and other areas of mathematics and physics. (c) We have provided a considerable amount of geometric and physical results on 2 and 3 dimensional lightlike surfaces and hypersurfaces, respectively, of Lorentz manifolds, as an attempt to bring closer mathematicians and physicists.

Chapters 1 and 2 contain most of the prerequisites for reading the rest of the book.

Chapter 3 deals with the fundamental existence and uniqueness theorem of null curves in Lorentz manifolds followed by some results when the ambient manifold is spacetime of general relativity or n-dimensional Minkowski space.

Chapter 4 is the core of this book, introducing the most general differential geometric technique to deal with all the induced geometric objects on the lightlike hypersurfaces under study. Key result in this chapter is proof of the existence of lightlike hypersurfaces. For physical use, we have provided considerable information on Monge hypersurfaces of Minkowski spaces.

Chapters 5 and 6 are devoted to the general theory of lightlike submanifolds and Cauchy-Riemann (CR) submanifolds respectively. The results of these chapters are primarily based on the present author's papers (Bejancu-Duggal [3] and Duggal-Bejancu [1,2,3]). The background material of Chapter 6 comes from the works of Bejancu [1] and Chen [1,2,3].

Chapters 7, 8 and 9 have been specifically written to apply the theory of lightlike hypersurfaces (cf. chapter 4) to relativity. The background material of these chapters comes from Yano [3], Adler et al. [1], Hawking-Ellis [1], Kramer et al. [1] and Duggal [7,9-12]. It is important to mention that, since this is the first book on lightlike submanifolds, the scope of applications has been limited to interaction with some results on Killing horizon, electromagnetic and radiation fields and asymptotically flat spacetimes. Also, scattered through these chapters there are some new physical results to stimulate interest for further research.

It is our hope that the audience of this book will include graduate students and researchers who have a basic acquaintance with semi-Riemannian geometry and its submanifolds and interest to learn its lightlike counterpart. In general, this book should be appropriate for a two semester graduate course in mathematics and or physics, provided the instructor exercises some selection in the most difficult areas. In particular, for those interested in mathematical physics, the instructor may skip chapters 5 and 6 and with further appropriate selections may cover the rest of the material in one semester. As a reference book, it should be readable to graduate students, research assistants and faculty.

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Both authors are grateful to all authors of books and articles whose work they have used in preparing this book.

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