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FUZZY RELATION EQUATIONS AND THEIR APPLICATIONS TO KNOWLEDGE ENGINEERING

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Foreword

It took many decades for Peirce's concept of a relation to find its way into the microelectronic innards of control systems of cement kilns, subway trains, and tunnel-digging machinery. But what is amazing is that the more we learn about the basically simple concept of a relation, the more aware we become of its fundamental importance and wide ranging ramifications. The work by Di Nola, Pedrycz, Sanchez, and Sessa takes us a long distance in this direction by opening new vistas on both the theory and applications of fuzzy relations – relations which serve to model the imprecise concepts which pervade the real world.

Di Nola, Pedrycz, Sanchez, and Sessa focus their attention on a central problem in the theory of fuzzy relations, namely the solution of fuzzy relational equations. The theory of such equations was initiated by Sanchez in 1976, in a seminal paper dealing with the resolution of composite fuzzy relational equations. Since then, hundreds of papers have been written on this and related topics, with major contributions originating in France, Italy, Spain, Germany, Poland, Japan, China, the Soviet Union, India, and other countries. The bibliography included in this volume highlights the widespread interest in the theory of fuzzy relational equations and the broad spectrum of its applications.

In the context of applications, the importance of the theory of fuzzy relational equations derives from the fact that human knowledge may be viewed as a collection of facts and rules, each of which may be represented as the assignment of a fuzzy relation to the unconditional or conditional possibility distribution of a variable. What this implies is that knowledge may be viewed as a system of fuzzy relational equations. In this perspective, then, inference from a body of knowledge reduces to the solution of a system of fuzzy relational equations. This basic idea underlies the theory of approximate reasoning based on fuzzy logic as well as various versions of fuzzy Prolog and, in particular, Professor Baldwin's language FRIL, which is a Prolog-based language for inference from fuzzy relations.

The work of Di Nola, Pedrycz, Sanchez, and Sessa has a dual purpose: first, to present an authoritative and up-to-date account of the theory in a rigorous, thorough, and complete fashion; and second, to describe its applications, especially in the realm of knowledge-based systems.

The theoretical part addresses the major issues, among them: fuzzy relational equations in residuated lattices, the lower solutions of max-min equations, the measures of fuzziness of solutions, the max-min decomposition, and fuzzy relational equations with triangular norms. In a transition to applications, the authors consider an issue which is of high intrinsic importance, namely, the approximate solution of fuzzy relational equations. In this and other chapters, the authors make the reading easier for the non-mathematician by describing solution algorithms and applying them to well-chosen examples.

The second part, which deals with applications, develops a systematic approach to knowledge representation and inference based on the theory developed in the earlier chapters. In addition to the applications to knowledge-based systems, the authors present a lucid account of the basic ideas underlying the analysis and design of fuzzy logic controllers. Such controllers have proved

to be highly successful in a variety of applications ranging from industrial process control and robotics to medical diagnosis and traffic control.

An important issue which is addressed in the chapters dealing with knowledge-based systems is that of the validation of production rules and the related problems of reduction and reconstruction. In these chapters, there is a great deal that is new in the application of the theory of fuzzy relational equations to the problem of inference.

The authors of this volume have played a leading role in the development of the theory of fuzzy relational equations and its applications. Not surprisingly, the book reflects their high expertise and expository skills. Much of the material is new; the writing is lucid and well-motivated; and the references are a model of thoroughness and organization. Di Nola, Pedrycz, Sanchez, and Sessa deserve our thanks and congratulations for authoring an outstanding text which is certain to become an important landmark in the development of the theory of fuzzy sets and its applications.

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Preface

The growing interest to knowledge engineering has forced the burning need to search novel tools which mimic human processes of perception, decision-making, object recognition, etc.

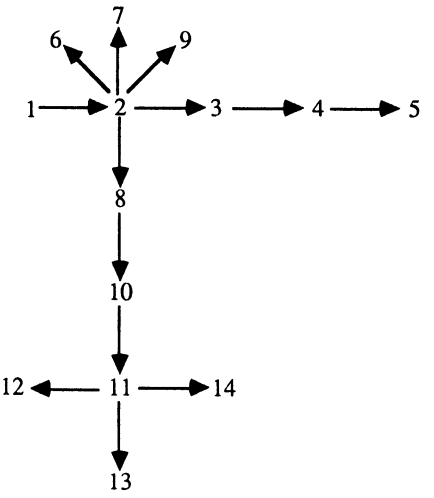
Fuzzy set theory is an approach studied extensively in this area. On the other hand, the notion of fuzzy set, i.e. a set with not sharply defined boundaries, is very natural for a human being. Moreover, this approach is very convenient since the user-friendly man-system communication is performed in a linguistic (not numerical) fashion.

The aim of this monograph is to provide the reader with fundamentals of fuzzy relations indicating clearly their applications to knowledge engineering as, e.g., verification of a knowledge base (in the sense of its consistency and relevancy), designing of inference mechanisms, reduction of a knowledge base, propagation of uncertainty in different reasoning schemes.

This book is organized in 15 Chapters. Chapters 1÷9 contain theoretical backgrounds of the theory of fuzzy relation equations. In these Chapters, the fuzzy sets are defined and studied on several types of lattice: of course, this in the spirit of the symbolic computations, characteristic for Artificial Intelligence. In Chapters 10÷14, containing the above mentioned applications to knowledge engineering, the fuzzy sets are expressed in the real unit interval in order to process the vague information coming from the expert.

Chapter 15 contains two useful and extensive bibliographies of papers on fuzzy equations, fuzzy relations and related topics. Each Chapter has its appropriate references.

The following diagram illustrates some main routes for studying the Chapters of this book. Of course, the reader should consider this diagram only as indicative.



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List of Abbreviations

Ch(s).	Chapter(s)
Corol.	Corollary
Def(s).	Definition(s)
Eq(s).	Equation(s)
Ex(s).	Example(s)
Fig(s).	Figure(s)
KB	Knowledge Base
Prop(s).	Proposition(s)
Sec(s).	Section(s)
Thm(s).	Theorem(s)
[x, Ch.y]	Reference x of Chapter y