

## MULTIELEMENT SYSTEM DESIGN IN ASTRONOMY AND RADIO SCIENCE

# ASTROPHYSICS AND SPACE SCIENCE LIBRARY

---

VOLUME 268

---

## EDITORIAL BOARD

### *Chairman*

W. B. BURTON, *Sterrewacht, Leiden, P.O. Box 9513, 2300 RA Leiden, The Netherlands*  
Burton@strw.leidenuniv.nl

### *Executive Committee*

J. M. E. KUIJPERS, *Faculty of Science, Nijmegen, The Netherlands*  
E. P. J. VAN DEN HEUVEL, *Astronomical Institute, University of Amsterdam,  
The Netherlands*  
H. VAN DER LAAN, *Astronomical Institute, University of Utrecht,  
The Netherlands*

## MEMBERS

I. APPENZELLER, *Landessternwarte Heidelberg-Königstuhl, Germany*  
J. N. BAHCALL, *The Institute for Advanced Study, Princeton, U.S.A.*  
F. BERTOLA, *Università di Padova, Italy*  
J. P. CASSINELLI, *University of Wisconsin, Madison, U.S.A.*  
C. J. CESARSKY, *Centre d'Etudes de Saclay, Gif-sur-Yvette Cedex, France*  
O. ENGVOLD, *Institute of Theoretical Astrophysics, University of Oslo, Norway*  
R. McCRAY, *University of Colorado, JILA, Boulder, U.S.A.*  
P. G. MURDIN, *Royal Greenwich Observatory, Cambridge, U.K.*  
F. PACINI, *Istituto Astronomia Arcetri, Firenze, Italy*  
V. RADHAKRISHNAN, *Raman Research Institute, Bangalore, India*  
K. SATO, *School of Science, The University of Tokyo, Japan*  
F. H. SHU, *University of California, Berkeley, U.S.A.*  
B. V. SOMOV, *Astronomical Institute, Moscow State University, Russia*  
R. A. SUNYAEV, *Space Research Institute, Moscow, Russia*  
Y. TANAKA, *Institute of Space & Astronautical Science, Kanagawa, Japan*  
S. TREMAINE, *CITA, Princeton University, U.S.A.*  
N. O. WEISS, *University of Cambridge, U.K.*

*A list of titles in the series can be found at the end of this volume.*

# MULTIELEMENT SYSTEM DESIGN IN ASTRONOMY AND RADIO SCIENCE

*by*

LAZARUS E. KOPILOVICH

*Institute of Radio Physics and Electronics,  
National Academy of Sciences of Ukraine,  
Kharkov, Ukraine*

and

LEONID G. SODIN

*Institute of Radio Astronomy,  
National Academy of Sciences of Ukraine,  
Kharkov, Ukraine*



SPRINGER-SCIENCE+BUSINESS MEDIA, B.V.

A C.I.P. Catalogue record for this book is available from the Library of Congress.

ISBN 978-90-481-5846-1      ISBN 978-94-015-9751-7 (eBook)  
DOI 10.1007/978-94-015-9751-7

---

*Front coverpicture:*  
*Radio telescope UTR-2: The North-South arm. A view from the ground.*

*Printed on acid-free paper*

All Rights Reserved  
© 2001 Springer Science+Business Media Dordrecht  
Originally published by Kluwer Academic Publishers in 2001

No part of the material protected by this copyright notice may be reproduced or utilized in any form or by any means, electronic or mechanical, including photocopying, recording or by any information storage and retrieval system, without written permission from the copyright owner.

# CONTENTS

<b>Preface</b>	<b>IX</b>
<b>PART I OPTIMIZATION OF RADIO AND OPTICAL INTERFEROMETERS</b>	<b>1</b>
<b>1 Linear Interferometers</b>	<b>3</b>
1.1 Introduction .....	3
1.2 Interferometers with the Number of Aerials up to 11 .....	4
1.3 The Number of Aerials from 12 up to 30.....	5
1.4 The Number of Aerials Exceeding 30.....	8
1.5 Conclusion.....	12
<b>2 Interferometers with Complete Coverage of Rectangular Domains in the <math>U, V</math> -Plane</b>	<b>21</b>
2.1 Introduction .....	21
2.2 Crosses and T systems.....	22
2.3 Configurations of Enhanced Efficiency. ....	24
2.3.1 Direct Product of Two Linear Bases.....	26
2.3.2 Configurations Based on 2-D Difference Sets.....	28
<b>3 Interferometers with Complete Coverage of Symmetrized Domains in the <math>U, V</math> -Plane</b>	<b>35</b>
3.1 Introduction .....	35
3.2 Keto's Configurations .....	36
3.3 Hexagonal Configurations.....	38
3.3.1 Basic Characteristics .....	40
3.3.2 Configurations of Enhanced Efficiency .....	45
3.3.3 Lowering Sidelobe Level .....	49
3.4 Conclusion.....	51
<b>4 Interferometers Having Non-Redundant Apertures</b>	<b>53</b>
4.1 Introduction .....	53
4.2 Estimates of the Number of Mask Elements .....	54
4.3 $n$ -Element Masks on $n \times n$ Grids.....	57
4.4 Masks Based on Cyclic Difference Sets .....	59
4.5 Systems of Matched Masks.....	63
4.6 Masks on Hexagonal Grids .....	65

<b>PART II NON-EQUIDISTANT PHASED ANTENNA ARRAYS</b>	<b>71</b>
<b>5 General Reasons and Statement of the Problem</b>	<b>73</b>
<b>6 Linear Non-Equidistant Antenna Arrays</b>	<b>77</b>
6.1 Introduction .....	77
6.2 Arrays Based on Cyclic Difference Sets .....	79
6.3 Optimization of CDS-Based Arrays .....	83
6.3.1 <i>A Priori</i> Estimates of the Sidelobe Level .....	83
6.3.2 Half-Thinned Arrays .....	86
6.3.3 Heavily Thinned Arrays .....	90
6.4 Sectioned Arrays .....	90
6.5 Comparison with Other Data .....	93
<b>7 Planar Antenna Arrays Based on Difference Sets</b>	<b>97</b>
7.1 Introduction .....	97
7.2 Derivation of Principal Relationships .....	98
7.3 Characteristics of Arrays Based on 2-D Difference Sets .....	104
<b>PART III CODED MASKS (APERTURES) FOR IMAGING IN X-RAY AND GAMMA-RAY RANGES</b>	<b>111</b>
<b>8 Methods for Synthesizing Coded Masks</b>	<b>113</b>
8.1 Introduction .....	113
8.2 Principles of Synthesis of Coded Masks .....	114
8.2.1 Masks Based on 2-D Difference Sets .....	118
8.2.2 Masks of Other Types .....	122
8.3 Decoding Matrix .....	124
8.3.1 Masks of Types 1 to 4 .....	124
8.3.2 Other Mask Types .....	127
8.4 Conclusion .....	131
<b>9 Sensitivity of X-Ray and Gamma-Ray Telescopes with Coded Apertures</b>	<b>135</b>
9.1 Starting Relationships .....	135
9.2 Calculation of the Signal-to-Noise Ratio .....	136
9.2.1 Masks Having Unimodular Decoding Matrices .....	136
9.2.2 Masks Based on 2-D Difference Sets .....	137
9.2.3 Masks as Direct Products of Cyclic Difference Sets .....	139
9.2.4 Masks as Direct Products of Regular Sequences .....	142
9.3 Conclusion .....	144

<b>MATHEMATICAL APPENDICES</b>	<b>145</b>
<b>A Difference Sets on Linear Grids</b>	<b>147</b>
A.1 Cyclic Difference Sets.....	147
A.1.1 Properties of Cyclic Difference Sets and Associated Notions .....	148
A.1.2 Types of Cyclic Difference Sets .....	150
A.2 Relative Difference Sets.....	153
A.3 Generalizations of Relative Difference Sets .....	155
<b>B Difference Sets on Rectangular Grids</b>	<b>157</b>
B.1 Definition and Properties of Two-Dimensional Difference Sets .....	157
B.2 Difference Sets on Grids with Coprime Sidelengths. ....	158
B.3 2-D Noncyclic Difference Sets.....	159
B.3.1 Hadamard Sets .....	159
B.3.2 Difference Sets of Other Types.....	162
B.4 Generalizations of 2-D Difference Sets .....	163
B.4.1 Generalized Difference Sets with $\Lambda_x \leq \Lambda$ , $\Lambda_y \leq \Lambda$ ...	164
B.4.2 Generalized Difference Sets with $\Lambda_x > \Lambda$ , $\Lambda_y > \Lambda$ ...	167
<b>C Tables of Difference Sets</b>	<b>169</b>

## PREFACE

The multielement systems have been widely used in many fields of astronomy and radio science in the last decades. This is caused by the increasing demands on the resolution and sensitivity of such systems over the wide range of the electromagnetic wavelengths, from gamma up to radio. The ground-based optical and radio interferometers, gamma-ray and X-ray orbital telescopes, antenna arrays of radio telescopes and also some other radio devices belong to scientific instruments using multielement systems. Therefore, the current problems of the optimal construction of such systems, or precisely, those of searching for the best arrangement of the elements in them, were formulated. A rather large number of scientific papers, including those of the authors, is devoted to these problems, and we believe that the time has come to integrate the basic results of the papers into the monograph.

The offered book consists of three parts. The first part is concerned with the optimal synthesis of optical and radio interferometers of various types and purposes; the synthesis of non-equidistant antenna arrays is considered in the second part; and the methods for the construction of coded masks for X-ray and gamma-ray orbital telescopes are expounded in the third one. Since in the text combinatorial constructions which are little known to astronomers are used, the necessary information is given in the appendices. Various tables containing the parameters of the systems considered are also represented. In a number of cases, they will enable one to facilitate their designing and development.

We hope that the book shall be of interest both to scientists and engineers who are concerned with developing astronomical instruments of a high spatial resolution. The materials given in the second part may also be of help to the designers of large antenna arrays in radio wave range, e.g. of antennas for the long-range radar and radio communication.

We are thankful to V. Pazyinin who let us become acquainted with his unpublished results.

We are also grateful to E. Usachev for the assistance in preparing the computer typescript.

L.Kopilovich  
Institute of Radio Physics and  
Electronics  
of National Academy of Sciences  
Kharkov, Ukraine

L.Sodin  
Institute of Radio Astronomy  
of National Academy of Sciences  
Kharkov, Ukraine