

Algebraic Model Theory

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Algebraic Model Theory

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PREFACE

The NATO Advanced Study Institute on “Algebraic Model Theory” was held at the Fields Institute in Toronto, Canada, August 19–30, 1996. There were over 80 participants who came from Canada, Colombia, Estonia, Finland, France, Germany, Hungary, Italy, Japan, Poland, Portugal, Russia, Spain, Turkey, the United States and the United Kingdom. 16 lecturers gave 47 hours of lectures over the two week period.

Certain branches of model theory have, over the last few years, grown at an enormous rate and several important contributions have been made outside of mathematical logic. It was one of the expressed aims of this advanced study institute to bring together leaders in the most current fields of model theory to exposit some of the recent advances. The format that was adopted for the institute was to highlight five separate topics: geometric stability theory, the model theory of analytic functions, the model theory of permutation groups, the spectra of countable theories, and the structure of finite algebras.

The papers in this volume written by Bouscaren, Chatzidakis and Pillay constitute the session on geometric model theory. Specifically the goal was to give a sketch of the model theoretic proof of the Manin-Mumford conjecture by Ehud Hrushovski. Bouscaren’s lectures provided much of the background stability theory necessary to understand the model theoretic content of the proof. Chatzidakis and Pillay provided specific details on the theory of algebraically closed fields with an automorphism, ACFA: the key theory where all the action takes place in Hrushovski’s proof. Chatzidakis’ paper includes the critical characterization of modular groups in ACFA. Finally, Pillay’s paper in this volume gives an outline of Hrushovski’s proof.

The session devoted to the model theory of analytic functions provided the most sensational talks of the institute. Wilkie announced and gave an outline of a proof that the real field expanded by all Pfaffian functions is o-minimal. This long standing conjecture had been one of the goals of the subject and its proof was all the more remarkable in that it was proved without showing quantifier elimination or model completeness. It relied on Khovanskii’s theorem that quantifier-free definable sets in the Pfaffian expansion of the reals have finitely many connected components and this number is bounded uniformly in parameters. Marker’s paper contains a proof of this result. For this volume, Wilkie includes a proof that the reals with exponentiation is decidable modulo Schanuel’s conjecture. Van den

Dries also lectured in this session and gave talks on the o-minimality of certain power series expansions of the reals but unfortunately, due to time constraints, was unable to submit anything for publication.

The model theory of permutation groups session had two distinct threads. Lachlan, Macpherson and Cherlin gave an exposition of work on smoothly approximable structures providing both a historical perspective and an overview of the monumental, as yet unpublished, work of Cherlin and Hrushovski on this subject. Lachlan gave a series of lectures outlining the theory of stable finitely homogeneous structures, a precursor of the theory of smoothly approximable structures. Macpherson addressed two topics: his own proof of the key theorem on finitely homogeneous structures and the characterization theorem for primitive smoothly approximable structures. Cherlin's lectures constituted a précis of his work with Hrushovski. Given that the paper for this volume is over 50 pages, one can only guess at how long the complete paper will be! Evans lectured on interesting connections between finite covers and cohomology but was unable to submit anything for this volume due to prior commitments.

Some years ago, Hrushovski announced the completion of the calculation of all the uncountable spectra of countable theories. This proof was never written down and for this institute, Hart and Laskowski endeavoured to give both some history of the problem and the main ingredients of Hrushovski's proof.

The talks given by Kiss, Valeriote, Willard and Ziegler focussed on two major topics from the study of equationally definable classes of algebraic structures. Kiss provided an introduction to a local structure theorem for finite algebras (tame congruence theory) which has turned out to be an essential tool in the investigation of questions dealing with the residual character and decidability of equational classes. In his paper, Valeriote describes what is known about the structure of algebras belonging to equational classes whose first order theory is decidable. An important special case of this was considered by Ziegler in his lectures. He discussed Geisler's recent work on the spectrum of a tame quiver and its connection to the study of the decidability of the first order theory of modules. Willard's paper deals with recent work on the residual character of equational classes. The highlight of his paper is a presentation of recent counterexamples due to McKenzie and the connection with the solution to Tarski's Finite Basis problem.

The Advanced Study Institute was generously sponsored by NATO and I would like to thank them for their support. I would also like to thank the Fields Institute for Research in the Mathematical Sciences for its support. In particular, without the efforts of the Fields Institute staff, in particular, Becky Sapping, Karen Walker and Alesia Zuccala, the institute would not

have been the success it was.

A special vote of thanks goes to Bradd Hart and Matthew Valeriote who conceived the plan for this ASI as an event that would fit very well with the program in algebraic model theory to be held at the Fields Institute during the 1996-97 academic year. Throughout the preceding year they both worked tirelessly on all aspects of the organization of the study institute. Thanks are also due to the members of the organizing committee not yet mentioned: Elisabeth Bouscaren, David Marker, and Anand Pillay.

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