

MATHEMATICAL INTUITIONISM AND
INTERSUBJECTIVITY

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MATHEMATICAL INTUITIONISM AND INTERSUBJECTIVITY

A Critical Exposition
of Arguments for Intuitionism



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What a mathematician is inclined to say about the objectivity and reality of mathematical facts, is not a philosophy of mathematics, but something for philosophical treatment.

Wittgenstein, *Philosophical Investigations*, §254

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T.P.

WORKS FREQUENTLY QUOTED

The list below includes titles of works that are often referred to in the book together with the abbreviations used. With these exceptions, works are referred to in the usual way, that is by giving the name of the author, year of publication and (if needed) the page number. If no confusion is likely, however, the author's name is dropped.

TE	M. Dummett: <i>Truth and Other Enigmas</i>
LBM	M. Dummett: <i>The Logical Basis of Metaphysics</i>
PBIL	M. Dummett: 'The Philosophical Basis of Intuitionist Logic'
WTM	M. Dummett: 'What Is a Theory of Meaning II'
EI	M. Dummett: <i>Elements of Intuitionism</i>
LU	E. Husserl: <i>Logische Untersuchungen</i>
VS	W.P. van Stigt: <i>Brouwer's Intuitionism</i>