

# Complex Analysis through Examples and Exercises

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# Complex Analysis through Examples and Exercises

by

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# Preface

The book *Complex Analysis through Examples and Exercises* has come out from the lectures and exercises that the author held mostly for mathematician and physicists . The book is an attempt to present the rather involved subject of complex analysis through an active approach by the reader. Thus this book is a complex combination of theory and examples.

Complex analysis is involved in all branches of mathematics. It often happens that the complex analysis is the shortest path for solving a problem in real circumstances. We are using the (Cauchy) integral approach and the (Weierstrass) power series approach .

In the theory of complex analysis, on the hand one has an interplay of several mathematical disciplines, while on the other various methods, tools, and approaches. In view of that, the exposition of new notions and methods in our book is taken step by step. A minimal amount of expository theory is included at the beginning of each section, the *Preliminaries*, with maximum effort placed on well selected examples and exercises capturing the essence of the material. Actually, I have divided the problems into two classes called *Examples* and *Exercises* (some of them often also contain proofs of the statements from the Preliminaries). The examples contain complete solutions and serve as a model for solving similar problems given in the exercises. The readers are left to find the solution in the exercises; the answers, and, occasionally, some hints, are still given. Special sections contain so called *Composite Examples* which consist of combinations of different types of examples explaining, altogether, some problems completely and giving to the reader an opportunity to check his entire previously accepted knowledge.

The necessary prerequisites are a standard undergraduate course on real functions of real variables. I have tried to make the book self-contained as much as possible. For that reason, I have also included in the *Preliminaries* and *Examples* some of the mathematical tools mentioned.

The book is prepared for undergraduate and graduate students in mathematics, physics, technology, economics, and everybody with an interest in complex analysis.

We have used for some calculations and drawings the mathematical software



packages *Mathematica* and *Scientific Work Place v2.5*.

I am grateful to Academician Bogoljub Stanković for a long period of collaboration on the subject of the book, to Prof. Arpad Takači for his numerous remarks and advice about the text, and to Ivana Štajner for reading some part of the text. I would like to express my thanks to Marčičev Merima for typing the majority of the manuscript. It is my pleasure to thank the Institute of Mathematics in Novi Sad for working conditions and financial support. I would like to thank Kluwer Academic Publishers, especially Dr. Paul Roos and Ms. Angelique Hempel for their encouragement and patience.

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