

DESIGN AND ANALYSIS OF SHELL STRUCTURES

SOLID MECHANICS AND ITS APPLICATIONS

Volume 16

Series Editor:

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Design and Analysis of Shell Structures

by

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Springer-Science+Business Media, B.V.

Library of Congress Cataloging-in-Publication Data

Farshad, Mehdi.

Design and analysis of shell structures / Mehdi Farshad.
p. cm. -- (Solid mechanics and its applications ; 16)

Includes index.

ISBN 978-90-481-4200-2

ISBN 978-94-017-1227-9 (eBook)

DOI 10.1007/978-94-017-1227-9

1. Shells (Engineering)--Design and construction. 2. Structural analysis (Engineering) I. Title. II. Title: Shell structures. III. Series.

TA660.S5F42 1992

624.1'7762--dc20

92-18175

ISBN 978-90-481-4200-2

Printed on acid-free paper

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Originally published by Kluwer Academic Publishers in 1992

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Subject Index

Preface

Shell Structures present immense structural and architectural potential in various fields of civil, mechanical, architectural, aeronautical, and marine engineering. Examples of shell structures in civil and architectural engineering are: varieties of concrete shell roofs, liquid retaining structures and water tanks, concrete silos, cooling towers, containment shells of nuclear power plants, and concrete arch dams. In mechanical engineering, shell forms are used in piping systems, curved panels, and in pressure vessel technology. Aircrafts, spacecrafts, missiles, ships, and submarines are examples of shells used in aeronautical and marine engineering. Shells are found in various biological forms such as the eye and the skull, plants, and animal shapes. Thus, another application of shell engineering would be the field of Biomechanics.

Shell structures developed since ancient times and now are being increasingly used in various industries. Shells are used in the covering of large spans, liquid retaining installations, silos, and containment structures. They are also used in the construction of light-weight vehicles, pressure vessels, and space structures. Advent of such materials as ferro-cement, fiber-reinforced concrete, composite materials, and reinforced polymers have all enhanced the domain of shell technology. With the development of new prefabrication schemes as well as the need for recycling of materials, the potential of shell applications has further increased. In addition to mechanical advantages, such as durability, high strength and stability, shell structures enjoy the unique position of having extremely high aesthetic value in various architectural designs.

In spite of all these features and potential applications, many engineers and architects are unacquainted with shells as well as the aspects of shell behavior and design. The purpose of this book is to familiarize the engineering and architectural students, as well as practicing engineers and architects, with the behavior and design aspects of shell structures. The goal of this book is to present three aspects: the physical behavior, the structural analysis, and the

design of shells in a simple, integrated, and yet concise fashion. Thus, the book contains three major aspects of shell engineering. These are: (1) physical understanding of shell behavior, (2) use of applied shell theories, (3) development of design methodologies together with shell design examples.

To achieve these goals, simplified shell theories have been discussed in this book and have been immediately applied to actual problems. In this sense, the book bridges the gap between the elaborate theoretical treatments of shells, on the one hand, and, the practical aspects of the analysis and design of shells, on the other hand. Being aware of a wide variety of existing numerical routines for shells analysis, we have, nevertheless, made use of simple analytical schemes of shell analysis so that the designer can understand the analysis procedure and to perform parametric studies. The theoretical tools required for rational analysis of shells are kept at a modest level so that engineering and architectural students, as well as practicing engineers and architects, can grasp the fundamentals of shell behavior and, at the same time, understand the related theory and be able to apply it to actual design problems. To achieve a physical understanding of complex shell behavior, quantitative presentations are supplemented by *qualitative* discussions so that the reader can grasp a "physical feeling" of shell behavior. To make the book useful as a reference manual, a number of analysis and detailed design examples are also worked out in various chapters.

The actual design of shells, involves the use of appropriate codes of practice. Thus, while making use of some existing codes on shells, in order to provide a text that could be used in various countries, we have attempted to present the designs apart from the existing codes. In some cases, the common guidelines provided by several standards, including ACI, BS, DIN, and IS, have been used.

This book can be used as a text book, and / or a reference book in undergraduate as well as graduate university courses in the fields of civil, mechanical, architectural, aeronautical, and materials engineering. It can also be used as a reference and design-analysis manual for the practicing engineers and architects. To make the book useful to design engineers and architects, the text is supplemented by a number of appendices containing tables of shell analysis and design charts and tables. Metric system is used throughout this book.

The material of this book have been developed through many years of teaching at the Universities of Shiraz and Tehran, University of Toronto, and the Swiss Federal Institute of Technology (ETHZ) as well as through research and practical design experience by the author. Thus, in the development of this text, various viewpoints and experiences have been extremely constructive.

The author would like to thank the Swiss Federal Laboratories for Materials Testing and Research (EMPA, Dübendorf) and, in particular, Professor F. Eggimann and Professor U. Meier and Hr. H. Fritz for supporting this project. Special thanks are due to Hr. P. Flüeler who has given great encouragement and support in bringing this book to its present publication. The author would also like to thank professor Gladwell for his useful comments on the manuscript. This book is dedicated to my family (Gowhar, Anahita, and Mazda) who have shown great patience during the long period of manuscript preparation.

M. Farshad

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