Random Evolutions and their Applications

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New Trends

by

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To my family: wife Maria, son Victor, and daughter Julia

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PREFACE

The book is devoted to the new trends in random evolutions and their various applications to stochastic evolutionary sytems (SES). Such new developments as the analogue of Dynkin's formulae, boundary value problems, stochastic stability and optimal control of random evolutions, stochastic evolutionary equations driven by martingale measures are considered.

The book also contains such new trends in applied probability as stochastic models of financial and insurance mathematics in an incomplete market.

In the famous classical financial mathematics Black–Scholes model of a (B,S)-market for securities prices, which is used for the description of the evolution of bonds and stocks prices and also for their derivatives, such as options, futures, forward contracts, etc., it is supposed that the dynamic of bonds and stocks prices are set by a linear differential and linear stochastic differential equations, respectively, with interest rate, appreciation rate and volatility such that they are predictable processes. Also, in the Arrow–Debreu economy, the securities prices which support a Radner dynamic equilibrium are a combination of an Ito process and a random point process, with the all coefficients and jumps being predictable processes.

We suppose that in our stochastic models of a (B,S)-market for securities prices the interest rate, appreciation rate, volatility and jumps depend on a Markov or semi-Markov process, which is independent of the standard Wiener process. In this way our models have an additional source of randomness, namely, that of a Markov or semi-Markov process, besides the Wiener process. For this reason the securities market is incomplete.

In the famous classical insurance (or actuarial) mathematics only the claims processes were modelled as stochastic. One main stream in present–day insurance mathematics undertakes to incorporate financial risk in the form of stochastic interest.

In our models of insurance mathematics the risk processes, which are the summary capitals of some insurance companies, are described by the functionals of Markov or semi-Markov process. It involves not only the claims processes, but also the interest processes. The source of risk is the entire insurance portfolio of the insurance company. Such an insurance market is also incomplete.

The above mentioned stochastic models of financial and insurance mathematics in an incomplete market are the realizations of continuous and discontinuous Markov or semi-Markov REs.

Contingent claim valuation, forecast of stocks prices, distributions of some financial portfolios, and hedging of European options in an incomplete market are investigated.

We also develop the new area as the statistics of random evolution processes and their applications to financial stochastic models, as well as stochastic stability and optimal control of financial models.

The stochastic stability, optimal stochastic control, and filtering, interpolation and extrapolation of the dynamics of stocks prices (including small perturbations) are studied.

Risk processes in insurance mathematics in an incomplete market will be studied. There will also be investigated averaged, diffusion, normal deviated risk processes and their ruin probabilities in an incomplete market.

The book is also devoted to the study of stochastic stability and optimal stochastic control of random evolutions and their various applications to stochastic evolutionary systems (SES), in particular, stability and control of stochastic models arising in finance and insurance in an incomplete market.

In the last few years many works have been devoted to the study of the qualitative properties of deterministic evolutionary systems, and to some stochastic processes, with the help of Lyapunov functions. Many efforts have also been devoted to the application of the Lyapunov's functions method to the solution of the problem of obtaining a sufficient conditions of optimality of a given control (Hamilton–Bellman–Jacobi's equations, or the dynamic programming principle), and to the solution of stability problems for such systems.

In this book we consider the method of stochastic Lyapunov functions for studying the qualitative properties of RE, SES, and controlled RE and SES. Roughly speaking, stochastic Lyapunov function is the function of random evolution process, which (if it is considered as a random function) has a supermartingale property. With the help of these functions and martingale methods we study the stochastic stability and optimality control properties of RE.

The book will be useful for experts in random processes, applied probability, stochastic stability and optimal control, also for experts in finance and insurance, and to those who may be interested in the new trends and new applications of random evolutions.

The book continues the new series of mathematical monographs which is being produced at the International Mathematical Center of the National Academy of Sciences of Ukraine.

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Kiev, Ukraine

April, 1999 **Anatoly Swishchuk**

LIST OF NOTATIONS

$(\Omega, \mathfrak{F}, \mathsf{P})$	—	probability space
$(X, \mathfrak{X})$	—	measurable phase space
$(x_n; n \geq 0)$	-	Markov chain in X
$\mathbb{R}_+$	:=	$[0, +\infty)$
$(x_n; \theta_n; n \geq 0)$	—	Markov renewal process, $\theta_k \in \mathbb{R}_+$
τ_n	:=	$\sum_{k=0}^{n} \theta_k$
$\nu(t) := \max\{ n : \tau_n \leq t \}$	—	counting provess
$x(t) := x_{\nu(t)}$	—	semi-Markov process
$\gamma(t)$	:=	$t - \tau_{\nu(t)}$
$\rho(dx)$	—	stationary probabilities of a Markov chain $(x_n; n \geq 0)$
$P(x, dy)$	:=	$\mathsf{P}\{ \omega : x_{n+1} \in dy / x_n = x \}$
$G_x(t)$	:=	$\mathsf{P}\{ \omega : \theta_{n+1} \leq t / x_n = x \}$
$m(x)$	:=	$\int_0^\infty t G_x(dt)$
m	:=	$\int_X \rho(dx) m(x)$
$\pi(dx)$	:=	$\frac{\rho(dx) m(x)}{m}$
$(U, \mathfrak{U})$	—	merged phase space
X_u	—	uth ergodic component of X, $\forall\, u \in U$
$\rho_u(dx)$	—	stationary probabilities in X_u, $\forall\, u \in U$
$m(u)$	:=	$\int_{X_u} \rho_u(dx) m(x)$
$\pi_u(dx)$	:=	$\frac{\rho_u(dx) m(x)}{m(u)}$
$W(t)$	—	standard Wiener process
$(\mathbf{B}, \mathfrak{B}, \lVert \cdot \rVert)$	—	separable Banach space
$\mathbf{B}^*$	—	dual space of $\mathbf{B}$
l	—	linear continuous functional, $l \in \mathbf{B}^*$

$\Gamma(t), \Gamma_x(t)$	—	semigroups of operators of t, $\forall x \in X$
$\Gamma, \Gamma(x)$	—	infinitesimal operators, $\forall\, x \in X$
$V(t), V^\varepsilon(t), v_\varepsilon(t)$	—	initial random evolutions and multiplicative operator functionals
$\hat{V}(t), V^0(t), V_0(t)$	—	limiting random evolutions (averaged or merged)
$z(t), z^\varepsilon(t), z_\varepsilon(t), u^\varepsilon(t), \ldots$	—	initial evolutionary stochastic systems
$\hat{z}, z^0(t), z_0(t), \hat{u}(t), \ldots$	—	limiting evolutionary stochastic systems (averaged or merged)
$W(t, A)$	—	Wiener martingale measure, $t \in \mathbb{R}_+$, $A \in \mathfrak{X}$
$w(t)$	—	Wiener process, $t \in \mathbb{R}_+$
(B, S)	—	Black–Scholes model for securities market
(B, S, X)	—	(B, S)–securities market in random media X
$B_t, B(t)$	—	dynamics of bonds prices, $t \in \mathbb{R}_+$
$S_t, S(t)$	—	dynamics of stocks prices, $t \in \mathbb{R}_+$
$r(x)$	—	interest rate, $x \in X$
$\mu(x)$	—	appreciation rate, $x \in X$
$\sigma(x)$	—	volatility, $x \in X$
RE	—	random evolutions
REP	—	random evolution processes
MOF	—	multiplicative operator functionals
SES	—	stochastic evolutionary systems
SMRE	—	semi-Markov random evolutions
BVP	—	boundary value problems
ADF	—	analogue of Dynkin formula
SOC	—	stochastic optimal control